

CHEMICAL ABSORPTION EFFECT ON MAGNETO  
HYDRODYNAMIC (MHD) CONVECTION FLOW  
PAST A VERTICAL PLATE WITH MASS  
TRANSFER

BY

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NOVEMBER 2018

**CHEMICAL ABSORPTION EFFECT ON MAGNETO HYDRODYNAMIC (MHD)  
CONVECTION FLOW PAST A VERTICAL PLATE WITH MASS TRANSFER**

**By**

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**A Project Submitted to the Department of Mathematical Science Federal University  
Gusau, Zamfara State.**

**In partial fulfillment for the requirement of the award of the Degree of Bachelor of  
Science**

**(B.sc) in Mathematics Science.**

**NOVEMBER. 2018**

### Declaration

I hereby declare that this project was written by Usman Auwal is a record of my own research work. It has not been presented before in any previous application for a Bachelor degree. All references cited have been duly acknowledged.



Usman Auwal

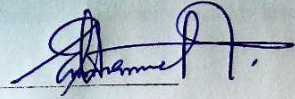
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Date



### Certification

I certify that this long essay was carried out by Usman Auwal (1410309013) has met the requirements for the Award of the Degree of Bachelor of Science (mathematics) of the Federal University Gusau and is approved for its contribution to knowledge.



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### **Dedication**

I dedicate this work to Allah whom out of His infinite mercy made it easy for me to complete my programme. And to my dearest Father and Mother, Alhaji Usman and Hajiya Fatima who gave birth to me and trained and guided me in the culture, pattern and system of Islam and its education, and My lovely Brother Alh. Isah Abbakar.



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### Nomenclature and Greek symbols

$K_r$  = the chemical reaction parameter

$\omega$  = A frequency parameter

$Gr$  = the thermal Grashof number

$G_m$  = the mass Grashof number

$M$  = magnetic parameter

$D_c$  = mass Diffusion term

$T$  = the temperature of the fluid

$V$  = the suction parameter

$g$  = acceleration due to gravity

$C_p$  = Heat capacity at constant pressure

$u$  = the velocity of the fluid

$\omega$  = A frequency parameter

$\rho$  = the fluid Density

$t$  = Time

$\varepsilon$  = Perturbation parameter

$B_0$  = the magnetic induction

$\beta$  = the thermal conductivity

$Sc$  = The Schmidt number

$M$  = magnetic parameter

$Q_0$  = the heat sink term

$k$  = Thermal conductivity



### Abstract

Chemical Absorption Effects on Magnetohydrodynamic (MHD) convection flow through vertical plate with mass transfer is investigated. As the problem is governed by coupled nonlinear system of partial differential equations (PDEs). Dimensionless quantities were introduced to transform the partial differential equation into ordinary differential equations (ODEs) and the equations are then solved using the perturbation method. Graphical representations of the numerical results are illustrated in Figure. 1 to Figure. 9 In this study, The influence of the effects of physical parameters such as Prandtl number, Schmidt number, thermal Grashof number for heat transfer, Grashof number for mass transfer and chemical reaction have been investigated separately in order to clearly observe their respective effects on the velocity, temperature, and concentration profiles of the flow.

## CHAPTER ONE

### 1.0 Background of the Study

#### 1.1 Introduction

fluid can be defined as a shear thinning which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear Dorsh *et al.* (2003) fluid flow model was introduced by Casson in 1959 for the predictions of the flow behaviors of pigment-oil suspension so for the flow, the stress-magnetic of fluid flow needs to exceed the yields shear stress, or else the fluid behaves as a rigid body. This type of fluids can be marked as a purely viscous fluid with high viscosity, fluid flow is based on the structure model of the interactive behaviors of solid and liquid phases of two-phase suspension some famous flow fluids include jelly, honey, sound, light and concentrated fruit juice, human blood can also be treated as fluid flow due to the presence of several substances such as fibrinogen, protein, albumin in aqueous-based plasma and human red blood cells. Veerach *et al.* (2017), a heat source is anything that can heat up a spacecraft, heat source can be external (from outside the spacecraft) or internal (from the inside of the spacecraft) external heat sources include the sun, reflected sunlight from planets and moon, heating by frictions when traveling through an atmosphere or gas cloud and released heat from planets, internal heat is often generated by orbital propulsion or electrical system. If a craft has parts that move against each other, friction can provide heating as well. Many natural substances like rocks and soil (e.g., aquifers, petroleum reservoirs) and man-made materials such as porous media (many of their important properties can only be rationalized by considering them to be porous media (Dutta and Gupta, 2003).



The term perturbation methods refer to a variety of methods to prove existence, uniqueness or regularity for equations that are in senses close to an equation that well understood. if an equations can be written as sum of well understood equations plus an extra term (perturbation that is small with respects to some relevant quantity, then some time a property of the well understood term can be transferred to the original equation. Williams *et al.* (2010).

Convection cannot take place in most solids because neither bulk current flows nor significant diffusion of matter can take place in rigid solids, but that is called heat conduction. Convection, however, can take place in soft solids or mixtures where solid particles can move past each other. Convective heat transfers in one of the major types of heat transfer and convection is also a major mode of mass transfer in fluids .convective heat and mass transfers take place both by diffusion .the random Brownian motion of individual particles in the fluid and by advection in which matter or heat is transported by the large-scale motion of current of fluids .in the context of heat and mass transfer, the term convection is used to refers to some of advection and diffusive transfers. However, mechanics the concept used the word convection is the general senses and different type of convection should be qualified for clarify.

Convection can be qualified in term of being natural, forced, gravitational granular, or thermo magnetic .it may also be said to be due to combustion, capillary action, or Marangoni and Soret effects. Heat transfer by natural convection plays a role in the structure of earth's atmosphere, its oceans and its mantle. Discrete convection cells in the atmosphere can see as clouds, with stronger convection resulting in thunderstorms .natural convection also play a role in stellar physics (Cengel and Yulus).

## 1.1 Statement of the problem

The governing equations in this study are the continuity, momentum, energy and diffusion equations, where the velocity, temperature and concentration fields are to be determined. Also the boundary conditions were modified.

## 1.2 Aim and objectives of the Study

The aim of study is to examine Chemical Absorption Effect on Magneto hydrodynamic (MHD) convection flow through a vertical plate with mass transfer analytically using perturbation method. The objectives of the study are:

1. To transform the dimensional governing partial differential equations (PDEs) into non dimensional form using some variable.
2. To obtain analytical solutions for the velocity, temperature and concentration.
3. To compute the skin friction, Nusselt and Sherwood number coefficients.
4. To examine the influence of thermo-physical quantities associated with the flow, with the help of graphs and tables.

## 1.3 Scope and Delimitation of the Study

This research work mainly focuses on heat source and mass absorption effects on fluid flow and is not extent to other kind's fluid.

## 1.4 Significance of the Study

This research work help us to examine the influence of thermo-physical quantities associated with fluid flow with the help of graph and table and obtain analytic solution for the velocity, temperature and concentration.

## 1.5 Definition of Basic Term



### 1.5.1 Radiation:

Radiation is the energy that comes from a source and travels through some material or through space, light, heat and sound are type of radiation.

### 1.5.2 Heat:

Heat is energy that is in a process of transform between a systems and its surrounding, other than as work or with the transform of matters.

### 1.5.3 Fluid:

Fluid is a substance, as liquid or gas that is capable of flowing and that changes its shapes at a steady rate when acted upon by a force leading change its shapes.

### 1.5.4 Fluid flow:

Fluid flow is motion of a fluid subjected to unbalanced force or stresses.

### 1.5.5 Magnetohydrodynamic (MHD):

Magnetohydrodynamic (MHD) (magneto fluid dynamics or hydro magnetic) is the study of properties of electrically conducting fluids .examples of such magneto-fluids include plasmas, liquid metals, and salt water or electrolytes, the word magneto hydrodynamics (MHD) is derived from magneto-meaning magnetic field, hydro- meaning water , and dynamics meaning movement.

### 1.5.6 Convection flow:

Chemical reaction encompasses changes that only involve the position of electrons in the forming and breaking of chemical bonds between atoms.

**1.5.7 Chemical reaction:**

Convective flow is the motion of fluid due to differences in density. These differences in density commonly occur due to temperature gradients. For example the circulating air flows room with a hot radiator



## CHAPTER TWO

### 2.0 Review of related literature

The study of radiation and heat absorption effect on MHD convection flow past a vertical plate is attracting the attention of many scholars. Fluid flow is one of such that has distant feature and little illustration recently. So far the flow, the shear stress or magnetic of fluid needs to exceed the yield shear stress, or else the fluid we have as a rigid body. this kind of fluids can be marked as a purely, viscous fluid with high viscosity flow model is based on a structure model of the interactive behaviors of solid and phase liquid phases of two phases of suspension .so the famous example of fluid fluids flow include jelly sauce, tomato, honey, soup and concentration fruits juice. Human blood can also be treated as a fluid flow due to presence of several substances such as fibrinogen, protein, globulin, in aqueous base plasma and human are either obtained by using approximate methods or by any numerical schemes.

There are many case in which the exact analytical solution of fluid flow are obtained. Vajravelu and Mukhopadhyay (2013) studies diffusion of chemicals reactive species on fluid flow over an unsteady permeable stretching surface Das *et al.* (1996) analyzed fluid flow in pipe filled with a homogeneous porous medium Hayat *et al.* (2012) investigation effects on magneto hydro dynamic flow of fluid Mustafa *et al.* (2011) analyzed unsteady boundary layer flow of fluid due to an impulsively started moving flat plate, Mustafa *et al.* (2014) discussed stagnation point flow and heat transfer of a fluid towards stretching sheet

The analyses of a boundary layer flow of a unsteady heat and mass transfer fluid has transfer Pines has been the focus of extensive and research by various scientists due to its importance in continuo's casting glass, blowing paper production, polymer extensions and several other one

of my refers to recent investigation by *Hayat and Qasim* (2012) influence of thermal radiation and joule heating on MHD flow of max well fluid in presence of thermophoresis fang et al ( 2010) analyzed a new family of unsteady layer over a stretching surface. *Khan and pop* (2010) considered boundary flow of nano-fluid past a stretching sheet, and *Kandhaswamy et al.* (2011) researched on scaling group transformation for (MHD) boundary layer flow of a nano-fluid past a vertical stretching surface in presence of suction/injection. on the other hand, mass transfer in impact due to it appearance in many scientific discipline that involve convention transfer of this phenomenon are evaporation of water separation chemicals in distillation process natural or artificial sources etc

*Mustafa et al.*(2011) investigated the boundary layer flow of a fluid due to an impulsively started moving flat plate, *rao et al* (2013) studied the thermal hydrodynamic slip condition on heat transfer flow of fluid past a semi- infinite vertical plate. Heat transfer of a fluids past a permeable shrinking sheet with viscous dissipation was considered by *Qasim and Norveen* (2014) , *Chamkha et al.* (2004) studied the unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption.

The study of heat mass transfer with chemical reaction is of considerable importance in chemicals and hydro metallurgical industries. The study of heat generation or absorption in moving fluid is important in many problems with are related with chemicals reaction and these concerned dissociating fluid. The effect of non uniform heat generation effect may change the temperature distribution and consequently rate of particle deposition in nuclear reactors.

*Lulukdar et al* (2010) studied perturbation analysis of unsteady magneto hydrodynamic convective heat and mass transfer in a boundary radiation and chemicals reaction. *Ganesan et al.*



(2001) studied first order chemicals reaction on flow past a vertical permeable plate with thermal radiation and chemicals uniform heat and mass flux. Dulal pal (1999) studied effect of chemicals reaction on the dispersion chemicals physiology of flood flow regulation by red cells was discussed by single and Stemler (2005). Analytical solution for heat and mass transfer by laminar flow of a Newtonian viscous, electrically condition and heat generation or absorbing fluid on a continuously vertical permeable surface in the presence of radiation, a first order homogenous chemicals reaction mass flux are repeated by Ibrahim *et al* (2003).

Wike *et al.* (2001) investigated the effect of chemicals reaction and diffusion in an isothermal laminar flow along a soluble flat plate. Devika *et al.* (2012) studied the influence of chemicals reaction effect on MHD free convection flow in a irregular channels with porous medium, Raji *et al.* (2017) studied transfer effects on a unsteady MHD free convective flow a vertical plate with chemical reaction using finite element methods . Raji *et al* (2016) found both analytical and numerical solutions of chemical acceleration finite vertical plate in magnetic field and variable temperature via Laplace transform and finite element techniques. Omokhuale *et al.* (2016) studied effect of Heat Absorption on Steady/Unsteady MHD Free Convection Heat and Mass Transfer Flow Past an Infinite Vertical Permeable Plate with Mass Absorption and Variable Suction.Omokhuale *et al.* (2017) examined Natural Convective Heat Mass Transfer Fluid Flow through a Porous Medium with Variable Pressure and Thermal Radiation.

Motivated by the above studied and applications mentioned, in this study Chemical Absorption Effect on Magnetohydrodynamic (MHD) convection flow past a vertical plate with Mass transfer investigated

## CHAPTER THREE

### 3.0 Methodology

#### 3.1 Formulation of the problem

Consider the flow of viscous incompressible, electrically conducting, visco-elastic, second order well-know, non-Newtonian fluid namely Rivlin-Erickson fluid past an impulsively started semi-infinite vertical plate in the presence of homogeneous chemical reaction, thermal radiation, thermal diffusion, radiation absorption and heat absorption. The  $x'$ -axis is taken along the plate in the vertically upward direct and the  $y'$ -axis is chosen normal to the plate.

Initially the temperature of the plate and the fluid is assumed to be  $T'_{\infty}$ , and the species concentration at plate is  $C'_w$  and in the fluid throughout  $C'_{\infty}$  are assume. At time  $t' > 0$ , the plate temperature is changed to  $T'_w$  causing convection current to flow near the plate and mass is supply at the costant rate to the plate and they started moving upward due to the impulsive motion, gaining a velocity of  $u_0$ . A uniform magnetic field of strength  $B_0$  is applied in the  $y$ -direction. Therefore velocity and magnetic field are given  $q = (u, 0, 0)$  and  $B = (0, B_0, 0)$ . The flow being slightly conducting the magnetic Reynolds number is much less than unity hence the induced magnetic field can be neglected in comparison with the applied magnetic field.

In the absence of any input electric field, the flow is governed by the following equation

Equation of momentum:

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = gB(T' - T'_{\infty}) + gBc(C' - C'_{\infty}) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\delta \mu_0^2 B_0^2 u'}{\rho} \quad (3.1)$$

Equation of energy:

$$\rho c_p \left( \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = k \frac{\partial^2 T'}{\partial y'^2} - Q_0(T' - T'_{\infty}) + Q_1(C' - C'_{\infty}) \quad (3.2)$$

Equation of concentration:

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = Dm \frac{\partial^2 C'}{\partial y'^2} - K\tau^*(C' - C'_{\infty}) \quad (3.3)$$



Where  $u'$  is the velocity of the fluid along the plate in  $x'$  - direction,  $t'$  - is the time,  $g$

Is the acceleration due gravity is the coefficient of volume  $B'$  is the coefficient of thermal

Expansion with concentration,  $T'$  is the temperature of fluid near the plate,  $T'_\infty$  is the temperature of fluid,  $c'$  is the species concentration in fluid near the,  $c'_\infty$  is the species concentration in fluid far away from the plate,  $\nu$  is kinematic viscosity,  $k'_0$  is coefficient of kinematic visco-elastic parameter,  $\delta$  is the electrical conductivity of the fluid  $\mu_e$  is the magnetic permeability,  $B_0$  is the strength of applied magnetic field  $\rho$  is the density of the fluid,  $u$  is the viscosity of fluid  $D$  is the molecular diffusivity  $D_1$  is the thermal diffusivity  $Q_0$  is the heat source /sink,  $Q_1$  is the radiation absorption parameter,  $k'_r$  is the chemical reaction parameter, and  $u_0$  is velocity of the plate.

The corresponding initial and boundary condition as follows

$$\left. \begin{aligned} u &= 1 + \varepsilon e^{int} \text{ at } y = 0, \quad u \rightarrow 0, \text{ as } y \rightarrow \infty \\ \theta &= 1 + \varepsilon e^{int} \text{ at } y = 0, \quad \theta \rightarrow 0 \text{ as } y \rightarrow \infty \\ c &= 1 + \varepsilon e^{int} \text{ at } y = 0, \quad c \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (3.4)$$

On introducing the following non-dimensional quantities

$$u = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y' u_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{c' - c'_\infty}{c'_w - c'_\infty} \quad (3.5)$$

In term of the above non dimensional quantities equation (3.1) to (3.3) reduce to the

From (3.1)

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = gB(T' - T'_\infty) + gBc((C' - C'_\infty)) + V \frac{\partial^2 v'}{\partial y'^2} - \frac{\delta \mu_e^2 B_0^2 u'}{\rho}$$

$$\frac{u_0^3 \partial u}{\nu \partial t} + v \frac{u_0^2 \partial u}{\nu \partial y} = g\beta(T'_w - T'_\infty)\theta + g\beta c(C'_w - C'_\infty)c \frac{\nu u_0^3 \partial^2 u}{\nu^2 \partial y^2} - \frac{\sigma \mu_e^2 B_0^2 u u_0}{\rho}$$

Multiply through by  $\frac{\nu}{u_0^3}$

$$\frac{\partial u}{\partial t} - \frac{v_0}{u_0} \frac{\partial u}{\partial y} = \frac{v g \beta (T'_w - T'_\infty) \theta}{u_0^3} + \frac{v g \beta c (C'_w - C'_\infty) c}{u_0^3} + \frac{\partial^2 u}{\partial y^2} - \frac{v \delta \mu_e^2 B_0^2 u}{\mu_0^3 \rho}, \quad \text{since } v' = -v_0$$

$$\text{since } \frac{v_0}{u_0} = \lambda, Gr = \frac{vg\beta(T'_\omega - T'_\infty)\theta}{u_0^3}, Gm = \frac{vg\beta c(C'_\omega - C'_\infty)c}{u_0^3}, M = \frac{v\delta\mu_0^2\beta_0^2 u}{\mu_0^2\rho}$$

$$\frac{\partial u}{\partial t} - \lambda \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC - mu \quad (3.6)$$

From (3.2)

$$\rho c p \left( \frac{\partial T'}{\partial t} + v \frac{\partial T'}{\partial y'} \right) = k \frac{\partial^2 T'}{\partial y'^2} - Q_0(T' - T'_\infty) + Q_1(C' - C'_\infty) \quad (3.2)$$

$$\begin{aligned} U_0^2 \rho c p \frac{(T'_\omega - T'_\infty) \partial \theta}{v} \frac{\partial \theta}{\partial t} - U_0 v_0 \rho c p \frac{(T'_\omega - T'_\infty) \partial \theta}{v} \frac{\partial \theta}{\partial y} \\ = u_0^2 k \frac{(T'_\omega - T'_\infty) \partial^2 \theta}{v^2 \partial y^2} - Q_0(T'_\omega - T'_\infty)\theta + Q_1(C'_\omega - C'_\infty) \end{aligned}$$

Multiplying through by  $\frac{v}{u_0^2 \rho c p} (T'_\omega - T'_\infty)$

$$\frac{\partial \theta}{\partial t} - \frac{v_0}{u_0} \frac{\partial \theta}{\partial y} = \frac{k}{\rho c p v} \frac{\partial^2 \theta}{\partial y^2} - \frac{v}{u_0^2 \rho c p} + \frac{Q_1(C'_\omega - C'_\infty)}{u_0^2 \rho c p (T'_\omega - T'_\infty)}$$

$$\text{Where } \lambda = \frac{v_0}{u_0}, \frac{1}{Pr} = \frac{k}{\rho c p v}, \phi = \frac{v}{u_0^2 \rho c p}, \delta = \frac{Q_1(C'_\omega - C'_\infty)}{u_0^2 \rho c p (T'_\omega - T'_\infty)}$$

(3.7)

$$\frac{\partial \theta}{\partial t} - \lambda \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \phi \theta + \delta c$$

From (3.3)

$$\frac{\partial c'}{\partial t'} + v' \frac{\partial c'}{\partial y'} = Dm \frac{\partial^2 c'}{\partial y'^2} - Kr^*(C' - C'_\infty)$$

$$U_0^2 \frac{(C'_\omega - C'_\infty) \partial c}{v} \frac{\partial c}{\partial t} - u_0 v_0 \frac{C'_\omega - C'_\infty \partial c}{v} \frac{\partial c}{\partial y} - u_0^2 Dm \frac{C'_\omega - C'_\infty \partial^2 c}{v^2 \partial y^2} - k^* r c (C'_\omega - C'_\infty)$$

Multiplying through by  $\frac{v}{u_0^2 (C'_\omega - C'_\infty)}$

$$\frac{\partial c}{\partial t} - \frac{v_0}{u_0} \frac{\partial c}{\partial x} - \frac{Dm \partial y}{v} \frac{\partial c}{\partial x} - \frac{v}{u_0^2} k^* r c$$



where  $\frac{v_0}{u_0} = \lambda$ ,  $\frac{Dm}{v}g = \frac{1}{sc}$ ,  $\frac{v}{u_0^2}krC = K^*rC$

$$\frac{\partial C}{\partial t} - \lambda \frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} - k^*rC \quad (3.8)$$

### 3.2 Method of solution

Equations (3.6) to (3.8) are coupled non linear partial differential equation and are to be solved by using the initial and boundary condition.

However exact solution is not possible for this set of equation and hence we solve these equation by using perturbation method of solving differential equation.

The equivalence perturbation method scheme of equation for (3.6) to (3.8) is follows

From

$$\frac{\partial u}{\partial t} - \lambda \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC - mu \quad (3.9)$$

$$U(y, t) = u_0(y) + \varepsilon u_1(y) e^{i\omega t}$$

$$\frac{\partial u}{\partial t} = i\omega u_1(y) e^{i\omega t}$$

$$\frac{\partial u}{\partial y} = u_0'(y) + \varepsilon u_1'(y) e^{i\omega t}$$

$$\frac{\partial^2 u}{\partial y^2} = u_0''(y) + \varepsilon u_1''(y) e^{i\omega t}$$

Substituting into (5)

$$i\omega u_1(y) e^{i\omega t} - \lambda u_1'(y) - \lambda \varepsilon u_1'(y) = u_0''(y) + \varepsilon u_1''(y) + Gr\theta_0(y) + \varepsilon Gr\theta_1(y) e^{i\omega t} + GmC_0(y) +$$

$$\varepsilon M u_1(y) e^{i\omega t} - M u_0(y) - \varepsilon M u_1(y) e^{i\omega t}$$

$$\varepsilon^0 e^0 = -\lambda U_0'' = U_0'' + Gr\theta_0 + GmC_0 - M U_0$$

$$U_0'' + \lambda U_0' - M U_0 = -Gr\theta_0 - GmC_0$$

(3.10)

$$\varepsilon e^{int} = inu_{1(y)} - \lambda u_{1(y)}^* + Gr\theta_{1(y)} + GMc_{1(y)} - Mu_{1(y)}$$

$$i \dots \lambda u_{1(y)}^* + (in - m)U_1 = -Gr\theta_1 - GMc_1 \quad \text{since } (in-m)=0$$

$$U_1^* + \lambda U_1' - \phi U_1 = -Gr\theta_1 - GMc_1 \quad (3.11)$$

... (3.11)

$$\frac{\partial \theta}{\partial t} - \lambda \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{pr \partial y^2} - \phi \theta + \phi c \quad (3.7)$$

$$\theta(y, t) = \theta_0 + \varepsilon \theta_1 e^{int}$$

$$\frac{\partial \theta}{\partial t} = in\varepsilon \theta_{1(y)}$$

$$\frac{\partial \theta}{\partial y} = \theta_0' + \varepsilon \theta_1' e^{int}$$

$$\frac{\partial^2 \theta}{\partial y^2} = \theta_0'' + \varepsilon \theta_1'' e^{int}$$

Substituting into (3.7)

$$in\theta_{1(y)} - \lambda \theta_0' - \lambda \varepsilon \theta_{1(y)}' e^{int} = \frac{1}{pr} \theta_0'' + \frac{1}{pr} \varepsilon \theta_{1(y)}'' e^{int} + \phi \theta_0 - \varepsilon \phi \theta_{1(y)} e^{int} + \lambda c_0 + \varepsilon \lambda c_{1(y)} e^{int}$$

$$e^a = -\lambda \theta_0' = \frac{1}{pr} \theta_0'' - \phi \theta_0 + \lambda c_0$$

Multiplying through by pr

$$\theta_0'' + \lambda pr \theta_0' - \phi pr \theta_0 = -\lambda pr c_0 \quad (3.13)$$

$$\varepsilon e^{int} = in\theta_1 - \lambda \theta_1' = \frac{1}{pr} \theta_1'' - \phi \theta_1 + \lambda c_1$$

Multiplying by pr



$$\begin{aligned}
 \theta_1'' + \lambda r \theta_1' - (\phi - in) r \theta_1 & \\
 = -\lambda p r C_1, \text{ since } (\phi - in) & = \beta \\
 \theta_1'' + \lambda p r \theta_1' - \beta r \theta_1 & \\
 = -\phi p r C_1. &
 \end{aligned}
 \tag{3.14}$$

From (3.8)

$$\frac{\partial C}{\partial t} - \lambda \frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} - k^* r C$$

$$C(y, t) = C_0(y) + \varepsilon C_1(y) e^{int}$$

$$\frac{\partial C}{\partial t} = in \varepsilon C_1 e^{int}$$

$$\frac{\partial C}{\partial y} = \varepsilon C_1' e^{int}$$

$$\frac{\partial^2 C}{\partial y^2} = \varepsilon C_1'' e^{int}$$

Substituting into (3.8)

$$in \varepsilon C_1 e^{int} - \lambda \varepsilon C_1' e^{int} = \frac{1}{S_c} C_1''(y) - S_c k^* C_0(y)$$

Multiplying through by  $S_c$

$$C_1'' + \lambda S_c C_1' + S_c k^* r C_1 - \varepsilon e^{int} = 0
 \tag{3.15}$$

$$\varepsilon e^{int} = in C_1(y) - \lambda S_c C_1'(y) = \frac{1}{S_c} C_1''(y) - K^* r C_1(y)$$

Multiplying through by  $S_c$

$$S_c in C_1 - \lambda S_c C_1' = C_1'' - S_c k^* r C_1$$

$$C_1'' + \lambda S_c C_1' - (K^* r - in) S_c C_1, \text{ Let } (K^* r - in) = \phi$$

$$C_1'' + \lambda Sc C_1' - \varphi Sc C_1 = 0 \quad (3.16)$$

$$U_0'' + \lambda U_0' - MU_0 = -Gr\theta_0 - GmC_0 \quad (3.11)$$

$$U_1'' + \lambda U_1' - \Phi U_1 = -Gr\theta_1 - GM C_1 \quad (3.12)$$

$$\begin{aligned} \theta_0'' + \lambda pr \theta_0' - \Phi pr \theta_0 \\ = -\lambda pr c_0 \end{aligned} \quad (3.13)$$

$$\theta_1'' + \lambda pr \theta_1' - \beta pr \theta_1 = -\lambda pr c_1 \quad (3.14)$$

$$C_0'' + \lambda Sc C_0' + Sc K^* r = 0 \quad (3.15)$$

$$\begin{aligned} C_1'' + \lambda Sc C_1' - \varphi Sc C_1 \\ = 0 \end{aligned} \quad (3.16)$$

From (3.14)

$$C_0'' + \lambda Sc C_0' + Sc K^* r = 0$$

Using method solving characteristic equation

a 1, b scλ, sck\*r

$$M^2 + scM - sck^*r$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad \frac{-\lambda Sc \pm \sqrt{(\lambda sc)^2 - 4(sc k^* r)}}{2}$$

$$\frac{-\lambda sc}{2} \pm \sqrt{\frac{(\lambda Sc)^2 - 4(Sc \lambda)}{4}}$$

$$\text{Since } \sqrt{\frac{(\lambda sc)^2 - 4(Sc \lambda)}{4}} = n_1$$

$$-\lambda sc \pm n_1, \quad M_1 = -\lambda Sc + n_1, \quad M_2 = -\lambda Sc - n_1$$



$$C_{o(y)} = A_1 e^{m_1 y} + A_2 e^{-m_2 y} \quad (3.17)$$

Applying the boundary condition

$C = 1$ , at  $y = 0$

$$1 = A_1 e^0 + A_2 e^0 \quad \text{it implies } A_1 + A_2 = 1 \quad (3.18)$$

$C \rightarrow 0$ , as  $y \rightarrow \infty$

$$0 = A_1 e^{0\infty} + A_2 e^{-\infty}, \text{ it implies } A_2 = 0 \quad (3.19)$$

Substituting  $A_1$  into (3.17)

$$A_2 = 1 \quad (3.20)$$

Substituting (3.18) and (3.19) in (3.17)

$$C_{o(y)} = e^{-m_2 y} \quad (3.21)$$

From (3.13)

$$C_1'' + \lambda S c C_1' - \varphi S c C_1 = 0, \quad a = 1, b = \lambda s c, c = \varphi s c$$

$$M^2 + \lambda S c M - \varphi S c = 0$$

$$\frac{-\lambda S c \pm \sqrt{(\lambda S c)^2 - 4(\varphi S c)}}{2}, \text{ since } \sqrt{\frac{(\lambda S c)^2 - 4(\varphi S c)}{4}} = n_2$$

$$\lambda S c \pm n_2$$

$$M_3 = -\lambda S c + n_2$$

$$M_4 = -\lambda S c - n_2$$

(3.22)

$$C_{1(y)} = A_3 e^{m_3 y} + A_4 e^{-m_4 y}$$

Applying the boundary condition

$$\theta_{0(x)}^{(y)} = V_2 e^{-m_2 y}$$

$$\theta_{0(x)}^{(y)} = -\lambda p r e^{-m_2 y} \quad \text{since } -\lambda p r = V_2$$

$$\theta_{0(x)}^{(y)} = -\lambda p r c_0$$

$$\theta_{0(x)}^{(y)} h = V_5 e^{m_5 y} + V_6 e^{-m_6 y}$$

(3.27)

$$-\lambda p r \pm m_3 \frac{2}{2} M_5 = \frac{-\lambda p r}{2} + m_3, M_6 = \frac{-\lambda p r}{2} - m_3$$

$$-\lambda p r \pm \sqrt{(\lambda p r)^2 - 4(\phi p r)} \quad \text{, since } \sqrt{(\lambda p r)^2 - 4(\phi p r)} = m_3$$

$$M^2 + \lambda p r M - \phi p r = 0, a = 1, b = \lambda p r, c = \phi p r$$

(Use method solving characteristic equation)

$$\theta_{0(x)}^{(y)} h = \theta_0'' + \lambda p r \theta_0' - \phi p r \theta_0 = 0$$

$$\theta_0'' + \lambda p r \theta_0' - \phi p r \theta_0 = -\lambda p r c_0$$

From (3.12)

$$C_1(x) = c_{m_2}$$

(3.26)

Substituting (3.23) and (3.24) in (3.21)

$$A_4 = 1$$

(3.25)

Substituting (3.23) in (3.22)

$$0 = A_3 + A_4, \therefore A_3 = -A_4 = 0$$

(3.24)

$$C \rightarrow 0, \text{ as } y \rightarrow \infty$$

(3.23)

$$1 = A_3 e^0 + A_4 e^0, \text{ it's implies } A_3 + A_4 = 1$$

C.H.M.Y. 0



$$\theta'_{o(y)} p = -M_2 A_7 e^{-m_2 y}$$

$$\theta''_{o(y)} p = M_2^2 A_7 e^{-m_2 y}$$

Substituting into 3.12)

$$A_7 = \frac{-\lambda p r}{M_2^2 - \lambda p r m_2 - \Phi p r} \quad (3.28)$$

$$\theta_{o(y)} h + \theta_{o(y)} p = A_5 e^{m_5 y} + A_6 e^{-m_6 y} + A_7 e^{-m_2 y} \quad (3.29)$$

$$\theta_{o(y)} = A_5 e^{m_5 y} + A_6 e^{-m_6 y} + A_7 e^{-m_2 y}$$

Applying the boundary condition

$$\theta = 1 \text{ at } y = 0$$

$$1 = A_5 e^0 + A_6 e^0 + A_7 e^0, \text{ it implies } A_5 + A_6 + A_7 = 1 \quad (3.30)$$

$$\theta \rightarrow 0, \text{ as } y \rightarrow \infty$$

$$0 = A_5 e^{\infty} + A_6 e^{-\infty} + A_7 e^{-\infty}, A = 0 \quad (3.31)$$

Substituting (3.30) in (3.28)

$$\theta_{o(y)} = A_6 e^{-m_6 y} + A_7 e^{-m_2 y} \quad (3.32)$$

From (3.13)

$$\theta_1'' + \lambda p r \theta_1' - \beta p r \theta_1 = -\phi p r c_1$$

$$\theta_{1(y)} h = \theta_1'' + \lambda p r \theta_1' - \beta p r \theta_1 = 0$$

$$M^2 + \lambda p r M - \beta p r = 0 \quad a = 1, b = \lambda p r, c = \beta p r$$

$$\frac{-\lambda p r \pm \sqrt{(\lambda p r)^2 - 4(\beta p r)}}{2}, \text{ since } \sqrt{\frac{(\lambda p r)^2 - 4\beta p r}{4}} = n_4$$

$$\frac{-\lambda p r}{2} \pm n_4, M_7 = \frac{-\lambda p r}{2} + n_4, M_8 = \frac{-\lambda p r}{2} - n_4$$

$$U_0 + \lambda U_0' - MU_0 = -Gr\theta_0 - GmC_0$$

From (3.10)

Substituting (3.37) in (3.35)

$$0 = V_8 e^\infty + V_9 e^\infty + V_{10} e^{-\infty}, \text{ it implies } V_8 = 0 \quad (3.37)$$

$$\theta \rightarrow 0, \text{ as } \gamma \rightarrow \infty$$

$$V_8 + V_9 + V_{10} = 1 \quad (3.36)$$

$$1 = V_8 e^0 + V_9 e^0 + V_{10} e^0$$

$$\theta = 1, \quad \text{as } \gamma = 0$$

Applying the boundary condition

$$\theta_{1(\gamma)} = V_8 e^{m\gamma} + V_9 e^{-m\gamma} + V_{10} e^{-m\gamma} \quad (3.35)$$

$$\theta_{1(\gamma)} h + \theta_{1(\gamma)} p = V_8 e^{m\gamma} + V_9 e^{-m\gamma} + V_{10} e^{-m\gamma}$$

$$V_{10} = \frac{M_2^2 - \lambda p r M_4 - \beta p r}{-\lambda p r} \quad (3.34)$$

Substituting into equation (3.13)

$$\theta_{1(\gamma)} p = M_2^2 V_{10} e^{-m\gamma}$$

$$\theta_{1(\gamma)} p = -M_4 V_{10} e^{-m\gamma}$$

$$\theta_{1(\gamma)} p = V_{10} e^{-m\gamma}$$

$$\text{since } -\lambda p r = V_{10}$$

$$\theta_{1(\gamma)} p = -\lambda p r e^{-m\gamma}$$

$$\theta_{1(\gamma)} p = -\lambda p r e^{-m\gamma}$$

$$\theta_{1(\gamma)} h = V_8 e^{m\gamma} + V_9 e^{-m\gamma}$$

(3.33)



$$U_{0(y)}h = U_0'' + \lambda U_0' - MU_0 = 0$$

Use methods of solving characteristic equation

$$M^2 + \lambda M - M = 0, a = 1, b = \lambda, c = m$$

$$\frac{-\lambda \pm \sqrt{\lambda^2 - 4(m)}}{2}, \quad \text{since } \sqrt{\frac{\lambda^2 - 4m}{4}} = n_5$$

$$\frac{-\lambda}{2} \pm n_5, \quad M_9 = \frac{-\lambda}{2} + n_5, \quad M_{10} = \frac{-\lambda}{2} - n_5 \quad (3.39)$$

$$U_{0(y)}h = A_{11}e^{m_9y} + A_{12}e^{-m_{10}y}$$

$$U_{0(y)}p_1 = -GmC_0$$

$$U_{0(y)}p_1 = -Gme^{-m_2y}, \text{ since } -Gm = A_{13}$$

$$U_{0(y)}p_1 = A_{13}e^{-m_2y}$$

$$U_{0(y)}' p_1 = M_2 A_{13} e^{-m_2y}$$

$$U_{0(y)}'' p_1 = M_2^2 A_{13} e^{-m_2y}$$

Substituting into (9) and dividing by coefficient of  $A_{13}$

$$A_{13} = \frac{-Gm}{M_2^2 - \lambda m - m} \quad (3.40)$$

$$U_{0(y)}p_2 = -Gr\theta_0$$

$$U_{0(y)}p_2 = -Gr A_6 e^{-m_6y} - Gr A_7 e^{-m_2y}$$

since  $-Gr A_6 = A_{14}$  and  $-Gr A_7 = A_{15}$

$$U_{0(y)}p_2 = A_{14}e^{-m_6y} \quad \text{and} \quad A_{15}e^{-m_2y} \quad (3.41)$$

$$U_{0(y)}p_2 = A_{14}e^{-m_6y}, \quad U_{0(y)}' p_2 = -M_6 A_{14} e^{-m_6y}, \quad U_{0(y)}'' p_2 = M_6^2 A_{14} e^{-m_6y}$$

Substituting into equation (9), and dividing by coefficient  $A_{14}$

$$A_{14} = \frac{-GrA_6}{m_6^2 - \lambda m - m} \quad (3.42)$$

$$U_{0(y)}P_2 = A_{15}e^{-m_6y}, \quad U'_{0(y)}P_2 = -M_2A_{15}e^{-m_6y}, \quad U''_{0(y)}P_2 = M_2^2A_{15}e^{-m_6y}$$

Substituting into equation (9) and dividing by coefficient of  $A_{15}$

$$A_{15} = \frac{-GrA_7}{m_2^2 - \lambda m - m} \quad (3.43)$$

$$U_{0(y)}h + U_{0(y)}P_1 + U_{0(y)}P_2 = A_{11}e^{m_0y} + A_{12}e^{-m_{10}y} + A_{13}e^{-m_2y} + A_{14}e^{-m_6y} + A_{15}e^{-m_2y}$$

$$U_{0(y)} = A_{11}e^{m_0y} + A_{12}e^{-m_{10}y} + A_{13}e^{-m_2y} + A_{14}e^{-m_6y} + A_{15}e^{-m_2y} \quad (3.44)$$

Applying the boundary condition

$$U = 1 + e^{int} \text{ at } y = 0$$

$$1 = A_{11}e^0 + A_{12}e^0 + A_{13}e^0 + A_{14}e^0 + A_{15}e^0 \quad (3.45)$$

$$A_{11} + A_{12} + A_{13} + A_{14} + A_{15} = 1$$

$$U \rightarrow 0 \text{ as } y \rightarrow \infty$$

$$0 = A_{11}e^{\infty} + A_{12}e^{-\infty} + A_{13}e^{-\infty} + A_{14}e^{-\infty} + A_{15}e^{-\infty} \quad (3.46)$$

$$A_{11} = 0$$

$$\text{Substituting (3.45) into (3.43)} \quad (3.47)$$

$$U_{0(y)} = A_{12}e^{-m_{10}y} + A_{13}e^{-m_2y} + A_{14}e^{-m_6y} + A_{15}e^{-m_2y}$$

From (3.11)

$$U_1 + \Delta U_1 - \Phi U_1 = -Gr\theta_1 - GMc_1$$

$$U_{1(y)}h = U_1 + \Delta U_1 - \Phi U_1 = 0$$

Using characteristic method of solving homogeneous equation



$$m^2 + \lambda m - \Phi = 0, a = 1, b = \lambda, c = \Phi$$

$$\frac{-\lambda \pm \sqrt{\lambda^2 - 4\Phi}}{2}, \quad \frac{-\lambda}{2} \pm \sqrt{\frac{\lambda^2 - 4\Phi}{4}} = n_6$$

$$\frac{-\lambda}{2} \pm n_6, \quad M_{11} = \frac{-\lambda}{2} + n_6, M_{12} = \frac{-\lambda}{2} - n_6$$

$$U_{1(y)}h = \Lambda_{16}e^{m_{11}y} + \Lambda_{17}e^{-m_{12}y} \quad (3.48)$$

$$U_{1(y)}p_1 = Gm c_1$$

$$U_{1(y)}p_1 = -Gme^{-m_{14}y}, \text{ since } -Gm = \Lambda_{18}$$

$$U_{1(y)}p_1 = -\Lambda_{18}e^{-m_{14}y}, \quad U'_{1(y)}p_1 = m\Lambda_{18}e^{-m_{14}y}, \quad U''_{1(y)}p_1 = -m^2\Lambda_{18}e^{-m_{14}y}$$

Substituting into (10), and dividing by coefficient of  $\Lambda_{18}$

$$\Lambda_{18} = \frac{-Gm}{m_4^2 - \lambda m - \Phi} \quad (3.49)$$

$$U_{1(y)}p_2 = -Gr\theta_1$$

$$U_{1(y)}p_2 = -Gr\Lambda_9e^{-m_{18}y} - Gr\Lambda_{10}e^{-m_{14}y}, \text{ let } -Gr\Lambda_9 = \Lambda_{19}, \text{ and } -Gr\Lambda_{10} = \Lambda_{20}$$

$$\text{it's imply } \Lambda_{19}e^{-m_{18}y} \quad \text{and } \Lambda_{20}e^{-m_{14}y}$$

$$U_{1(y)}p_2 = \Lambda_{19}e^{-m_{18}y}, \quad U'_{1(y)}p_2 = -m_{18}\Lambda_{19}e^{-m_{18}y}, \quad U''_{1(y)}p_2 = m_{18}^2\Lambda_{19}e^{-m_{18}y}$$

Substituting into (3.11), and dividing by coefficient of  $\Lambda_{19}$

(3.50)

$$\Lambda_{19} = \frac{-Gr\Lambda_9}{m_{18}^2 + \lambda m - \Phi}$$

$$U_{1(y)}p_2 = \Lambda_{20}e^{-m_{14}y}, \quad U'_{1(y)}p_2 = -m_{14}\Lambda_{20}e^{-m_{14}y}, \quad U''_{1(y)}p_2 = m_{14}^2\Lambda_{20}e^{-m_{14}y}$$

Substituting into (3.11), and dividing by coefficient of  $\Lambda_{20}$

(3.51)

$$\Lambda_{20} = \frac{-Gr\Lambda_{10}}{m_4^2 - \lambda m - \Phi}$$

$$U_{1(y)}h + U_{1(y)}p_1 + U_{1(y)}p_2 = A_{16}e^{m_{11}y} + A_{17}e^{-m_{12}y} + A_{18}e^{-m_{4y}} + A_{19}e^{-m_{8y}} + A_{20}e^{-m_{4y}}$$

$$U_{1(y)} = A_{16}e^{m_{11}y} + A_{17}e^{-m_{12}y} + A_{18}e^{-m_{4y}} + A_{19}e^{-m_{8y}} + A_{20}e^{-m_{4y}} \quad (3.57)$$

Apply the boundary condition

$$u = 1 + e^{int}, \quad y = 0$$

$$1 = A_{16}e^0 + A_{17}e^0 + A_{18}e^0 + A_{19}e^0 + A_{20}e^0$$

$$A_{16} + A_{17} + A_{18} + A_{19} + A_{20} = 1 \quad (3.53)$$

$$U \rightarrow 0 \text{ as } y \rightarrow \infty$$

$$0 = A_{16}e^{\infty} + A_{17}e^{-\infty} + A_{18}e^{-\infty} + A_{19}e^{-\infty} + A_{20}e^{-\infty}$$

$$A_{16} = 0 \quad (3.54)$$

Substituting (3.52) in (3.51)

$$U_{1(y)} = A_{17}e^{-m_{12}y} + A_{18}e^{-m_{4y}} + A_{19}e^{-m_{8y}} + A_{20}e^{-m_{4y}} \quad (3.55)$$

Velocity

$$U_{(y,t)} = A_{12}e^{-m_{10}y} + A_{13}e^{-m_{2y}} + A_{14}e^{-m_{6y}} + A_{15}e^{-m_{2y}} \\ + \varepsilon(A_{17}e^{-m_{12}y} + A_{18}e^{-m_{4y}} + A_{19}e^{-m_{8y}} + A_{20}e^{-m_{4y}})e^{int} \quad (3.56)$$

Temperature

$$\theta_{(y,t)} = (A_6e^{-m_{6y}} + A_7e^{-m_{2y}}) + \varepsilon(A_9e^{-m_{8y}} + A_{10}e^{-m_{4y}})e^{int} \quad (3.57)$$

Concentration

$$C_{(y,t)} = (e^{-m_{2y}}) + \varepsilon(e^{-m_{4y}})e^{int} \quad (3.58)$$

The skin friction, Nusselt number and of mass transfer are important physical parameter for type of boundary layer flow.

The skin-friction at the plate in the non-dimensional give by



$$\begin{aligned}
 u(y, t) = & m_{10}A_{12} + m_2A_{13} + m_6A_{14} + m_2A_{15} \\
 & + \varepsilon(m_{12}A_{17} + m_4A_{18} + m_8A_{19} \\
 & + m_4A_{20})e^{int}
 \end{aligned}
 \tag{3.59}$$

The Nusselt number is given by

$$\theta(y, t) = m_5A_6 + m_2A_7 + \varepsilon(m_8A_9 + m_4A_{10})e^{int}
 \tag{3.60}$$

The rate of mass transfer Sh is given by

$$\varepsilon(y, t) = m_2 + \varepsilon(m_4)e^{int}
 \tag{3.61}$$

## CHAPTER FOUR

### 4.0 Result and discussion

The Chemical Absorption Effect on Magnetohydrodynamic (MHID) Convection Flow through vertical with Mass transfer has been formulated and solved analytically. In order to point out the effects of physical parameters namely Visco-elastic parameter  $\lambda$ , magnetic parameter  $M$ , Heat parameter  $Q$ , Thermal Grashof number  $Gr$ , Mass Grashof number  $Gc$ , magnetic number  $Pr$  Schmidt number  $Sc$  and Chemical reaction parameter  $Kr$ , on the flow patterns, the computation of the flow field are carried out.

The Velocity profile has been studied and presented in figure 4.6 to 4.9 the velocity profile are studied for different values of magnetic parameter ( $M=1.00, 2.00, 3.00, 5.00$ ) and is presented in figure 4.5 From the graph we observed that velocity is increase with decrease in magnetic parameter. The velocity profiles is studied for different values of Mass Grashof number ( $Gm=2.00, 4.00, 6.00, 8.00$ ) and presented in figure 4.9 it is observed that the velocity increase with decrease in Mass Grashof number the profiles studied for different value of modified Grashof number ( $Gr=1.00, 2.00, 5.00, Gc=3.00$ ) and is presented in figure 4.8 from the graph its observed that the velocity increase with increase in modified Grashof number. The temperature

Profiles have been studied presented in figure 4.4 to 4.5 the effect temperature for different values of heat sink ( $Q=-1.00, 2.00, 3.00, 5.00$ ) is presented in figure 4.5 from the graph, it is shown that Temperature increase with decrease in. The temperature profiles for different values of Prandtl number ( $Pr=0.71, 1.00, 3.00, 7.00$ ) is presented in figure 4.4 we observed from the graphs that temperature increase with decrease Prandtl number. The temperature profiles for different value of the heat absorption parameter ( $Q=1.00, 2.00, 3.00, 5.00$ ) is presented in figure 4.5 from the graph it's observed that the temperature is increasing with decrease in.

The Concentration profile has been studied is presented in figure 4.1 to 4.3 the effects of concentration for different value of Visco-elastic parameter ( $\lambda=1.00, 2.00, 3.00, 4.00$ ) is presented in figure 4.1 from the graphs it is observed that the temperature increase with decrease in. The effect of concentration for different value of chemical reaction parameter ( $Kc=0.50, 1.00, 1.50, 3.00$ ) is presented in figure 4.2 from the graph it is observed that the temperature is decreasing with decrease in chemical reaction parameter. The effect of concentration for different



value of Schmidt number ( $Sc=0.30, 0.60, 1.00, 2.00$ ) is presented in figure 4.3 from the graph it is observed that the temperature is increase with decrease in Schmidt number.

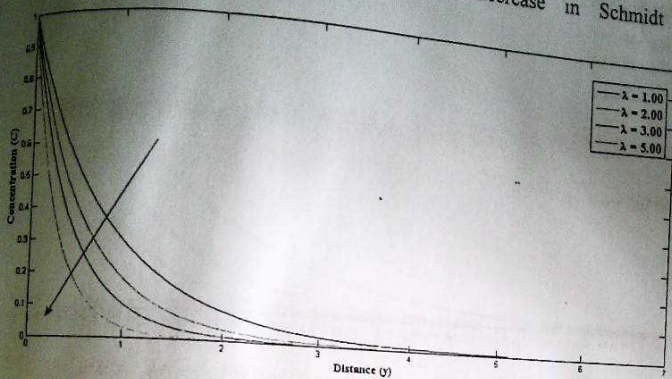


Figure 4.1 effect of visco-elastic parameter on nusselt number

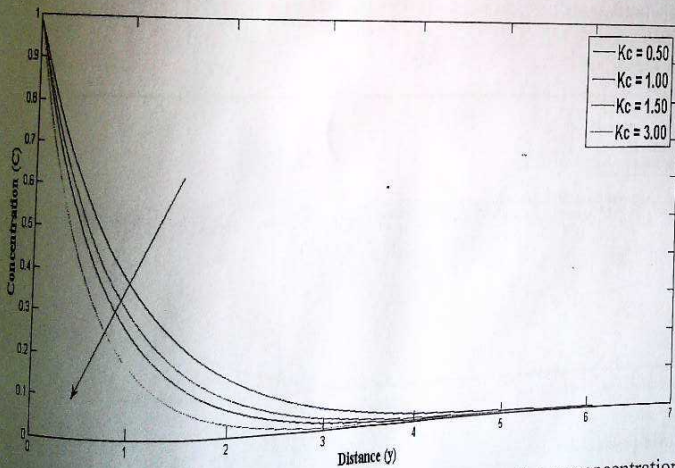


Figure 4.2 effect of increase the values of chemicals reaction on concentration.

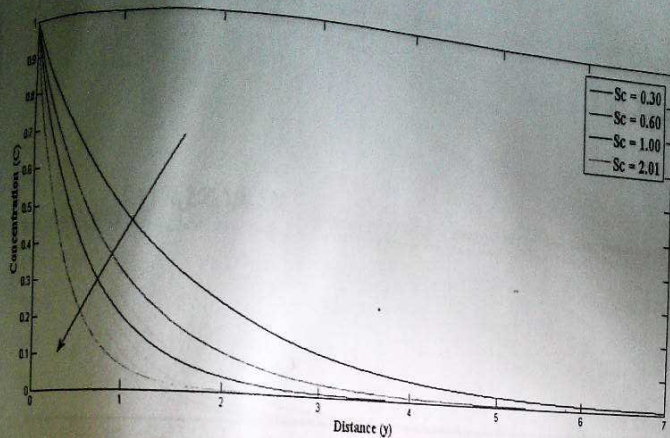


Figure 4.3 effect of Schmidt number on concentration.

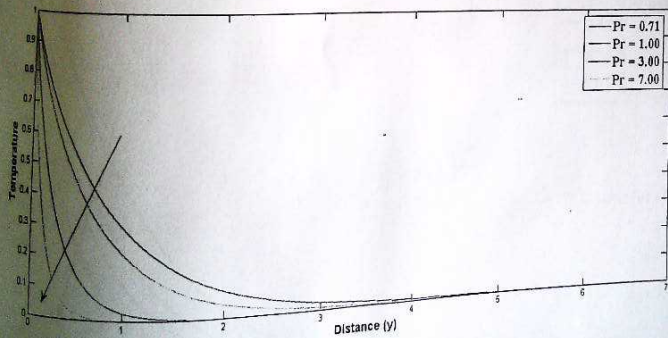


Figure 4.4 effect of Prandtl number on temperature.



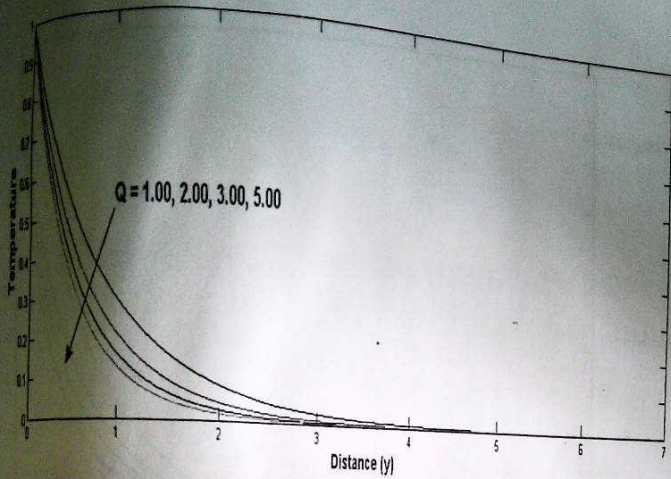


Figure 4.5 effect of heat absorption parameter on temperature.

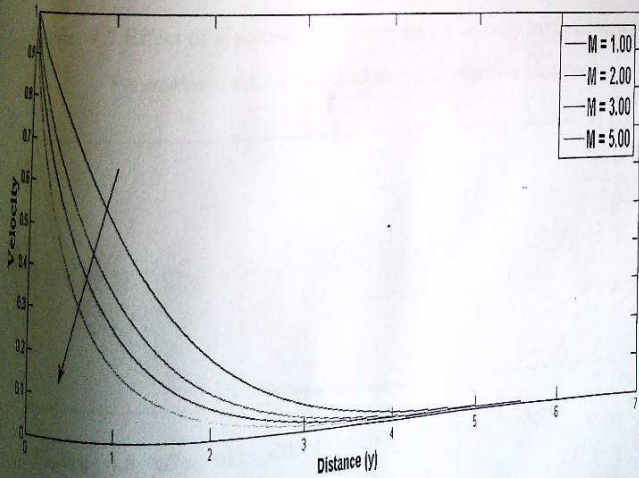


Figure 4.6 effect of magnetic parameter on velocity.

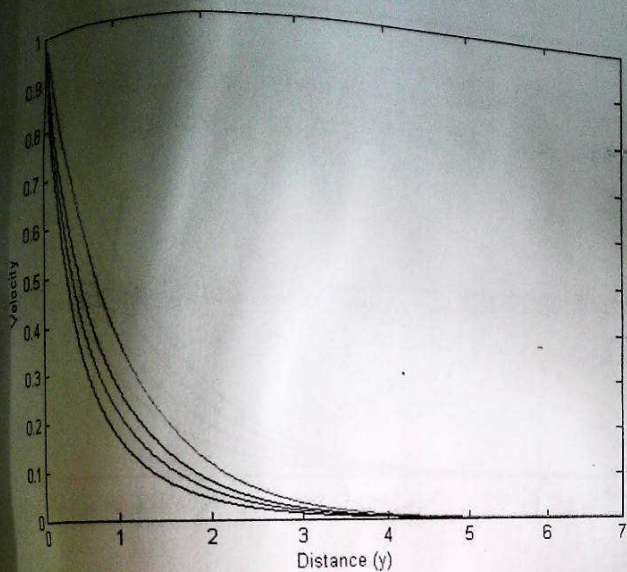


Figure 4.7 Effect of visco-elastic parameter on velocity in the absence of radiation  
 Parameter solet number and chemical reaction parameter

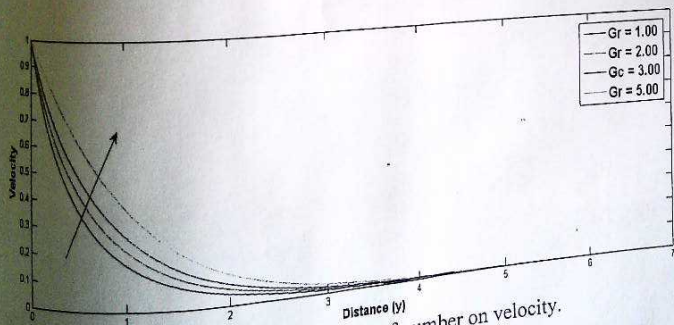


Figure 4.8 effect of modified Grashof number on velocity.



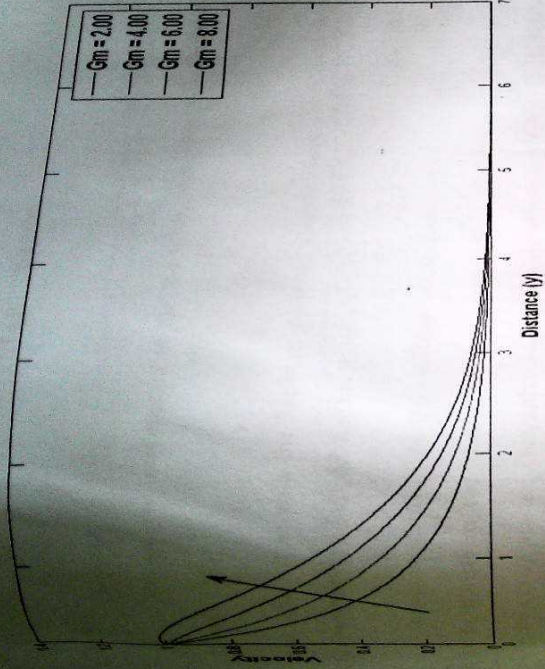


Figure 4.9 effect of the mass Grashof number on velocity

## CHAPTER FIVE

### 5.1 Summary

In chapter one we see the introduction of the, the aims and objective as well as definition of some important terms.

We started chapter two with the literature review. Chapter three we formulated the equation governing the flow. It's the problems is governed by:

Coupled nonlinear systems of partial differential equations (pdes), dimensional quantities were introduced to transform the partial differential equation into ordinary differential equations (odes), and the equations are then solved using the perturbation method. In chapter four we see the numerical representation for different values of velocity, temperature, concentration. In chapter five we see the summary, conclusion, reference.

### 5.2 Conclusion

We investigated the MHD free convection flow of a non-Newtonian fluid past an impulsively started vertical plate in the presence of chemical reaction, thermal diffusion, radiation absorption, thermal radiation, and heat absorption, with constant mass flux, the governing boundary layer equations are formulated with appropriate boundary conditions. The governing boundary layer equation is simplified and non-dimensionalized. The dimensionless equations are solved by using the finite difference method, the effect of various physical parameter such as Grashof number ( $G_r$ ), modified Gr, magnetic field parameter ( $M$ ), Schmidt number ( $Sc$ ), Prandtl number ( $Pr$ ), heat absorption parameter ( $\phi$ ), chemical reaction parameter ( $K_r$ ) are considered on the dimensionless velocity, temperature, and concentration. Computation on the variation of



local skin friction, Nusselt number and Sherwood number are also recorded. From the graphs plotted we discover that:

1. Velocity decrease with an increase in magnetic field parameter, Schmidt number, Prandtl number and visco-elastic parameter while it increases with an increase in Grashof number, modified Grashof number and Soret number.
2. Temperature increases with an increase radiation absorption parameter, visco-elastic parameter, Eckert number while it decreases with an increase in radiation parameter, Prandtl number and heat absorption parameter.
3. Concentration increase with an increase in Soret number while it decreases with an increase in Schmidt number and chemical radiation parameter.

## References

- Balier A.Y. and Dorsh R.S.R.(1996) ;Thermal radiation and effect on horizontal surfaces in saunated porous medium Transport porous media ,vol,23 pp.357-61.
- Chamkha. ^ J. (2004); MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation, absorption and a chemical reaction. Int. Commun. Heat Mass Transfer 30 (3) 413-422.
- Dorsh M.A. (2004); Combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation, Int. Eng.Sci.42 (2) 699-713.
- Depal, H. Mondal. (2011); Effects of Soret Dufour, chemical reaction and thermal radiation on MHD non-Darcy unsteady mixed convective he T. Hayyat, M. Nawaz,(2011), Soret and Dufour effect on the mixed convection flow of second grade fluid subject to Hall and ionslip current, Int. J Numer meth.vol.3 pp.7-12.
- Hayyat, M. Nawaz,(2011), Soret and Dufour effect on the mixed convection flow of second grade fluid subject to Hall and ionslip current, Int. J Numer meth.vol.3 pp.7-12.
- Kandasamy. K. Periasamy. K.K.S. Prabhu.(2005) Effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection,Int. J. Heat Mass Transf. 48(7); p 1388-1394
- Mostafa A.and Mahmoud A.(2009); Thermal radiation effect on unsteady MHD free convection flow past vertical plate with temperature dependent ; The Canadian journal of Engineerring,vol.87pp-52.



- Omokhuale, E., Usman, H. and Altine, M. M. (2016). Effect of Heat Absorption on Steady/Unsteady MHD Free Convection Heat and Mass Transfer Flow Past an Infinite Vertical Permeable Plate with Mass Absorption and Variable Suction. *Journal of the Nigerian Association of Mathematical Physics*, Volume 38 (Nov., 2016). 433 – 442.
- Omokhuale, E., Okoro, F. M., Asibor, R. E. and Usman, H. (2017). Natural Convective Heat Mass Transfer Fluid Flow Through a Porous Medium with Variable Pressure and Thermal Radiation. *Journal of the Nigerian Association of Mathematical Physics*, Volume 39 (Jan., 2017). 129 – 138.
- Patil, P. S. Kulkarni. (2008); Effect of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation, *Int. journal Therm. Sci.* 47(8) 1043-1054
- Raptis A. (1986); flow through a porous medium in the presence of magnetic field. – *International Journal. Energy Research* vol. 10, pp.97-101.
- Raju M.S.Liu X.R.and Law C.K..K.(1982) A formation of combined force and free convection past horizontal and vertical surface. *Int. J. Heat Mass Transfer* ,vol.27 pp.2215-2224.
- Seddeek, A.A Darwish, M.S. Abdelmeguid. (2007) ;Effect of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media with radiation *Commun. Nonlinear Sci.Numer Simul* 12 (2) 195-2123.
- Vajranvelu M.H.(1990); Radiative and free convection effects on the oscillatory flow past a vertical plate. *Astrophysics and space, Science* ,vol. 166,pp.26 -75

## Appendix

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10) a01-7:
1  sqrt((a*Sc)^2+4.*b*Sc)/4;
2  sqrt((a*Sc)^2+4.*Sc*(b+i*n))/4;
3  sqrt((a*Pr)^2+4.*Q*Pr)/4;
4  sqrt((a*Pr)^2+4.*Pr*(Q+i*n))/4;
5  (a*Sc)/2+a1;
6  exp(-m2*y);
7  (a*Sc)/2+a2;
8  exp(-m4*y);
9  (a*Pr)/2+a3;
10 exp(-m6*y);
11 (a*Pr)/2+a4;
12 exp(-m8*y);
13 sqrt((a)^2+4.*M)/4;
14 sqrt((a)^2-4.*(M+i*n))/4;
15 (a)/2+a5;
16 (a)/2+a6;
17 Car/(m6^2-a^2*m6-M);
18 C/m/(m2^2-a^2*m2-M);
19 B3-B4;
20 B3*exp(-m10*y)+B3*exp(-m6*y)+B4*exp(-m2*y);
21 (m8^2-a^2*m8-(M+i*n));
22 C/m/(m4^2-a^2*m4-(M+i*n));
23 B7-B8;
24 B6*exp(-m12*y)+B7*exp(-m8*y)+B8*exp(-m4*y);
25
26 T1*epsilon*exp(i*n);
27 C0-C1*epsilon*exp(i*n);
28 a0*a1*epsilon*exp(i*n);
29
30 R*linewidth';
31 Distance (Y)
32 Velocity

```