NUMERICAL MODELLING OF A P-N JUNCTION DEPLETION REGION

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BY

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CERTIFICATION

This is to certify that this research work was carried out under my supervision by Felix W. Burari, in the Department of Physics, University of Ibadan, Ibadan.

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DEDICATION

This work is dedicated to my father Mr. Wilfred Pammo Burari and my mother Mrs. Emy W. Burari.

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HISTORICAL DEVELOPMENT OF PHYSICAL DEVICE MODELLING

Prior to the widespread availability of digital computers, solid state-devices were theoretically characterised using closed - form analytical techniques based on approximate solutions to the carrier transport processes. A well known example of this type of analysis was described by schockly (1) in his paper on Unipolar FETS in 1952. This approach usually proceeds by dividing the device into regions in which simplified linearised approximations are applied, joined by appropriate boundary conditions (1,2, 3). This method was originally applied to one - dimensional models, but was later extended to include two - dimensional effects in both silicon and Gallium Arsenide devices (4,5,6).

Important effects such as carrier velocity saturation, absent from some of the very early models, were included in later analyses (7,8). The closed - form analysis technique proved very effective in characterising large geometry unipolar devices and has continued to be used in many applications which take advantage of the relative simplicity and ease of programming inherent in this approach.

However, although this approach allows rapid analysis and provides a basic insight into the device physics, it is unsuitable for modelling devices where the transport process is other than largely one - dimensional and where the electric field varies rapidly throughout the device. This implies that a closed - form solution is unsuitable for modelling sub-micron devices and many planar devices, such as FETS, found in a wide range of discrete and integrated forms.

Interest in the numerical simulation of semiconductor devices, using physical device models, began over twenty years ago. In 1964, Gummels (9) successfully demonstrated that this approach could be used to characterise a silicon bipolar transistor, using a one-dimensional steady - state model. The limited computer resources available at this time meant that device simulations had to be restricted to one- dimension. McCumber and Chynoweth (10) demonstrated Gunn instabilities, in what was one of the first reported one-dimensional electron temperature models for a unipolar GaAs Sample. De Mari (11, 12) apply one -dimensional numerical models to P - n junctions. In 1969 Scharfetter and Gummel (13) reported a one - dimensional simulation used to model silicon Read (IMPATT) diodes.

Their numerical scheme for accurately solving the continuity equation has now become an established technique in many simulations and is still used in many two-dimensional simulations.

Two - dimensional numerical simulations were developed to obtain a more realistic representation of planar and three terminal devices. Two-dimensional models also allow other important phenomena such as current crowding and high level injection in bipolar junction transistors (B J T s) and short and this channels in FETs to be investigated, which is not possible for one-dimensional models. Kennedy and O'Brien (14) reported a two - dimensional simulation for silicon junction (FETs) in 1970. Considerable effort has been directed at simulating FETs. In particular metal - Oxide semiconductor FETs and metal semiconductor FETs have received much attention.

Three - dimensional simulations have been recently developed to account for three - dimensional effects found in small devices with narrow widths and non-uniformities in the active regions. Small geometry very large scale integration (VLSI) MOSFETS with channel widths of this order of the gate length cannot be accurately modelled using two-dimensional models and three -dimensional simulations have been used to investigate non-uniformities in this channel, fringing field effects, breakdown voltage and threshold voltage variations.

ABSTRACT

The historical development of physical device modelling was discussed and the physics of semiconductor devices reviewed with particular emphasis on PN junction. The depletion layer of a PN junction was studied extensively. The poisson equation was solved and an expression was obtained which enables us to obtain the width of a depletion layer.

Newton - Raphson iteration method was used to obtain the extent of the depletion layer under a specified biasing potential.

INTRODUCTION

Semiconductor devices form the fundamental of modern electronics, being used in applications extending from computers to Satellite communication systems. A wide variety of devices are available, fabricated from a range of semiconductor materials. The most common active devices found in electronic systems include bipolar and field effect transistors, diodes, thyristor and triacs. Silicon is one of the most commonly used semiconductor material for both discrete and integrated devices, although we have other materials like GaAs and InP.

Modelling plays an important role in the design of the semiconductor devices. The development of solid state devices has involved a largely empirical design process with many iterations of the fabrication stage being required to achieve the desired specification.

Device modelling could be able to predict characteristics such as current - voltage, capacitance - voltage relationship and voltage breakdown, all of which are functions of the design parameters.

NOTATION

q	Electronic charge 1.6×10^{19}C
E = E (Er, Eo)	Dielectric constant.
Me, Mn	Electron and hole effective masses
h	Planck's constant, 6.626×10^{-34} J - S
k	Boltzmann's constant, 1.38×10^{-23} J/K
Ei; Eg	Electron energy at Fermi lenel; Energy gap
T	Temperature
Q	Boltsmann factor (q/KT)
ni	Intrinsic free electron density
n,p	Electron and holw densities
/	Potential
Øn, Øp	Electron and hole quasi-Fermi Potentials
E	Electric field in vector and scalar, E = grad 4
Un, Up	Electron and hole mobilities
Dn, Dp	Electron and hole diffusion constants
Jn, Jp, jn, jp	Electron and hole current densities in vector and scalar time
t	time
G, U	Generation and recombination
Ln, Lp	Electron and hole lifetimes
Mo	Free electron mass 9.11×10^{-31} Kg

Mce	Conductive effective mass of electrons
Ne, Nv	Effective density of states in conduction and valence Band.
Vin	Built - in potential.

LIST OF TABLES

PAGES

LIST OF FIGURES

TABLE OF CONTENTS

PAGES

CHAPTER 2:

CHAPTER 3: PN JUNCTIONS

CHAPTER 5: DISCUSSIONS AND SUGGESTION

CHAPTER ONE

CARRIER CONCENTRATION AND ENERGY BANDS IN SEMICONDUCTOR DEVICES.

1.1 Semiconductors

At absolute zero a pure, perfect crystal of most semiconductors will be an insulator. The characteristic semiconducting properties are brought about by thermal excitation, impurities, lattice defects, or departure from non-chemical composition. Semiconductor materials are distinguished by having their specific electrical conductivity somewhere between that of a good conductors ($10^6\ (\Omega cm)^{-1}$) and that of a good insulators ($10^{-5}\ (\Omega cm)^{-1}$) ; hence the name. Among those materials, by far the most important in engineering use is silicon (Si). Of quite lesser importance is Germanium (Ge), which like silicon is an element belonging to Group IV of the periodic table. Becoming more important daily are the compound semiconductors, usually, compounded of two elements (but sometimes more) of Groups III and V or II and VI of the periodic table.

Among the compound semiconductors Gallium Arsenide (GaAs) is the most important. Also in use for specific purposes are:-

Indium Antimonide (InSb), Gallium Phosphide (GaP), Cadmium Sulphide (Cds), lead - Tin - Telluride (PbSnTe) and others.

1.2 CARRIER CONCENTRATION

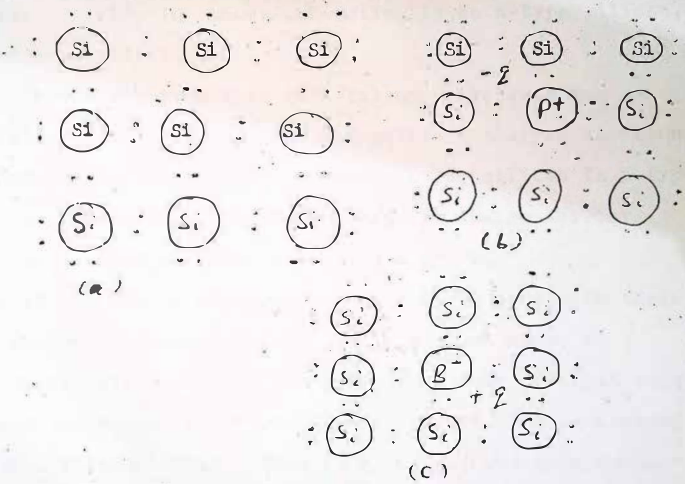


Fig. 1 Three basic bond pictures of a semiconductor
(a) Intrinsic Si with neglible impurities
(b) n - type Si with donor (phosphorus)
(c) P - type Si with acceptor (boron),

Figure 1, shows three basic bond pictures of a semiconductor. Fig 1a shows intrinsic silicon, which is very pure and contains a neglible small amount of impurities; each silicon atom shares its four valence electrons with the four neigbouring atoms, forming four covalent bonds. Fig. 1b shows schematically an n-type silicon, where a substitutional

Phosphorus atom with five valence electrons has replaced a silicon atom, and a negative - charged electron is donated to the conduction band. The silicon is n-type because of the addition of the negative charge carrier and the phosphorus atom is called a donor.
Fig. 1C Similarly shows that when a Boron atom with three valence electrons substitutes for a silicon atom, an additional electron is accepted to form four covalent bonds around the boron, and a positive - charged hole is created in this valence band. This is P - type, and this Boron is an acceptor.

1.2.1 Intrinsic Semiconductor

We now consider the intrinsic case. The number of occupied conduction band levels is given by

$$n = \int_{E_c}^{E_{top}} N(E)\,F(E)\,dE \qquad (1.1)$$

where E_c is the energy at the bottom of the conduction band and E_{top} is the energy at the top. The density of statis N(E) can be approximated by the density near the bottom of the conduction band for low - enough carrier densities and temperatures

$$N(E) = M_c \frac{\sqrt{2}}{\pi^2} \frac{(E - E_c)}{3}^{\frac{1}{2}} (M_{de})^{3/2} \quad \_\_\_\_\_\_ (1.2)$$

where M_c is the number of equivalent minima in the conduction band and M_{de} is the density-of-state effective mass for the electrons:

$$M_{de} = (M_1^* \; M_2^* \, M_3^*)^{1/3} \quad \_\_\_\_\_\_\_\_ (1.3)$$

where M_1^*, M_2^*, M_3^* are the effective masses along the principal axes of the ellipsoidal energy surface, an example is, in Silicon, $M_{de} = (M_1^* \; M_1^*)^{1/3}$ __

$$\_\_\_\_\_\_ (1.4)$$

The Fermi - Dirac distribution function F(E) is given by $$F_{(E)} = \frac{1}{1 + \exp \frac{(E-E_F)}{KT}} \quad (1.5)$$

where K is Boltzmann's constant, T the absolute temperature, and E_F the Fermi energy, which can be determined from the charge neutrality condition.

The integral in equation (1) can be evaluated to be

$$n = N_c \frac{2}{\sqrt{\pi}} F_{\frac{1}{2}} \frac{(E_F - E_c)}{KT} \qquad \underline{\hspace{4em}}(1.6)$$

where N_c is the effective density of states in the conduction band and is given by

$$N_c = 2 \frac{(2\pi m_{de} KT)^{3/2}}{h^2} M_c$$

$$\underline{\hspace{6em}}(1.7)$$

For the Fermi level several KT below E_c in non degenerate semiconductors, the integral approaches $\sqrt{\pi}\ e^{nf/2}$ and equation (1.6) becomes

$$n = N_c \exp\left(\frac{E_c - E_f}{KT}\right) \qquad \underline{\hspace{4em}}(1.8)$$

Similarly, we can obtain the hole density near the top of the valence band:

$$P = N_v \frac{2}{\sqrt{\pi}} F_{\frac{1}{2}} \left(\frac{E_v - E_f}{KT}\right)$$

$$\underline{\hspace{6em}}(1.9)$$

where Nv is the effective density of statis in the valence band and is given by

$$N_v = 2\left(\frac{2\pi m_{dh} KT}{h^2}\right)^{3/2}$$

$$\underline{\hspace{6em}}(1.10)$$

where M_{dh} is the density of state effective mass of the valence band

$$M_{dh} = (M^{*}_{lh}\ 3/2 + M^{*}_{lh}\ 3/2)^{2/3} \qquad 1.11$$

where the subscripts refer to light and heavy hole masses. Again under nondegenerate conditions.

$$P = N_v \exp\left(- \frac{E_f - E_v}{KT} \right) \qquad 1.12$$

For intrinsic semiconductors at finite temperatures continuous thermal agitation exists, which results in excitation of electrons from the valence band to the conduction band and leaves an equal number of holes in the Valence band, that is $n = P = n_i$, where n_i is the intrinsic carrier density. This process is balanced by recombination of the electrons in the conduction band with holes in the valence band.

The Fermi level for an intrinsic semiconductor is obtained by equating equations (1.8) and (1.12)

$$E_F = E_i = \frac{E_c + E_v}{2} + \frac{KT}{2} \ln \left(\frac{N_v}{N_c}\right)$$

$$= \frac{Ec + Ev}{2} + \frac{3KT}{4} \ln \left(\frac{M_{dh}}{M_{de}\ M_c{}^{2/3}} \right) \qquad 1.13$$

Hence the Fermi level E_i of an intrinsic semiconductor generally lies very close to the middle of the bandgap.

The intrinsic carrier density is obtained from equations (1.8), (1.12) and (1.13)

$$nP = n_i^2 = N_c N_v \exp(-Eg/KT)$$

$$\text{or} \quad n_i = \sqrt{N_c N_v}\, e^{-Eg/2KT} \quad \text{(1/14)}$$

1.2.2 Donors and Acceptors:

When a semiconductor is doped with donor or acceptor impurities, impurity energy levels are introduced. A donor level is defined as being neutral if filled by an electron, and positive if empty and negative if filled by an electron.

The simplest calculation of impurity energy levels is based on the hydrogen - atom model. The ionization energy for the hydrogen atom is

$$E_H = \frac{M_o q^4}{32\pi^2 E_o^2 \hbar^2} = 13.6 \text{ eV} \quad \text{(1.15)}$$

where E_o is the free - space permittivity.

The ionization energy for the donor Ed, can be obtained by replacing M_0 by the conductivity effective mass of electrons

$$M_{ce} = 3 \left(\frac{1}{M_1^*} + \frac{1}{M_2^*} + \frac{1}{M_3^*} \right)^{-1}$$

and by replacing E_0 by the permittivity of the E_s in eqn (1.15)

$$E_d = \left(\frac{E_0}{E_s} \right)^2 \left(\frac{M_{ce}}{M_0} \right) E_H \qquad \text{\_\_\_\_\_\_\_\_} (1.16)$$

The ionization energy for donors as calculated in Eq. 1. 16 is 0.006eV for Ge, 0.025eV for Si, and 0.07eV for GaAs. The hydrogen-atom calculation for the ionization level for the acceptors is similar to that for the donors. We consider the unfilled valence band as a filled band plus an imaginary hole in the central force field of a negativiely charged acceptor. The calculated acceptor ionization energy (measured from the valence - band edge) is 0.015eV for Ge, 0.05eV for Si and about 0.05eV for GaAs.

1.3 Energy bands in Semiconductors.

The band structure of a crystalline solid, that is, the energy - momentum (E - K) relationship, is usually obtained by solving the schroedinger equation of an appropriate one - electron problem

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(r) \right\} \phi_k(r) = E_k \, \phi_k(r)$$

The bloch theorem states that if a potential energy $V(r)$ is periodic with the periodicity of the lattice, then the solutions $\phi_k(r)$ of the Schroedinger equation are of the form

$$\phi_k(r) = e^{jk.r} U_n(K,r) = \text{Bloch function.}$$

where $U_n(K,r)$ is periodic in r with the periodicity of the direct lattice and n is the band index. From the Bloch theorem, it can be shown that the energy E_k is periodic in the reciprocal lattice, that is $E_k = E_{k+G}$, where

$$Ga = h\,a^* + k\,b^* + l\,c^*$$

h, k andl are integers

and a^*, b^* and c^* are reciprocal lattice vectors. For a given band index, to label the energy uniquely, It is sufficient to use only K's in a primitive cell of the reciprocal lattice.

The energy bands of solids have been studied theoretically using a variety of numerical methods.[1]

In the case of semiconductors, the three methods most frequently used are the orthogonalized plane-wave method, the Pseudopotential method and the K.P method.

The valence band in the zincblende consists of four subbands when spin is neglected in the schroedinger equation, and each band is doubled when spin is taken into account.

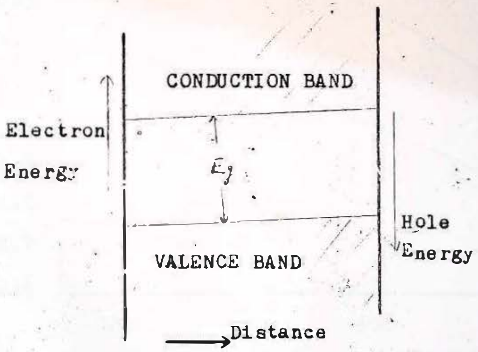


fig. 2: Energy band diagram

Above is the energy band diagram of a Semiconductor.

At room temperature and under normal atmosphere, the values of the bandgap are 0.66 eV for Ge, 1.12eV for Si, and 1.42eV for GaAs. These values are for high - purity materials. For highly doped materials the bandgaps become smaller. It has been observed experimentally that the bandgaps of most semiconductors decrease with increasing temperature.

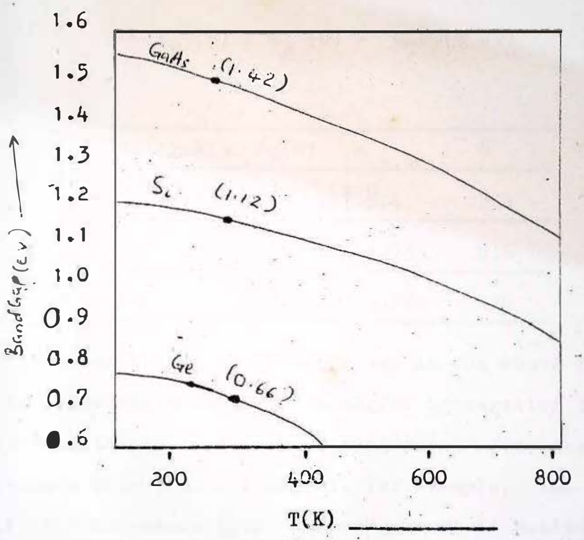


fig. 3: Energy bandgaps of Ge, Si and GaAs as a function of temperature.

Fig. 3 shows variations of bandgaps as a function of temperature for Ge, Si and Ge, Si and GaAs. The bandgap approaches 0.75, 1.17 and 1.52eV respectively, for the three semiconductors at OK.

The variation of bandgaps with temperature can be expressed by a function

$$E_g (T) = E_g (0) - Q_T^2/(T + b)$$

Materials	Eg(0)	a $(\times 10^{-4})$	b
GaAs	1.52	5.4	204
Si	1.17	4.73	636
Ge	0.75	4.774	235

where $E_g(o)$, a, b are given in the above table. The temperature coefficient dEg/dT is negative for the aforementioned three semiconductors. Some semiconductors have positive dEg/dT, for example, the bandgap of Pb.S increases from 0.286eV at OK to 0.41eV at 300K. The bandgaps of Ge and GaAs increases with pressure near room temperature.

1.4 GENERATION AND RECOMBINATION OF EXCESS CARRIERS

In view of the fact that nonzero excess - carrier concentrations represent a deviation from equilibrium

conditions, there arise physical mechanisms that endeavour to restore the equilibrium state.
More precisely, whenever the electron and hole concentrations i.e. n and P are positive, corresponding to total carrier concentrations greater than the equilibrium values, there is a tendency for the excess carriers to recombine or to disappear by mutual annihilation conduction electrons fall back into vacancies in the bond structure, thereby removing both themselves and an equal number of holes'P' from circulation. If on the other, electron concentration 'n' and hole concentration 'P' are negative, as they will be if the total concentrations are caused to be less than the corresponding equilibrium values, there is a tendency for excess carriers to appear or to be generate in pairs by the breaking of covalent bonds.

Recombination - generation process that comes into operation when the equilibrium is disturbed can be described in terms of a local rate of recombination which has a dimensions of pairs per cubic centimeter per second.

1.5 COMPENSATION AND DEEP IMPURITIES

A semiconductor doped by equal concentrations of donor and acceptor impurities is said to be fully compensated. The free electrons donated by the donors are grabbed by the acceptors since a free electron comes down in energy when it occupies an acceptor state, and each system tends in equilibrium towards its lowest possible energy.

In a fully compensated semiconductor, therefore, all the donated electrons have been caught by acceptor states and none is available for conduction. All the valence electrons stay in the valence levels since no acceptor state is left inoccupied, so there are no holes either. The number of available charge carriers will be very low, like in an intrinsic, undoped, semiconductor. Contrary to intrinsic materials, however, a compensate semiconductor has a lot of positively and negatively charged ions (the donors and acceptors) embedded in it and though macroscopically it is electrically neutral, these charges would affect its conductivity.

If an atom from Group II, like Zn, is used to dope a group IV semiconductor like Si, two bonds in the lattice will be missing in the vicinity of the Zn atom. This atom can therefore accept either a single electron from the valence band and become singly ionized, or accept two and become doubly ionized. The energy levels that these valence electrons must attain to be accepted, however, are much higher than for a Group III acceptor like boron. Such levels are called deep. Thermal energy is not sufficient to excite valence electrons into them but they can catch free electrons from the conduction level. This may enhances the recombination of such trapped electrons with holes that may come by. Thermal energy is not sufficient to liberate an electron trapped in a deep impurity. One therefore finds that deep impurities drastically reduce the number of electrons free for conductions and an N- type materials so doped behaves like an insulator.

Deep levels may be created in semiconductor by crystalline defects and by many heavy metal atoms. Especially useful are gold (Au), which in minute quantities is used in Si to enhance recombination and increase the operating speed of switching devices, and chromium (Cr), is used to dope P - type GaAs, create deep donor levels that cause compensation and turn it into a practical insulator called semi-insulating GaAs. Such GaAs has resistivities of uptp $10^9 \Omega cm^{-1}$ and is used as a single crystal substrate on which digital circuits or microwance devices of GaAs are made.

1.6 HIGH DOPING DENSITIES AND DEGENERATE SEMICONDUCTORS

In all our above discussion, there is one implicit assumption and that is that the impurity atoms are few and far between in the semiconductor crystal. If their concentration is made large enough for orbits of the fifth distant electrons of neighbouring donor atoms to start to overlap them they begin to be influenced by each other and then the semiconductor.

Properties, such as the behaviour of its conductivity with temperature, will change. Such a materials is called degenerate, that is, one can no longer[1] consider the allowed orbits of the electrons independently. They units into a single system in whcih the pauli exclusion principle, for-bidding electrons to have the same allowed state holds. We can obtain an estimate of the impurity concentration, N, above which energy levels of impurity atoms become affected by the nearness of other impurity atoms in their neighbourhood as follows:

The average distance between neighbouring impurity atoms is $N^{-1/3}$ if N is their concentration. If this distance becomes comparable to the diameter of the fifth electron orbit, degeneracy sets in . This leads to a value of approximately 10^{19} cm^{-3} as the limiting concentration (about three orders of magnitude less than the semiconductor atom concentration). The technological limit to impurity inclusion is usually higher and is called the solid solubility limit. This is property of the semiconductor, the impurity and the temperature at which the impurity is introduced. Attempts to increase the impurity concentration further will fail because the excess impurity will segrate form inclusions in the crystal and will not be electrically active, that is, will not contribute carriers. High doping densities also introduce mechanical stresses in the crystal because of accumulated differences in atomic sizes and increase the number of crystal faults.

CHAPTER TWO

MOBILITY, ELECTRICAL CONDUCTIVITY, EXCESS CARRIERS, LIFETIME, DIFFUSION AND TRANSPORT PHENOMENA.

2.1 SCATTERING MECHANISM

If a constant voltage source is connected to the two sides of a semiconductor chip, and electrical field E is created in it. This field acts upon the free charge carriers and causes them to drift in the direction of the force it applies, thereby creating a drift current.

When a charge carrier is acted upon by a constant electrical field in a vacuum, its ensuing acceleration a is $a = \frac{qE}{M_e}$

and its velocity V at time t, if it started from rest:

$$V = \int_0^t a\,dt = \frac{qEt}{M_e}$$

Inside a semiconductor, on the other hand, the movement of the charge carrier is not smooth but is perturbed by various obstacles, causing what is known as scattering. There are two main types of scattering mechanism:

(a) Lattice Scattering is caused by collisions of the moving carrier with distrubances in the periodic internal potential inside the semiconductor crystal. These disturbances are due to the vibrations of the crystal lattice atoms around their proper place in the lattice because of their thermal energy. The effect of the internal periodic potential itself, which exists in any crystal, can be taken into consideration by assigning an effective mass m^* to the moving electron or hole. These masses are different from the mass M_o of the electron outside the crystal. Therefore it is only the disruptions in the periodic potential, caused by the thermal vibrations of the atoms, that scatters the drifting free carriers: at a certain moment an electron can bump into a region where the crystal atoms are more densely packed than usual, yet a moment later it may find itself in a sparsely packed region.

The dense and sparse regions form pressure were existing inside the crystal. The vibrational wave - particl entities are called Phonons.

The wave or particle nature that phonons exhibit depends on the type of experiment performed. Their interaction with current carries inside the semiconductor results from the local disturbances introduced by the existence of phonons in the otherwise periodic lattice potential. The interactions can also be looked upon as collisions between the current carries and phonons. In these collisions, the total energy and momentum are conserved, but become redistributed, changing magnitude and direction of carrier velocity.

Following the collision the carrier is again accelerate in the direction of the field and, inevitably, scattered again, and this repeats all the time. It is obvious that lattice scattering will grow more severe with increasing temperature.

(b) Impurity Scattering is caused by the presence of ionized impurity atoms in various positions in the crystal lattice. Due to their net charge, they exert a force on the free carrier passing nearby, causing it to change its direction. This type of scattering is less severe if the free carrier is moving faster (that is, at higher temperatures), and spends less time in the vicinity of the ionized impurity atom.

2.2 AVERAGE DRIFT VELOCITY; MOBILITY

Consider a charge carrier under the effect of an electric field E, when there is a scattering mechanism.

Suppose we start with n_0 carriers at time $t = 0$, measured for each carrier from the moment of its last collision, then atime t later. There are still n(t) that have not suffered a second scattering collision and that are still accelerating in the direction of the field. Between t and t + dt an additional dn carriers will suffer a second collision and lose their momentum in the field direction. The number of still accelerating carriers will therefore be reduced by by dn which, to a first approximation, is proportional to the number of still uncollided carriers n and to the time increment dt but not to the moving carriers' energy

$$- dn = \frac{1}{\tau} ndt \quad \_\_\_\_\_\_\_\_ \quad 2.2.1$$

where $\frac{1}{\tau}$ is a constant of proportionality. The solution of equation 2.2.1, by separation of variables, is

$$n = n_0 \exp\left(- \frac{1}{\tau}\right) \quad \_\_\_\_\_\_\_\_ \quad 2.2.2$$

where τ has the dimensions of time, the average free time between collision. The probability for a collision during the period dt is dn/no. Therefore

$$- \frac{dn}{n_0} = \frac{1}{\tau} \exp\left(- \frac{1}{\tau}\right) dt \quad \_\_\_\_\_\_\_\_ \quad 2.2.3$$

The right - hand side of 2.2.3 gives the distribution of the time t that a carrier moves till it collides. For instance, a relatively large fraction of the starting n_0 carriers will collide again near $t = 0$, at the beginning of their movement, because at $t = 0$, the right - hand side of 2.2.3 is maximum. But there will be some, on the other hand, whose time t to the next collision will be very long because $\exp(-\frac{t}{\tau})$ never quite reaches zero in a finite time. Therefore some electrons or holes will be accelerated to very high velocities, while most will reach only a low velocity before colliding and being scattered again. To find an average time between collisions, we must sum the times to the second collision for the various carriers, assigning a proper weight to each time t depending on the relative numbers of carriers colliding at that time.

The carrier drift velocity $V_d = \frac{q\tau}{M^*} E$ we obtain a velocity that is proportional to the field and is constant in a constant field. The proportionality factor is called MOBILITY and denoted by μ.

$$\mu_{e,h} = \frac{q\tau}{M^*_{e,h}} \; ; \; V_d = \mu E.$$

where e, h refer to electron and hole respectively.

The net collision time τ between the lattice scattering collision time τ and the impurity collision time τo is:

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2}$$

The resulting mobility is $\frac{1}{\mu} = \frac{1}{\mu_l} + \frac{1}{\mu_i}$

where μ_l is the mobility from acoustic phonon lattice scattering and μ_i the mobility from ionized impurities. μ_L and μ_i are temperature dependant.

The dependance of μ on the temperature will therefore be determined by μ_i at low temperatures, and by μ_l at high temperatures. The higher the doping level the lower μ_i becomes, and consequently the lower the becomes. Infact it may become so small that even at room temperature and above μ_i would still determine the total mobility.

2.3 CONDUCTIVITY

The drift current densities of the holes and electrons are given by

$$Je\ (drift) = qn\ (Vd)e = +\ qn\mu_e E \quad \_\_\_\_\_ \quad 2.3.1$$

$$J_h \text{ (drift)} = qP(V_d)_h = + qn\mu_e E \quad \rule{2cm}{0.4pt} \; 2.3.2$$

where J_e is the electron drift current density and J_h is the hole drift current density. The total drift current density is also given as:

$$J = J_e + J_h = q(n\mu_e + P\mu_h)E \quad \rule{2cm}{0.4pt} \; 2.3.3$$

The specific conductance J of the semiconductor is defined as the ratio of the magnitudes of J and E.

$$\sigma = \frac{J}{E} = q(n\mu_e + P\mu_h) \quad \rule{2cm}{0.4pt} \; 2.3.4$$

In the case of intentionally doped extrinsic maternal, one of the terms in the bracket of equation 2.3.4 is usually negligible compared to the other. In the case of an intrinsic materials, where the number of electrons equals the number of holes.

$$\sigma_i = qn_i(\mu_e + \mu_h) \quad \rule{2cm}{0.4pt} \; 2.3.5$$

The specific conductivity varies with temperature for two main reasons: One is the dependance of the free charge carriers concentration or temperature which will be felt either at very low temperatures (when not all the impurity atoms are ionized) or at very high temperatures (when the rate of generation of thermally created free carriers becomes very high). The second reason is the dependance of mobility on temperature, which has less effect and determines the conductivity in the intermediate range if the doping is high and the material degenerate, the conductivity becomes more or less constant with temperature in the low and intermediate ranges. At low temperatures this results from the ability of the extra electron or hole associated with the dopant atom to drop from one impurity atom to its very near neighbour without the necessity for thermal energy to ionize it.

The mobility obtained from conductivity measurements is called conductivity mobility and refers to majority carrier movement. The mobility of the minority species is usually termed drift mobility, and is approximately equal to the conductivity mobility for the same doping.

2.4 EXCESS CARRIERS AND LIFETIME

The densities of carriers in the semiconductors and determined under thermal equilibrium, that is with temperature uniform and equal to the surrounding ambient temperature, -and with no external carrier injecting mechanisms, such as irradiation by photons, or forces, such as electricfied, applied the homogenuity of the doped semiconductor have been dealt with under this condition, only the temperature, dopant density and the semiconductor materials determine the carrier concentrations, which are uniform (Statistically) in time and position, since there is no reason for them to vary. Equilibrium, however, can be disturbed: carrier concentrations can be increased well above the values appropriate to the temperature. When this happens we say that the semiconductor contains EXCESS CARRIERS.

Let in denote the equilibrium value and n the excess value. Therefore the total carrier concentrations is $n = \bar{n} + n$ ______ 2.4.1a

Similarly for holes, the total carrier concentrations is $P = \bar{P} + P$ ______ 2.4.1b

A common method to generate excess carrier is to irradiate the semiconductor by photons with a wavelength short enough to ionize the valence electrons, thereby generating electron - hole pairs. In such a case $n = P$. However, if the semiconductor is doped, say type N, it has many more electrons than holes, so that the relative importance of the additional excess electrons is much smaller than that of the excess minority holes. The number of these may be increased by several orders of magnitude. One can therefore expect that in any such experiment on extrinsic material most striking effect will be the increase in minority carrier density. The total generation is the sum of the thermal generation G_{lh} and the photon generation G_{ph}. In thermal equilibrium, G_{lh} is balanced by the equilibrium recombination rate given by

$$R(T) = r(T)\,\bar{n}\,\bar{p} = G_{lh}\,(T) \qquad 2.4.2$$

when the semiconductor is not in equilibrium and the irradiation is suddently stopped. Sometimes will pass before the excess carriers recombine and the semiconductor returns to equilibrium. During that time the recombination rate, being proportional to the increased n and p is higher than thermal generation, which is unchanged because the temperature is kept constant. The excess carriers will therefore gradually disappear.

During this period the excess of recombination over generation is

$$R = R - \bar{R}(T) = R - G_{th} = rnp - \bar{r}\bar{n}\bar{p} \;\ldots\; 2.4.3$$

Substituting n, p from 2.4.1, we obtain

$$\hat{R} = r(\bar{p}\hat{n} + \hat{p}\bar{n} + \hat{P}\hat{N}) \quad \rule{4cm}{0.4pt} \quad 2.4.4$$

For N - type extrinsic material, where $\bar{n} \quad N_D$; $n = P \quad \bar{n}$, the only significant term in equation 2.4.4 is the one containing $\bar{n}$:

$$\hat{R} \simeq r\bar{n}\hat{p} \quad \rule{4cm}{0.4pt} \quad 2.4.5$$

This equation shows that excess recombination is proportional to excess minority concentration. If the light, after generating carriers for some time, is turned off at $t = 0$, then the decay of excess minority carriers (here holes) during the time segment dt, t seconds later, will be

$$dp(t) = -\hat{R}(t)\,dt = r\bar{n}\hat{p}\,dt \quad \rule{2cm}{0.4pt} \quad 2.4.6$$

Defining τ_h as

$$\tau_h = \frac{P(t)}{\hat{R}(t)} = \frac{1}{r\bar{n}} = \text{const.}, \quad \rule{3cm}{0.4pt} \quad 2.4.7$$

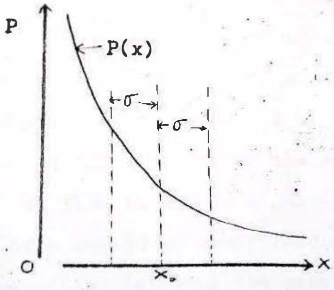
one gets $$\frac{dp}{dt} = -\frac{P}{\tau_h} \quad \rule{4cm}{0.4pt} \quad 2.4.8$$

where τ_h is called the excess holes lifetime. It controls the electronic behaviour of devices such as transistors and diodes at high frequencies and during switching.

2.5 DIFFUSION OF CARRIERS

Diffusion is a manifestation of the random thermal motion of particles; it shows up as a particle current that appears whenever mobile particles are non uniformly distributed in a system. We base our discussion of diffusion on the sketch below, showing the concentration of the carriers, taken to be holes for this development, as a function of the position coordinate x in a one - dimensional semiconductor bar. Consider a surface normal to the x coordinate at x_o

(a)



(b)

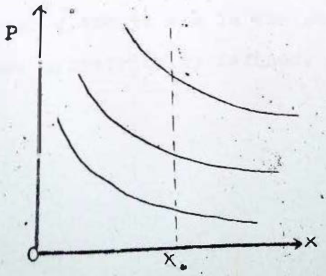


fig (a) Hole concentration

fig (b) Carrier distributions that produce the same diffusive flow at x_o.

For the hole distribution shown in fig (a) above, there is a diffusive flow of holes across this surface in the + x direction. This diffusive flow arises simply because the hole concentration to the left of x_o is greater than the hole concentration to the right of x_o.

We illustrate the dependance of the diffusive flow of carriers on the concentration imbalance by focussing on the carriers in the two bonlike volume elements of cross-sections area A and width 0^- which lie on either side of the plane at x_o. In unit time some fraction of the carriers in the volume element to the left of x_o flow in the + x direction across the plane at x_o as a result of their random thermal motion. In the same time interval the same fraction of the carriers in the volume elements are in the same thermal environment and are symmetrically defined.

There is a net flow of carriers from left to right because there are more carriers in the left volume element than in the right one. The net rate of flow of holes is proportional to the concentration imbalance if P_- denotes the hole contraction in the left volume element and P_+ the concentration in the right volume element, the rate at which holes across the boundary at x_0 is

$$\text{hole flux} = (\text{constant})\,(P_- - P_+) \quad \text{\_\_\_\_\_\_\_} 2.5.1$$

In the limit, as r approaches zero, this rate is proportional to dP/d_x at x_0. That is

$$\text{hole flux} = - D_h \left(\frac{dp}{dx}\right) x_0 \quad \text{\_\_\_\_\_\_\_\_} 2.5.2$$

The constant of proportionality is called the diffusion coefficient for holes. The negative sign appears because carriers flow down the concentration slope from regions of high concentration to regions of lower concentration.

The hole current density associated with diffusion is

$$J_h = - qD_h \frac{dp}{dx} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} 2.5.3$$

The electron current density associated with diffusion is

$$J_e = qDe \frac{dn}{dx} \quad \text{———} \quad 2.5.4$$

These are cases of one - dimensional concentration variations for holes and electrons.
Diffusion flows occurs simply because the number of carriers that have velocity components directed from the region of high concentration towards a region of lower concentration is greater than the number of carriers that have oppositely - directed velocity components.

The particle flux density that results from diffusion depends on the carrier - concentration gradient and not on concentration itself.

2.6 THE TRANSPORT AND CONTINUITY EQUATIONS

In the general case where both concentration gradients and electrical fields are present the current carried by each type of charge carrier has two components: diffusion due to concentration gradients and drift due to electric field. From equations 2.3.3, 2.5.4 and 2.5.4, these current densities are

$$J_h = q\mu_h PE - qD_h \frac{dP}{dx} \quad \text{———} \quad 2.6.1$$

$$J_e = q\mu enE - qDe\,\frac{dn}{dx} \qquad 2.6.2$$

and the total conduction current will be

$$J = J_h + J_e.$$

This is a current - transport equations. It forms a starting point for semiconductor device analysis.

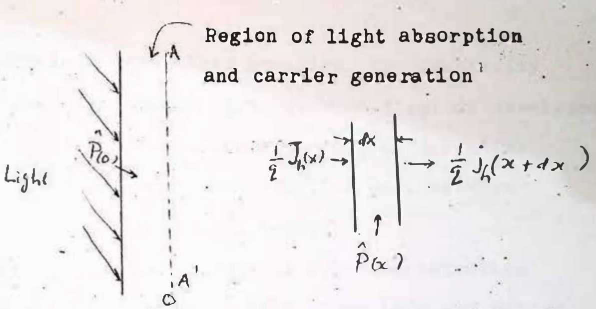

Region of light absorption and carrier generation

Fig. 2.6.1: Hole flows in and out of an infinitesimal region to demonstrate the continuity equation.

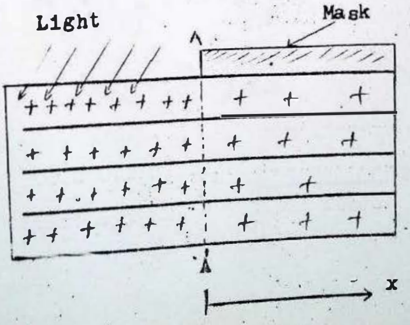


Fig 2.62.

Fig. 2.6.2: Type N Semiconductor, partially exposed to light. Electrons (the majority) are not marked. Holes (the minority) are marked with +.

Referring to fig 2.6.2, how deep the excess carriers can penetrate the covered region before their number is reduced back to equilibrium due to recombination.

In order to solve this question, the continuity equation for the charge carriers must first de developed. From fig. 2.6.1, we examine a narrow, differential section of our unit-are semiconductor bar, between x and x + dx in the covered region.

Equating the time change in hole concentration there to the difference in hole flows into and out of this section, adding the thermally generated new holes and subtracting those recombining, all per unit time, will get us the continuity equation.

$$\frac{dp}{dt}\, dx = \left(\frac{1}{q} J_h(x,t) - \frac{1}{q} J_h(x+dx,t) \right) + \left(G_{th} - \frac{p}{\tau_h} \right) dx \qquad 2.6.3$$

From equilibrium, we know that $G_{th} = \bar{P}/\tau_h$, where $\bar{P}$ is the time - independent equilibrium hole concentration. Substituting this and $P = P + \bar{P}$ into the equation above and dividing throughout by dx yields.

$$\frac{dp}{dt}(x,t) = -\frac{1}{q}\frac{dJ_h}{dx}(x,t) - \frac{P(x,t)}{\tau_h} \qquad 2.6.4$$

Similarly for electrons,

$$\frac{dn}{dt}(x,t) = +\frac{1}{q}\frac{dJ_e}{dx}(x,t) - \frac{n(x,t)}{\tau_e} \qquad 2.6.5$$

Restricting our attention to the static, time - independent, field - free situation of fig. 2.6.2. The concentration in the covered region at a depth x will be governed by equation 2.6.4, which reduced to

$$-\frac{1}{q}\frac{dJ_h}{dx} - \frac{P}{\tau_h} = 0 \qquad 2.6.6$$

substituting J_h from 2.6.1 we obtain

$$\frac{d^2P}{dx^2} - \frac{P}{\tau_h D_h} = 0 \qquad 2.6.7$$

The solution of equation 2.6.7 contains two integration constants:

$$P(x) = C_1 \exp\left(-\frac{x}{(\tau_h D_h)^{\frac{1}{2}}}\right) + C_2 \exp\left(+\frac{x}{(\tau_h D_h)}\right)$$

The quantity $(\tau_h D_h)^{\frac{1}{2}}$ has the dimensions of distance and is called the <u>diffusion length</u> for holes

$$L_h = (\tau_h D_h)^{\frac{1}{2}} \qquad \text{------------} \quad 2.6.8$$

The excess hole concentration drops to e^{-1} times its initial value at a distance of L_h and is practically back to equilibrium after three to four diffusion lengths.

2.7 INTERNAL FIELD IN A SEMICONDUCTOR WITH A NON-UNIFORM DOPING.

All along, only semiconductors, whose impurity doping density is uniform were discussed. Let us examine a case where there is a concentration gradient of the dopant along the X - axis.

If the outside current circuit is left open, then we must have

$$J = J_e + J_h = (0) \qquad \text{------------} \quad 2.7.1$$

If the semiconductor is also in thermodynamic equilibrium, then also the current of each individual carrier type must be zero.

$$J_e = J_h = 0 \qquad \text{———} \; 2.7.2$$

Substituting 2.6.1 and 2.6.2 into 2.7.2, we then find that in the case of a nonuniform, semiconductor at equilibrium, an internal 'built-in' electrical field must exist

$$E = \frac{D_h}{\mu_h} \frac{1}{p} \frac{dp}{dx} = - \frac{D_e}{\mu_e} \frac{1}{\bar{n}} \frac{d\bar{n}}{dx} \qquad \text{———} \; 2.7.3$$

Einstein's relations is written as

$$\frac{D_{e,h}}{\mu_{e,h}} = \frac{KT}{q} \qquad \text{———} \; 2.7.4$$

Applying Einstein's relations

$$E = \frac{KT}{q} \frac{1}{p} \frac{dp}{dx} = - \frac{KT}{q} \frac{1}{\bar{n}} \frac{d\bar{n}}{dx} \qquad \text{———} \; 2.7.5$$

The physical reasons for this internal field, dependent on carrier gradients, are as follows: Assume the dopant concentration, say that of donors, decreases with x. The free electrons contribute by the donors tends to diffuse towards the lower concentration, i.e. in the +x direction. The donors, which are positively ionized, cannot move from their lattice positions.

Consequently, a net positive charge will be created on the left, equal to $q\ (N_D(x) - \bar{n}(x)\ )$, while on the eight, there are more electrons than donors and the net charge will be negative, so that a field cause arise. The diffusion and the field cause two opposing currents and equilibrium is reached when the two effects just balance each other.

CHAPTER THREE

P N JUNCTIONS

3.1 Introductions

A PN junction occurs whenever the impurity concentration changes from a predominance of donors to a predominance of acceptors over a sufficiently small distance. Under such conditions a region develops in which the mobile - carrier concentrations are much smaller than the immobile - impurity concentrations and their local deviations from electrical neutrality, accompanied by intense electric fields. The characteristic behaviour associated with P N junctions is a consequence of these deviations from neutrality, in silicon and Germanium, this change must occur over a distance not significatly greater than about 10^{-7}m to produce a PN junction.

If the change in impurity concentration occurs more gradually, there will be no substantial deviation from neutrality, and the structure will behave simply as a two carrier conductor in which the carrier concentrations vary from point to point.

3.2 THE JUNCTION IN EQUILIBRIUM

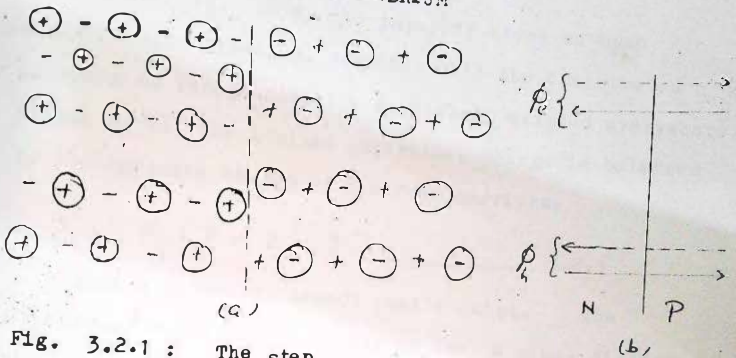


Fig. 3.2.1 : The step junction: (a) the junction vicinity as it would look if the carriers did not diffuse;
(+) ionized donors,
(−) ionized acceptors;
+ holes, − electrons;
(b) electron and hole flows; ------ diffusion flow, ________drift flow;
(c) distribution of ionized impurities and free carriers in equilibrium.

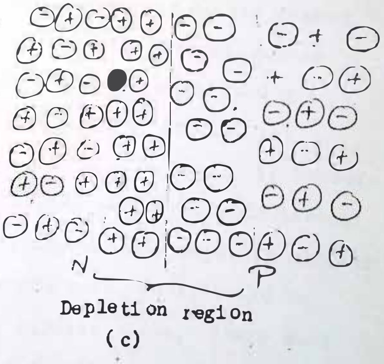


Fig. 3.2.1 (a) shows the impurity atoms on both sides of the junction, together with the contributed carriers as though complete neutrality existed everywhere. At any point the ionized impurities charge is balanced by the opposite charge of the free carriers.

$$N_D + \bar{p} = N_A + \bar{n} \quad \text{———} \quad 3.2.1$$

Such a situation cannot really exist. The electrons, which are in excess on the N side, diffuse into the P side, when there are few of them. Holes diffuse in the opposite direction. A momentary diffusion current flow arises, as marked by the dashed line in fig. 3.2.1 (b). But this flow causes an immediate loss of neutrality; the N side, losing electrons, is charged positive becuase of the net donor charge left behind, the P side similarly becomes negatively charged. A potential barrier builds across the junction which obstructs any further majority carrier diffusion and makes the current zero, as it would be in equilibrium. For each carrier type, there must be two current components, diffusion and drift, which balance each other to zero thermodynamic equilibrium.

A region must therefore exist, on both sides of the metallurgical junction, in which there is a built-in field in equilibrium and in which there is a depletion of mobile carriers, since the field sweeps them away. This region is called the DEPLETION LAYER. It is a space - charge region because of the net ionized impurity charge in it. This is shown in fig 3.2.1(c).

To obtain expressions for the <u>built in field</u> and the width of the depletion layer, we need the transport equations (2.6.1), (2.6.2), that is

$$J_h = q\mu_h PE = qD_h \frac{dp}{dx}$$

$$J_e = q\mu enE - qD_e \frac{dn}{dx}$$

and poisson's equation (in one dimension) which relates the rate of change of the electric field E to the net charge density (and dielectric constant EE_o

$$\frac{dE}{dx} = - \frac{d^2V}{dx2} = \frac{P}{EE_o} \qquad \text{———} \quad 3.2.2$$

where V is the potential. From the discussion above, we must have $J_h = J_e = 0$ in equilibrium.

From $J_h = o$ we get $Edx = \frac{D_h}{\mu_h} \frac{1}{P}dP$ ——————— 3.2.3

Integrating E from P to N side, between points far enough from the junction for its influence to be negligible and for neutrality to hold, where $P_p = N_A$ on the P side and $P_n = n_i^2/N_D$ on the N side, one gets the built - in voltage denoted by V_b.

$$V_B = - \quad Edx = - \frac{D_h}{\mu_h} \quad {}^{N} \frac{1}{P}dp = \frac{D_h}{\mu_h} \ln \frac{P_p}{P_n}$$

$$\therefore V_B = \frac{D_h}{\mu_h} \ln \frac{N_A N_D}{n_i^2} \qquad \text{———————} \quad 3.2.4$$

Similarly, if we start from the $J_e = 0$ equation we obtain similar result

$$V_B = - \quad Edx = \frac{D_e}{\mu_e} \quad \frac{1}{n} dn = \frac{D_e}{\mu_e} \ln \frac{N_A N_D}{n_i^2}:$$

——————— 3.2.5

Since it is the same V_B irrespective of whether (3.2.4) or (3.2.5) is used, we must have

$$\frac{D_h}{\mu_h} = \frac{D_e}{\mu_e} \qquad \text{———————} \quad 3.2.6$$

which is part of Einstein's relations.

The built-in voltage V_B depends on the concentrations of majority carriers on both sides of the junction. Because of n_i^2 in (3.2.5) V_B in Si is so much larger than in Ge, and it is even larger in GaAs (with all at the same temperature).

3.3 The Depletion Region in Equilibrium and under Reverse bias.

In order to find how the potential, electric field and charge depend on x, the distance from the injuction plans, poisson's equation must be used because of the net impurity charge in the depletion region.

$$E_F = E_{F_i} + \frac{KT}{2}\ln\frac{n}{p} \qquad 3.3.1$$

where E_F is fermi energy, E_{F_i} is the fermi level in the intrinsic semiconductor. Let us obtain the poisson's equation. Use of (3.2.7) yields for the N side of the junction.

$$qV_n = E_F - E_{Fi} = \frac{KT}{2}\ln\frac{N_n}{P_n} \qquad 3.3.2$$

where Vn depends on x. Use of $nP = n_i^2$ gives

$$qV_n = KT \ln \frac{N_n}{n_i} \qquad 3.3.3$$

Similarly, for the P side

$$P_p = n_i \exp\left(-\frac{qV_p}{KT}\right) \qquad 3.3.5$$

Equations (3.3.4) and (3.3.5) are called Boltzmann relations. Using them we can express the carrier concentrations as functions of the potentials Vn and Vp. The two separate potentials can be combined into one by taking the intrinsic Fermi and $\mathcal{E}_{F_i}$ at the junction plane x = 0 as the common reference level of zero potential. This combined potential V(x) would then revert to $V_n(x)$ for x o and to Vp (x) O. O The total charge density in the depletion region is therefore:

$$Q = q(N_D + P - N_A - n)$$

$$= q\left(N_D + N_A - 2 n_i \sinh \frac{qV(x)}{KT}\right) \qquad 3.3.6$$

where (3.3.4) and (3.3.5) have been used.

Poisson's equation can now be written for our one dimensional junction

This is a second - order nonlinear differential equation for $V(x)$. To solve it we must know the impurity profiles $N_A(x)$ and $N_D(x)$. Analytic solutions can be obtained only for simple cases such as the ideal step junction where

N_D = Constant $\quad N_A = 0$ for $x \angle 0$

$N_D = 0 \quad N_A$ = Constant for $x > 0$

or for linearly graded junction, where

$N_A = g(x) \quad (x > 0)$ and $N_D = - g(x) \quad (x \angle 0)$,

$g(m^{-4})$ is a positive constant.

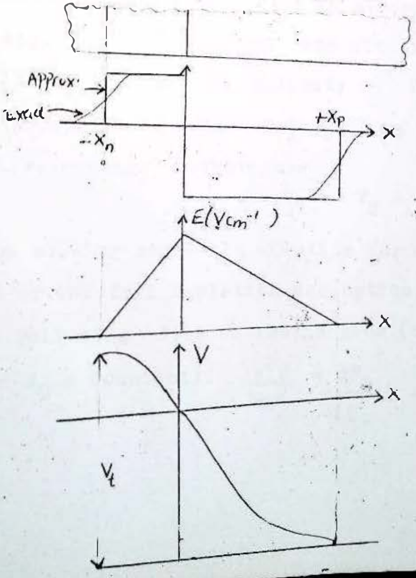


Fig. 3.3.1: The charge field intensity and potential in the depletion region of a step junction (or Abrupt junction).

The boundary condition for (3.3.7) is that the total voltage across the junction V_t from $x = -X_n$ on the N side to $x = X_p$ on the P side, is the sum of the external voltage V_A and the internal built in voltage V_B given by (3.2.5). In equilibrium $V_A = 0$. If an external voltage V_A is applied it changes V_t and with it X_n and X_p. If V_A has the same polarity as V_B (+ to N side, - to P side), it is called Reverse Bias and is then a negative number. Therefore

$$V_t = V_B - V_A$$

Now solving poisson's equation for any V_t. Because of the full depletion assumption, (3.3.7) has the following form on the N side ($n \simeq P \simeq 0$; $N_A = 0$; N_O = constant): $\frac{d^2V}{dx^2} = \frac{qN_D}{\epsilon\epsilon_o}$; $-X_n \leq x \leq 0$ ——3.3.8(a) and on the P side $(n \;\; P = 0;\; N_A = \text{constant};\; N_D = 0)$

$$\frac{d^2V}{dx^2} = \frac{qNA}{EE_o} \;;\; 0 \leq X \leq X_p \quad \text{———} \quad 3.3.8(b)$$

Integrating equations (3.3.8) yields

$$\frac{dV}{dx} = \frac{qN_D}{EE_o} X + C_1 ;\; - X_n \leq X < 0 \quad \text{———} \quad 3.3.9(a)$$

$$\frac{dV}{dx} = \frac{qN_A}{EE_o} X + C_2 ;\; 0 \leq X \leq X_p \quad \text{———} \quad 3.3.9(b)$$

The integration constants C_1 and C_2 may be replaced by expressions involving X_n and X_p because the field $E = - dV/dx$ becomes zero at the depletion edges $x = - X_n$ and $X = + X_p$. Therefore

$$C_1 = - \frac{qN_D}{EE_o} X_n, \quad C_2 = - \frac{qN_A}{EE_o} X_p$$

$$E = - \frac{dV}{dx} = - \frac{qN_A}{EE_o} (X - X_p);\; 0 \leq X \leq X_p \quad \text{———} \quad 3.3.10(b)$$

The field E is shown in fig 3.3.1.

A Second integration yields

$$V = - \frac{qN_D}{EE_o} (X_n X + \frac{X^2}{2}) + D_1 ;\; - X_n \leq X \leq 0 \quad \text{———} \quad 3.3.11(a)$$

$$V = \frac{qN_A}{EE_o} (\frac{X^2}{2} - X_p x) + D_2 ;\; 0 \leq X \leq X_p \quad \text{———} \quad 3.3.11(b)$$

At $X = 0$ the two equations yield the same potential. Therefore $D_1 = D_2$. Since we took $X = 0$ as the reference plane for zero potential we also have $D_1 = D_2 = 0$. The potential $V(x)$ is also shown in fig 3.3.1.

The potential difference V_t is given by that from $X = - X_n$ to $X = + X_p$, since it is only in this region that there is a field.

$$V_t = V(-X_n) - V(+X_p)$$

$$= \frac{q}{2EE_o}(N_D X_n^2 + N_A X_p^2) \qquad \text{———} \quad 3.3.12$$

In order to obtain X_n and X_p separately, we impose a condition that the net positive charge on the N side equals the net negative on the P side, or that the field at $X = 0$, obtained from 3.3.10(a), is the same as that obtained from 8.15(b) (which by Gauss's law amounts to the same thing).

$$\frac{Q}{A} = qN_D X_n = qN_A X_p \qquad \text{———} \quad 3.3.13$$

where A is the junction area, the charge Q is shown in fig. 3.3.1.

For e.g (3.3.12) and (3.3.13) X_n and X_p can be obtain:

$$X_n = \left\{ \frac{2E_oE}{q} V_t \frac{N_A}{N_D(N_A + N_D)} \right\}^{\frac{1}{2}} \quad \text{———} \quad 3.3.14(a)$$

$$X_p = \left\{ \frac{2EE_o}{q} V_t \frac{N_D}{N_A (N_A + N_D)} \right\}^{\frac{1}{2}} \quad \text{———} \quad 3.3.14(b)$$

The total length of the depletion layer is

$$X = X_n + X_p = \left\{ \frac{2EE_o V_t}{q(N_A + N_D)} \right\}^{\frac{1}{2}} \left[\left(\frac{N_A}{N_D} \right)^{\frac{1}{2}} + \left(\frac{N_D}{N_A} \right)^{\frac{1}{2}} \right]$$

It should be noted that X_n and X_p are proportional to $V_t^{\frac{1}{2}}$, which is characteristic of a step junction.

The maximum field occurs at the metallugical junction, at $X = 0$;

$$E_{max} = \frac{qN_D X_n}{EE_o} = \frac{qN_A X_p}{EE_o}$$

$$= \left(\frac{2qV_t}{EE_o} \frac{N_A N_D}{N_A + N_D} \right)^{\frac{1}{2}} .$$

3.4 THE JUNCTION UNDER FORWARD BIAS

If two ohmic contacts are attached to the opposite sides of the P and N regions and an external positive voltage V_A applied, the total potential barrier is reduced to $V_t = V_B - V_A$ ———— 3.4.1

This is forward - bias connection and it reduces the energy-bands bending.

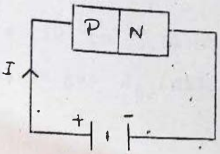


Fig. 3.4.1: The forward bias junction connection.

Even a small reduction of the barrier V_R by V_A imbalances the drift and diffusion components in the Je and Jn expressions, the drift being reduced and the diffusion inchanged. A net current I now flows in the circuit of fig. 3.4.1 because of diffusion of majority carriers. Let us examine the changes in minority carrier concentrations at the boundaries of the depletion layer. The imbalanced components in the current equations (2.6.1) and (2.6.2) are two very large quantities. Taking (2.6.1), for example, they are qU_hPE and $qD_h dp/dx$.

One can approximate dp/dx by $(Pn^- Pp)/Dx$, where Dx is the width of the depletion layer

$$J_h \text{ (diffsn)} - qD_h \frac{P_n - P_p}{Dx} \quad n \quad qD_h \frac{N_A}{Dx} \qquad \underline{\qquad} 3.4.2$$

Since $P_p \simeq N_A$ and $P_n = n_i^2 / N_D \ll N_A$.

Suppose, in equilibrium; $N_A = 10^{18} cm^{-3}$, $Dx = 10^{-4} cm$, and assuming $D_h = 15 cm^2 S^{-1}$ in Si, we get J_h (diffsn) $= 24000 A cm^{-2}$.

This very high current density is exactly balanced by

$$J_h \text{ (drift)} = qU_h E\bar{P} \quad \text{———} \quad 3.4.3$$

flowing in the opposite direction. With two such large quantities only a very small unbalance is necessary to obtain a sizable net current. We restrict our attention to such small umbalnces and we assume that the functional relationship between E and P or n remains the same in forward bias as it was in equilibrium when $Je = J_h = 0$. Equoations (2.6.1) and (2.6.2) then yield respectively.

$$E = \frac{D_h}{U_h} \frac{1}{P} \frac{dP}{dx} = \frac{KT}{q} \frac{1}{P} \frac{dP}{dx} \quad \text{———} \quad 3.4.4(a)$$

$$E = -\frac{De}{e} \frac{1}{n} \frac{dn}{dx} = -\frac{KT}{q} \frac{1}{n} \frac{dn}{dx} \quad \text{———} \quad 3.4.4(b)$$

Integrating from P side to the N side boundary of the depletion layer will yield the total voltage V_t as before:

$$V_t = V_B - V_A = -\int_P^N E dx$$

$$= -\frac{KT}{q} \quad \frac{1}{p}dp = \frac{KT}{q}\ln\frac{P_p}{P_n} \qquad \text{——— 3.4.5(a)}$$

$$V_t = \frac{KT}{q} \quad \frac{1}{n}dn = \frac{KT}{q}\ln\frac{N_n}{N_p} \qquad \text{——— 3.4.5.(b)}$$

But n_n and P_p are the majority concentrations at the depletion boundaries, and provided the forward bias is not too high, they are not affected by the flow and retain their equilibrium values, namely $n_n \quad N_D$ and $P_p \quad N_A$. Equations 3.4.5 can be used to obtain the enhanced monority concentrations at the depletion boundaries:

$$\left.\begin{aligned} P_n &= P_n \exp\left(\frac{-qV_t}{KT}\right) = N_A \exp\left(-\frac{qV_B}{KT}\right) \exp\frac{qV_A}{KT} \\ n_p &= n_n \exp\left(-\frac{qV_t}{KT}\right) = N_D \exp\left(-\frac{qV_B}{KT}\right) \exp\frac{qV_A}{KT} \end{aligned}\right\} \text{——— 3.4.6}$$

For positive V_A the minority concentrations are therefore increased. This is called carrier injection.

The excess minority concentrations near the depletion boundary must gradually decrease as distance from the junction increases, because they recombine with the majorities.

The carrier injection is called low when the increased minority concentration remains at least an order of magnitude lower than the majority on that side. Under low injection the majority concentration are practically unchanged from their equilibrium values.

The situation is different in the case of high injection, that is, when the forward bias voltage V_A is high enough to increase the excess minority on at least one side of the junction to the same order of magnitude as the majority on that side. In this case an appreciable field will build up in the bulk semiconductor on that side, taking up part of the externally applied voltage V_A so that the junction voltage is no longer $V_B - V_A$.

CHAPTER FOUR

NUMERICAL ANALYSIS FOR SEMICONDUCTOR DEVICES.

The semiconductor equations consist of a set of partial differential equations which must be solved subject to a pre-defined set of boundary conditions over a specified domain. Although generalized solutions are not available for all devices there are many analytical solutions available for a wide variety of devices. They are severely limited in their range of application and acuracy because of the multi-dimensional non-linear nature of most modern devices. A more generalized method of solution frequently applied to the semiconductor equations is to solve them using numerical techniques.

4.1 ANALYSIS OF A P N JUNCTION

A P N junction perform a wide variety of functions such as current rectification, amplification, switching, and oscillation. Hence a P N junction serves as a basic junction unit in constituting a semicinductor device. The two most common forms of a P N junction are the abrupt junction and a linearly graded junction.

Semiconductor device modelling requires that every semiconductor device problem should be solved by obtaining solutions for the full set of semiconductor equations.

Lets consider poisson's equation in one-dimension.

$$\frac{d^{2}\psi}{dx^{2}} = - \frac{q}{E} (N_d - N_a + P - n) \qquad \text{———} \quad 4.1.1$$

In this case of a linearly graded junction, the net impurity concentration is given by ;

$$N_d - N_a = mx \qquad \text{———} \quad 4.1.2$$

The hole - current equation

$$J_p = - q\mu_p P \text{ grad } \phi_p$$

reduces in the one - dimensional case to

$$\frac{d\phi_p}{dx} = - \frac{J_p}{q\mu_p P} \qquad \text{———} \quad 4.1.3$$

Unless the hole density P is extremely low, the gradient in ϕ_p is essentially zero for non-significant J_p. Thus, if voltage V is applied across the P N junction, then ϕ_p is flat and equal to V for most of the P - region. Similarly, ϕ_n is shown to be flat and equal to zero for most of the n region.

It must be stated here that the thermal - equilibrium conditions $\phi = \phi_n = V$ (P - region) and $\phi_p = \phi_n = 0$ (n - region) hold at locations that are sufficiently apart from the junction.

The election and hole densities in the depletion layer are written as;

$$n = n_i e^{Q\psi}, \quad P = n_{ie}Q(V - \psi) \qquad \text{———— } 4.1.4$$

where $Q = q/KT$

Substituting equations (4.1.2) and (4.1.4) into equation (4.1.1) we obtain

$$\frac{d^2\psi}{dx^2} = -\frac{q}{E}\left\{mx + n_{ie}Q(V-\psi) - n_i e^{Q\psi}\right\} \text{— } 4.1.5$$

Boundary conditions for ψ are established at locations that are sufficiently far from the junction located at $X = 0$ so that the space charge neutrality holds with good approximation locations for these points are determined as $X = \pm a$. Charge neutrality at these points are expressed as;

$$- ma' + n_i e \; Q\left(V-\psi(-a') \right) - n_{ie}^{\;Q\left\{\psi(-a')-V\right\}} = 0$$

$$\text{————— } 4.1.6(a)$$

and

$$ma' + n_i e^{-Q\psi(a')} = 0 \qquad \text{———— 4.1.6(b)}$$

Simultaneously, the thermal - equilibrium conditions are assumed at these points; that is $\phi_p(-a') = \phi_n(-a') = V$, and $\phi_p(a') = \phi_n(a') = 0$

Substituting this equation into equation (4.1.6) we obtain explicit formulas for $\psi(\pm a')$:

$$\psi(-a') = \frac{1}{Q}\ln\left\{\sqrt{\left(\frac{ma'}{2n_i}\right)^2 + 1} - \frac{ma'}{2n_i}\right\} + V \qquad \text{———— 4.1.8(a)}$$

$$\psi(a') = \frac{1}{Q}\ln\left\{\sqrt{\left(\frac{ma'}{2n_i}\right)^2 + 1} + \frac{ma'}{2n_i}\right\} \qquad \text{———— 4.1.8 (b)}$$

Neglecting the free carrier terms, equations (4.1.5) becomes

$$\frac{d^2\psi}{dx^2} = -\frac{q}{E}\,mx \; ; \; -a \leqslant x \leqslant a \qquad \text{———— 4.1.9}$$

where 'a' is the extent of the depletion layer. Integrating equations (4.1.9) yields:

$$\frac{d\psi}{dx} = -\frac{qm}{2E}\left(x^2 - a^2\right) \qquad \text{———— 4.1.10}$$

Again, integrating equation (4.1.10) leads to

$$\psi(x) = \frac{qm}{6E}\left(x^3 - 3a^2x - 2a^3\right) + \psi(-a) \qquad \text{———— 4.1.11}$$

A maximum electric field is located at $x = 0$, whose absolute value is given by;

$$E_{max} = \frac{d\psi}{dx}\ (x = 0) = \frac{qm}{2E}a^2 \quad \text{———} \quad 4.1.12$$

Putting x = a in equation 4.1.11 and replacing X = ± a' by ±a in equation 4.1.8 we obtain

$$\frac{2qm}{3\varepsilon}a^3 = \frac{1}{Q}\ln\left[\frac{\sqrt{(ma/2n_i)^2+1}+ma/2n_i}{\sqrt{(ma/2n_i)^2+1}-ma/2n_i}\right] - V \quad \text{-----} \quad 4.1.13$$

This is a nonlinear equation for a. It can be solved numerically using, for example, Newton-Raphson's iteration method. From the solution, we shall obtain the extent of the depletion layer under nonsignificant - current assumption

4.2 NEWTON - RAPHSON ITERATION PROCEDURE.

This method is a second order iteration procedure for simple roots.

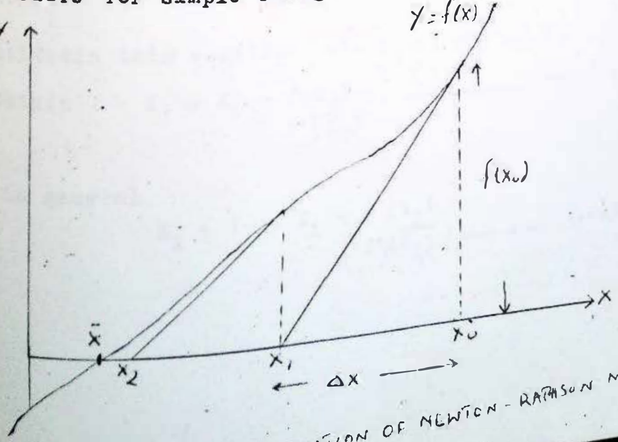


Fig 4·2·1 : DERIVATION OF NEWTON - RAPHSON METHOD

Fig. 4.2.1 Shows the original equation $F(x) = 0$; the desired root $\tilde{x}$ is the x coordinate where the curve $f(x)$ crosses the x axis.

We start by picking an initial guess value X_0. To get a new value X_1, we draw the tangent to the curve at the point $x = x_0$, $y = f(x_0)$ and follow the tangent down to its intersection with the x axis. To get x_2 we repeat the process, starting from x_1.

$$X_1 = X_o - D_x \qquad 4.2.1$$

The slope of the tangent to the curve at the point $(X_{0,}\ f(X_{01})$ is

$$f'(X_o) = \frac{f(X_o)}{D_x}$$

So that

$$D_x = \frac{f(X_o)}{f'(X_o)} \qquad 4.2.2$$

Substitutin this equation into equation 4.2.1,

we obtain

$$X_1 = X_o - \frac{f(X_o)}{f'(X_o)} \qquad 4.2.3(a)$$

or in general

$$X_1 + 1 = X_1 - \frac{f(X_1)}{f'(X_1)} \qquad 4.2.3(b)$$

This is Newton Raphson iteration formula

We can now apply this iteration procedure to equation (4.1.13)

$$\frac{2qm}{3E} a^3 = \frac{1}{q} \ln \frac{\sqrt{\left(\frac{ma}{2m_1}\right)^2 + 1} + \frac{ma}{2m_1}}{\sqrt{\left(\frac{ma}{2m_1}\right)^2 + 1} - \frac{ma}{2m_1}} - V$$

Let $C = \frac{2qm}{3E}$, $D = \frac{m}{2m_1}$.

Equation (4.1.13) becomes

$$Ca^3 = \frac{1}{q} \ln \left[\frac{\sqrt{(Da)^2 + 1} + Da}{\sqrt{(Da)^2 + 1} - Da} \right] - V \qquad \text{———} \ 4.2.4$$

Using Binomial expansion

$$(1 + x^2)^{\frac{1}{2}} = 1 + \frac{1}{2} x - \frac{1}{8} x^2 + \frac{3}{48} x^3 + \ldots\ldots\ldots \qquad 4.2.5$$

for X 1.

$(1 + (Da)^2)^{\frac{1}{2}}$ can be rewritten as

$$(1 + (Da)^2)^{\frac{1}{2}} \quad Da \left(1 + \frac{1}{(Da)^2} \right)^{\frac{1}{2}}$$

Let $\left(\frac{1}{Da}\right)^2 = R$ and $Da = S$

$S(1 + 12)^{\frac{1}{2}}$ can be expanded using Binomial expansion

as $$S(1 + R)^{\frac{1}{2}} = S \left(1 + \frac{1}{2}R - \frac{1}{8}R^2 + \frac{3}{48}R^3 + \ldots\ldots \right)$$

Neglecting higher terms are obtain

$$S(1 + R)^{\frac{1}{2}} = S\left(1 + \tfrac{1}{2}R\right)$$

$$= Da\left(1 + \frac{1}{2}(Da)^2\right.$$

Therefore

$$\left(1 + (Da)^2\right)^{\frac{1}{2}} = Da + \frac{1}{2(Da)} \qquad \text{\_\_\_\_\_\_} \quad 4.2.6$$

Substituting this equation into (4.2.4) yields

$$Ca^3 = \frac{1}{Q}\ln \frac{2(Da) + \frac{1}{2(Da)}}{2(\frac{1}{Da})} - V$$

$$= \frac{1}{Q}\ln\left(4(Da)^2 + 1\right) - V \qquad \text{\_\_\_\_\_\_} \quad 4.2.7$$

where $Q = q/KT$

Equation (4.2.7) is the simplified form of equation (4.1.13).

$$\frac{1}{Q}\ln\left(4(Da)^2 + 1\right) - Ca^3 - V = G(a) \qquad \text{\_\_\_\_} \quad 4.2.8$$

Differntiating equation (4.2.8) with respect to a, one obtains

$$G3(a) = \frac{dG(a)}{da} = \frac{1}{Q}\,\frac{8D^2a}{4D2_a{}^2 + 1} - 3Ca^2 \qquad \text{\_\_\_\_} \quad 4.2.9$$

From equation 4.2.3

$$a_1 = a_0 - \frac{\frac{1}{Q}\ln\left(4D^2a_0{}^2 + 1\right) - Ca_0{}^3 - V}{\frac{1}{Q}\,\frac{8D^2a_0}{(4D2_{a_0}{}^2 + 1)} - 3Ca_0{}^2}$$

4.3 COMPUTER PROGRAMS AND THE COMPUTED OUTPUT RESULTS

Three programs were written at different biasing voltage. The aim of the programs are to determine the widths of the depletion layer of a p - n junction diode with respect to the biasing voltage. Newton-Raphson iteration method was used to solve the equation

$$Ca^3 = \frac{1}{Q} \ln \left\{ 4(Da)^2 + 1 \right\} - V \qquad \text{—— 4.3.1}$$

The constants C, D and Q were introduced in the programs, where $C = \frac{2qm}{3E}$, $D = M/2ni$, $Q = q/KT$

An initial guess value of the depletion width 1.942×10^{-5} was used. An appropriate Do loop was used to ensure that the stipulated conditions were not exceeded.

$Y = A - G/G3$ in the Newton - Raphson iterative formula, where Y is the width of the depletion layer. Statement 35 in the programs ensures that the derivative of equation (4.3.1) is never zero else it goes back to $A = A + 1.0 \times 10^{-7}$.

Z is the difference between the new value of the depletion width and the guess value. If the difference is less than or equal to 1.0×10^{-8} then the new value

Y is equal to the old value A.

The last WRITE statements writes the values of the depletion layer width Y and the applied potentials. The procedure continues until the DO loop is exhausted. The statment STOP terminates the iteration and it implies that the values have converged.

The computer programs and the obtained results are shown on the next page.

PROGRAM 1: ZERO BIAS VOLTAGE CONDITION

```
NEWTON RAPHSON METHOD FOR FINDING 'A' FROM THE EQUATION C*A**3=
ALOG(4.0*(D*A)**2+1)/Q
Q=q1/K*T,C=2.0*q1*M/3.0*E,D=M/2.0*n1
FELIX.W.BURARI 64425

REAL M,n1,K,E,q1,T,A
OPEN(UNIT=3,FILE='FEL.OUT')
DATA q1,T,M,n1,E,K,A/1.602E-19,300.,1.0E21,1.40E10,1.064E-12,
*1.38E-23,1.942E-5/
D=3.57E10
C=1.0E14
Q=38.67
DO 70 I=1,10
V=(0.0)*(I-1)
WRITE(3,20)A
FORMAT(1H ,'THE ITERATION VALUE OF A=',E10.4)
A=A+1.0E-7
C1=4.0*(D*A)**2+1
B1=ALOG(C1)
B=B1/Q
G=B-C*A**3
G1=8.0*(D**2)*A
G2=G1/C1*Q
G3=G2-3.0*C*A**2
WRITE(3,30)A
FORMAT(1H ,'THE ITERATION VALUE OF A=';E10.4)
IF(G3.EQ.0.0)GO TO 25

PERFORMANCE OF NEWTON RAPHSON'S ITERATION PROCEDURE

Y=A-G/G3
Z=Y-A
IF(ABS(Z).LE.1.0E-8)GO TO 60
Y=A
WRITE(3,65)Y
WRITE(*,65)Y
FORMAT(1H ,'THE WIDTH OF THE DEPLETION LAYER Y=',E10.4)
CONTINUE
STOP
END
```

```
SER.NO.        ITERATED DEPLETION WIDTH
 1.0               1.942E-5

 2.0               1.963E-5

 3.0               1.973E-5

 4.0               1.982E-5

 5.0               1.992E-5

 6.0               2.002E-5

 7.0               2.012E-5

 8.0               2.022E-5

 9.0               2.032E-5

10.0              2.042E-5
```

PROGRAM II: FORWARD BIAS VOLTAGE CONDITION

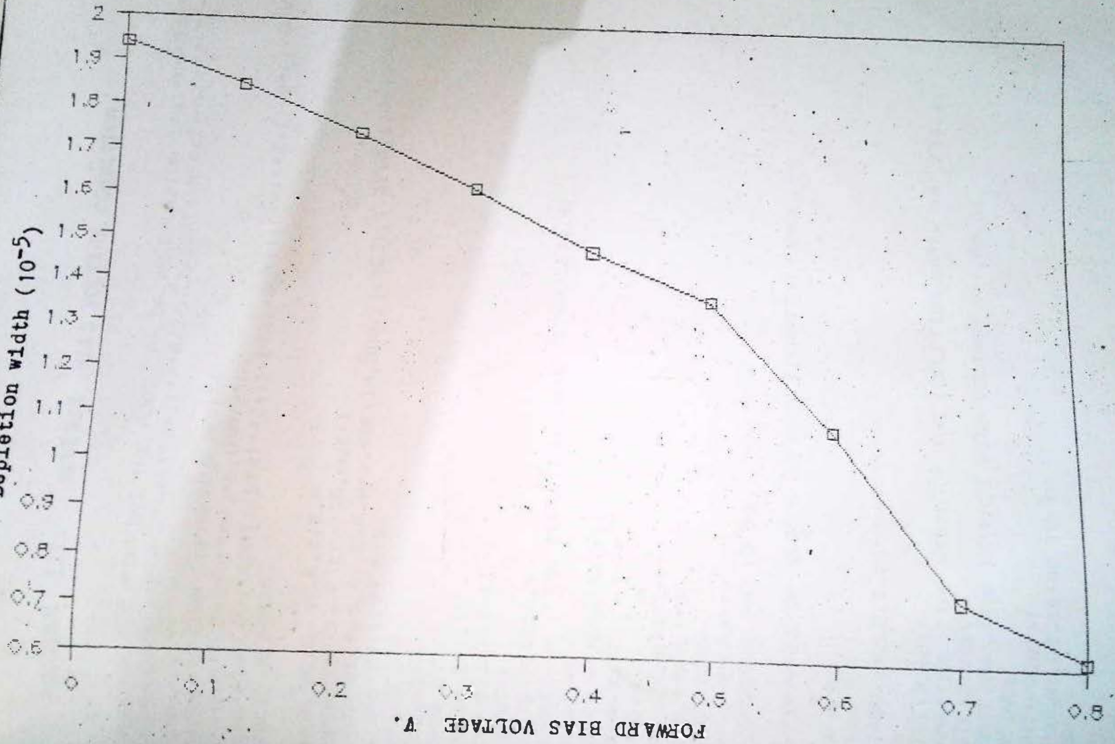
```
 PROGRAM FOR FINDING THE ROOT 'A' OF AN EQUATION 2*q1*M**3/3*E
 ALOG((SQRT((M*A/2*ni)**2+1)+M*A/2*ni)/SQRT((M*A/2*ni)**2+1)-M*A/
 2*ni)/Q-V
 USING NEWTON RAPHSON'S METHOD
 Q=q1/K*T,C=2*q1*M/3*E,D=M/2*ni
 C*A**3=ALOG((SQRT((D*A)**2+1)+D*A)/SQRT((D*A)**2+1)-D*A)/Q-V
 FELIX W BURARI

  REAL M,ni,q1,K,T,E,A
  OPEN(UNIT=3,FILE='BR.OUT')
  DATA q1,T,M,ni,E,K,A/1.60E-19,300.,1.0E21,1.4E10,1.06435E-12,
 *1.38E-23,1.942E-8/
  D=3.57E10
  C=1.0E14
  Q=38.67
10 DO 80 I=1,10
  V=(0.1)*(I-1)
  KOUNT=1
  WRITE(3,20)A,V
20 FORMAT(1H ,'THE ITERATION VALUE OF A=',E10.4)
30 A=A+1.0E-7
  C1=(4.0*(D*A)**2+1)
  B1=ALOG(C1)
  B=B1/Q
  G=B-((C*A**3)+V)
  G1=8.0*A*D**2
  G2=G1/C1*Q
  G3=G2-3*C*A**2
  IF(G3.EQ.0.)GO TO 30

 PERFORMANCE OF NEWTON RAPHSON ITERATION PROCEDURE

  Y=A-G/G3
  Z=Y-A
  IF(ABS(Z).LE.1.0E-8)GO TO 50
  Y=A
50 WRITE(3,60)Y
60 FORMAT(1H ,'THE WIDTH OF THE DEPLETION LAYER Y=',E10.4)
  WRITE(3,70)V
70 FORMAT(1H ,'THE FORWARD BIAS VOLTAGE V=',F8.3)
80 CONTINUE
  KOUNT=KOUNT+1
  IF(KOUNT.GT.10)GO TO 90
90 STOP
  END
```

```
SER.NO.  WIDTH OF DEPLETION LAYER 'Y' FORWARD BIAS VOLTAGE 'V'
              (1.0E-5)
  1.0          1.942
  2.0          1.8506                        0.0
  3.0          1.7470                        0.1
  4.0          1.6286                        0.2
  5.0          1.4898                        0.3
  6.0          1.3819                        0.4
  7.0          1.0931                        0.5
  8.0          0.73468                       0.6
  9.0          0.61851                       0.7
                                             0.8
```


Depletion width (10^-5)
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
FORWARD BIAS VOLTAGE V.

PROGRAM III: REVERSE BIAS VOLTAGE CONDITION.

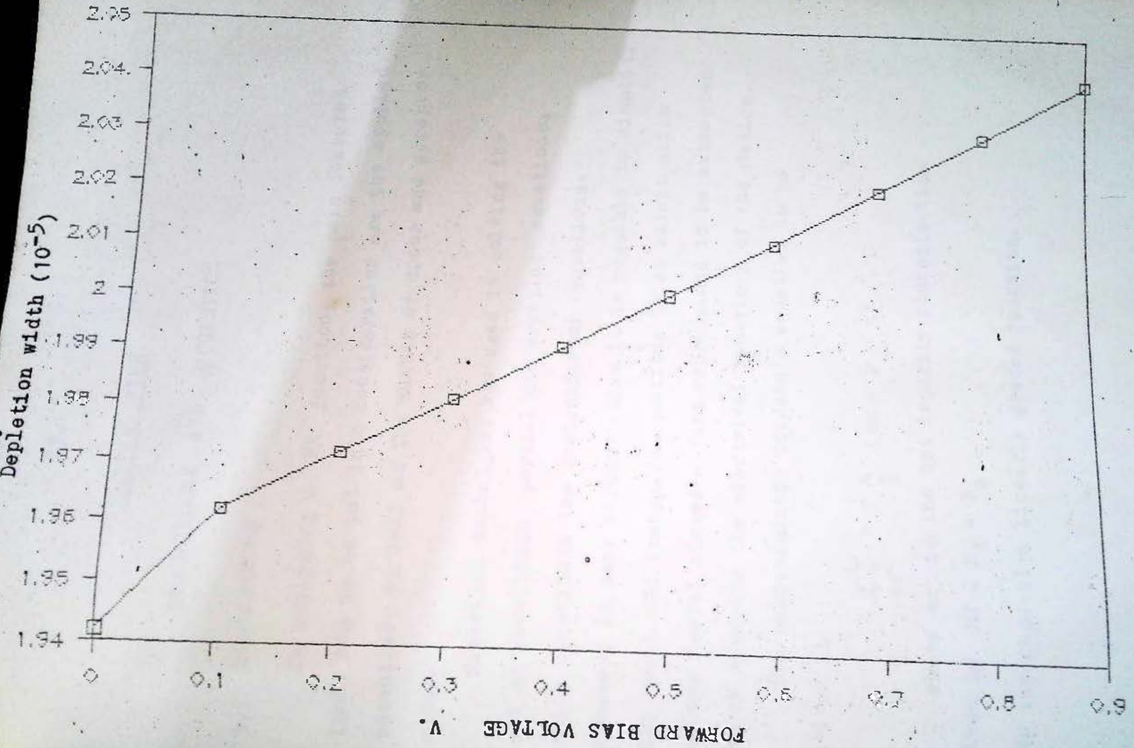
```
   PROGRAM FOR FINDING THE ROOT 'A' OF AN EQUATION 2*q1*M**3/3*E
   ALOG((SQRT((M*A/2*ni)**2+1)+M*A/2*ni)/SQRT((M*A/2*ni)**2+1)-M*A/
   2*ni)/Q-V
   USING NEWTON RAPHSON'S METHOD
   Q=q1/K*T,C=2*q1*M/3*E,D=M/2*ni
   C*A**3=ALOG((SQRT((D*A)**2+1)+D*A)/SQRT((D*A)**2+1)-D*A)/Q-V
   FELIX W BUKARI

   REAL M,ni,q1,K,T,E,A
   OPEN(UNIT=3,FILE='BR.OUT')
   DATA q1,T,M,ni,E,K,A/1.60E-19,300.,1.0E21,1.4E10,1.06435E-12,
  *1.38E-23,1.942E-5/
   D=3.57E10
   C=1.0E14
   Q=38.67
10 DO 80 I=1,10
   V=(-0.1)*(I-1)
   KOUNT=1
   WRITE(3,20)A,V
20 FORMAT(1H ,'THE ITERATION VALUE OF A=',E10.4)
30 A=A+1.0E-7
   C1=(4.0*(D*A)**2+1)
   B1=ALOG(C1)
   B=B1/Q
   G=B-((C*A**3)+V)
   G1=8.0*A*D**2
   G2=G1/C1*Q
   G3=G2-3*C*A**2
   IF(G3.EQ.0.)GO TO 30

   PERFORMANCE OF NEWTON RAPHSON ITERATION PROCEDURE

   Y=A-G/G3
   Z=Y-A
   IF(ABS(Z).LE.1.0E-8)GO TO 50
   Y=A
50 WRITE(3,60)Y
60 FORMAT(1H ,'THE WIDTH OF THE DEPLETION LAYER Y=',E10.4)
   WRITE(3,70)V
70 FORMAT(1H ,'THE REVERSE BIAS VOLTAGE V=',F8.3)
80 CONTINUE
   KOUNT=KOUNT+1
   IF(KOUNT.GT.10)GO TO 90
90 STOP
   END
```

```
SER.NO.   WIDTH OF THE DEPLETION LAYER Y  REVERSE BIAS VOLTAGE V
             (1.0E-5)
 1.0            1.942                          0.0
 2.0            1.962                          0.1
 3.0            1.972                          0.2
 4.0            1.982                          0.3
 5.0            1.992                          0.4
 6.0            2.002                          0.5
 7.0            2.012                          0.6
 8.0            2.022                          0.7
 9.0            2.032                          0.8
10.0            2.042                          0.9
```


Depletion width (10^{-5})
2.05
2.04
2.03
2.02
2.01
2
1.99
1.98
1.97
1.96
1.95
1.94
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
FORWARD BIAS VOLTAGE V.

CHAPTER FIVE

DISCUSSIONS AND SUGGESTION

5.1 Discussions

In analysing a PN junction, the main features that has to be put into consideration are the applied potentials as well as the number of donor and acceptor atoms.

Numerical techniques are used in solving the set of non-linean partial differential equations, which constitute the semiconductor equations. However, in many circumstances it is possible to simplify the model and transport equations to an extent which allows useful closed - form expressions to be extracted which describe the electrical behaviour of the device.

The semiconductor poisson's equation to be solved is

$$\frac{d^2\psi}{dx^2} = - \frac{q}{E} \ (mx + P - n)$$

where mx is the net impurity concentration given by $MX = N_d - N_a$

for the case of a linearly graded junction.

The appropriate boundary conditions were imposed on the differential equation and the necessary substitution were made. We then obtain an expression

$$\frac{2qm_a3}{3E} - \frac{1}{Q}\ln\left[\frac{\sqrt{(ma/2ni)^2+1} + ma/2ni}{\sqrt{ma/2ni)^2+1} - ma/2ni}\right] - V$$

The equation is non-linear in a.
However, this form of the equation may be very difficult to solve, hence need for further simplification.

$$Ca^3 = \frac{1}{Q}\ln\ (4\ (Da)^2 + 1\ ) - V$$

where the parameters C, D and Q have been defined in the previous chapter. The above equation was solved using Newton - Raphson Iteration method.

It was observed, however, that numerical integration tends to diverge rather than to converge unless the electric - field magnitude at the junction point is given an extraordinary accuracy. Even then, the solution is obtained only for a very localized region around the junction point, which suggest an unpromising future for numerical device analysis.

5.2 SUGGESTION

Semiconductor devices are modelled using electrical circuit models and physical device models. The importance of physical device modelling will increase as the demand for smaller and more complex device continues.

Although a lot of research has been done on device modelling, what remains is the need for a high performance computers to be able to yield solutions to a number of sophisticated new devices.

I suggest that researchers should use the knowledge of numerical analysis in contributing to predicting characteristics of some of the devices that will appear in the near future.

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APPENDIX 1
NUMERICAL SOLUTION OF THE CURRENT CONTINUITY EQUATION

The current continuity equation solutions are used to derive the electron and hole distributions n^{k+1} and p^{k+1} in terms of $n^k, p^k, \mu_n^k, \mu_p^k, E^k, D_n^k$ and D_n^k, where the superscript k is the time step index. Two-dimensional current continuity algorithms based on linearised and Scharfetter-Gummel schemes are described below.

A.1 Linearised Continuity Scheme

The linearised semi-implicit time-dependent continuity scheme for electrons for a uniform mesh is,

$$\frac{n_{i,j,k+1} - n_{i,j,k}}{\Delta t} = \frac{1}{2q}\left[\nabla J_{i,j}(n_{k+1}, \mu_k E_k) + \nabla J_{i,j}(n_k, \mu_k E_k)\right] \qquad (A.1.1)$$

Δt is the time step and q the electronic charge. Expanding this expression,

$$\begin{aligned} n_{i,j,k+1,L+1} = \frac{\Delta t}{2q}\Bigg[& \frac{J_{x_{i+1/2},j}(n_{k+1,L}, \mu_k E_{x,k}) - J_{x_{i-1/2},j}(n_{k+1,L}, \mu_k E_{x,k})}{\Delta x} \\ & + \frac{J_{y_{i,j+1/2}}(n_{k+1,L}, \mu_k E_{y,k}) - J_{y_{i,j-1/2}}(n_{k+1,L}, \mu_k E_{y,k})}{\Delta y} \\ & + \frac{J_{x_{i+1/2},j}(n_k, \mu_k E_{x,k}) - J_{x_{i-1/2},j}(n_k, \mu_k E_{x,k})}{\Delta x} \qquad (A.1.2) \\ & + \frac{J_{y_{i,j+1/2}}(n_k, \mu_k E_{y,k}) - J_{y_{i,j-1/2}}(n_k, \mu_k E_{y,k})}{\Delta y}\Bigg] + n_{i,j,k} \end{aligned}$$

where the superscript L refers to the iteration index in calculating n^{k+1}, k is the time step index, i and j are the x and y spatial indices. Re-expressing this equation by substituting for J_x and J_y and incorporating under-relaxation, the implicit continuity scheme becomes,

$$n_{i,j,k+1,L+1} = \omega\left[\frac{\Delta t}{2}\left\{A_{i,j}n_{i+1,j,k+1,L} + B_{i,j}n_{i,j+1,k+1,L+1} + C_{i,j}n_{i-1,j,k+1,L+1}\right.\right.$$

$$\left.\left. + F_{i,j}n_{i,j-1,k+1,L+1} + G_{i,j}n_{i,j,k+1,L} + H_{i,j}\right\} + n_{i,j,k}\right].$$

$$- (\omega - 1)n_{i,j,k+1,L}. \tag{A.1.3}$$

where ω is the relaxation factor, which is less than unity for successive-under-relaxation (SUR). The factors in equation (A.1.3) are given by,

$$A_{i,j} = \frac{(a_{x_{i,j}} + b_{x_{i,j}})}{2\Delta x^2}$$

$$B_{i,j} = \frac{(a_{y_{i,j}} + b_{y_{i,j}})}{2\Delta y^2}$$

$$C_{i,j} = \frac{(b_{x_{i-1,j}} - a_{x_{i-1,j}})}{2\Delta x^2}$$

$$F_{i,j} = \frac{(b_{y_{i,j-1}} - a_{y_{i,j-1}})}{2\Delta x^2} \tag{A.1.4}$$

$$G_{i,j} = \frac{(a_{x_{i,j}} - a_{x_{i-1,j}} - b_{x_{i,j}} - b_{x_{i-1,j}})}{2\Delta x^2}$$

$$+ \frac{(a_{y_{i,j}} - a_{y_{i,j-1}} - b_{y_{i,j}} - b_{y_{i,j-1}})}{2\Delta y^2}$$

$$H_{i,j} = \frac{(c_{x_{i,j}} - c_{x_{i-1,j}})}{2\Delta x^2} + \frac{(c_{y_{i,j}} - c_{y_{i,j-1}})}{2\Delta y^2}$$

and

$$a_{x_{i,j}} = 1/2(\psi_{i,j} - \psi_{i+1,j})(\mu_{i,j} + \mu_{i+1,j})$$

$$a_{y_{i,j}} = 1/2(\psi_{i,j} - \psi_{i,j+1})(\mu_{i,j} + \mu_{i,j+1})$$

$$b_{x_{i,j}} = D_{i,j} + D_{i+1,j}$$

$$b_{y_{i,j}} = D_{i,j} + D_{i,j+1} \tag{A.1.5}$$

$$c_{x_{i,j}} = (n_{i,j,k} + n_{i+1,j,k})a_{x_{i,j}} + b_{x_{i,j}}(n_{i+1,j,k} - n_{i,j,k})$$

$$c_{y_{i,j}} = (n_{i,j,k} + n_{i,j+1,k}).a_{y_{i,j}} + b_{y_{i,j}}\,(n_{i,j+1,k} - n_{i,j,k})$$

where n is the carrier concentration, D the diffusion coefficient, ψ the potential, μ the mobility, Δx the x space step and Δy the y space step.

This scheme is relatively easy to implement and gives satisfactory results in many applications. The accuracy of the above linearised scheme is discussed in Chapter 4, but is subject to significant errors for large time steps and in regions of rapid change in carrier concentration (or potential).

A.2 Steady-state 'Scharfetter-Gummel' Schemes

The popular Scharfetter-Gummel continuity scheme for the steady-state ($\partial n/\partial t = 0$) is expressed as:

For electrons:

$$\begin{aligned}
& n_{i,j-1}.D_{n_{i,j-1/2}}.B\left(\frac{\psi_{i,j-1} - \psi_{i,j}}{V_T}\right).\frac{a_{i-1} + a_i}{2b_{j-1}} \\
& + n_{i-1,j}.D_{n_{i-1/2,j}}.B\left(\frac{\psi_{i-1,j} - \psi_{i,j}}{V_T}\right).\frac{b_{j-1} + b_j}{2a_{i-1}} \\
& - n_{i,j}.\left[D_{n_{i,j-1/2}}.B\left(\frac{\psi_{i,j} - \psi_{i,j-1}}{V_T}\right).\frac{a_{i-1} + a_i}{2b_{j-1}}\right. \\
& + D_{n_{i-1/2,j}}.B\left(\frac{\psi_{i,j} - \psi_{i-1,j}}{V_T}\right).\frac{b_{j-1} + b_j}{2a_{i-1}} \\
& + D_{n_{i+1/2,j}}.B\left(\frac{\psi_{i,j} - \psi_{i+1,j}}{V_T}\right).\frac{b_{j-1} + b_j}{2a_i} \\
& \left.+ D_{n_{i,j+1/2}}.B\left(\frac{\psi_{i,j} - \psi_{i,j+1}}{V_T}\right).\frac{a_{i-1} + a_i}{2b_j}\right] \\
& + n_{i+1,j}.D_{n_{i+1/2,j}}.B\left(\frac{\psi_{i+1,j} - \psi_{i,j}}{V_T}\right).\frac{b_{j-1} + b_j}{2a_i} \\
& + n_{i,j+1}.D_{n_{i,j+1/2}}.B\left(\frac{\psi_{i,j+1} - \psi_{i,j}}{V_T}\right).\frac{a_{i-1} + a_i}{2b_j} \\
& - G_{i,j}.\frac{a_{i-1} + a_i}{2}.\frac{b_{j-1} + b_j}{2} = 0 \qquad \text{(A.2.1)}
\end{aligned}$$

For Holes:

$$
\begin{aligned}
& p_{i,j-1}.D_{p_{i,j-1/2}}.B\left(\frac{\psi_{i,j}-\psi_{i,j-1}}{V_T}\right).\frac{a_{i-1}+a_i}{2b_{j-1}} \\
& +p_{i-1,j}.D_{p_{i-1/2,j}}.B\left(\frac{\psi_{i,j}-\psi_{i-1,j}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_{i-1}} \\
& -p_{i,j}.\left[D_{p_{i,j-1/2}}.B\left(\frac{\psi_{i,j-1}-\psi_{i,j}}{V_T}\right).\frac{a_{i-1}+a_i}{2b_{j-1}}\right. \\
& +D_{p_{i-1/2,j}}.B\left(\frac{\psi_{i-1,j}-\psi_{i,j}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_{i-1}} \\
& +D_{p_{i+1/2,j}}.B\left(\frac{\psi_{i+1,j}-\psi_{i,j}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_i} \\
& +D_{p_{i,j+1/2}}.B\left(\frac{\psi_{i,j+1}-\psi_{i,j}}{V_T}\right).\frac{a_{i-1}+a_i}{2b_j} \\
& \left.+p_{i+1,j}.D_{p_{i+1/2,j}}.B\left(\frac{\psi_{i,j}-\psi_{i+1,j}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_i}\right] \\
& +p_{i,j+1}.D_{p_{i,j+1/2}}.B\left(\frac{\psi_{i,j}-\psi_{i,j+1}}{V_T}\right).\frac{a_{i-1}+a_i}{2b_j} \\
& -G_{i,j}.\frac{a_{i-1}+a_i}{2}.\frac{b_{j-1}+b_j}{2}=0 \qquad \text{(A.2.2)}
\end{aligned}
$$

V_T is the scaled thermal voltage. The half-point values of the diffusion coefficients and carrier mobilities are obtained by linear interpolation.

$$D_{n_{i+1/2,j}}=\frac{D_{n_{i,j}}+D_{n_{i+1,j}}}{2} \qquad \text{(A.2.3)}$$

In order to avoid computational problems (underflows, overflows and inaccuracies), it is necessary to pay particular care in the implementation of the Bernoulli function. There are several possible approaches to obtaining values for the Bernoulli expressions. Simplified approximations are available, although if the normal exponential function available on the computer is chosen it is necessary to ensure that underflow and overflow errors do not occur. A useful method of implementing the Bernoulli function is to use the following scheme [A1]:

$$B(x)=\begin{cases} x \le x_1 & -x \\ x_1 < x < x_2 & \dfrac{x}{exp(x)-1} \\ x_2 \le x \le x_3 & 1-\dfrac{x}{2} \\ x_3 < x < x_4 & \dfrac{x\,exp(-x)}{1-exp(-x)} \\ x_4 \le x < x_5 & x\,exp(-x) \\ x_5 \le x & 0 \end{cases} \tag{A.2.4}$$

The constants x_1 to x_5 depend on the individual computer hardware. They are defined using Selberherr's notation by:

$$'exp(x_1)'\;'-'\;'1'\;'='\;'-1' \tag{A.2.5}$$

$$x_2\,'/'\,('exp(x_2)'\;'-'\;'1')\;'='\;'1'\;'-'\,(x_2\,'/'\,'2')^{x_2} < 0 \tag{A.2.6}$$

$$'1'\;'-'\,(x_3\,'/'\,'2')\;'='\;x_3\,'\cdot'\,'exp('-'x_3)'\,'/'\,('1'\;'-'\,x_3)')^{x_3} > 0 \tag{A.2.7}$$

$$'1'\;'-'\;'exp('-'x_4)'\;'='\;'1' \tag{A.2.8}$$

$$'exp('-'x_5)'\;'='\;'0' \tag{A.2.9}$$

where the parameters inside quotation marks represent the computer implementation of the parameter.

A.3 Full Time-Dependent Schemes

The time dependent Scharfetter-Gummel continuity schemes are defined as:

For electrons:

$$n_{i,j-1,k+1}\cdot D_{n_{i,j-1/2,k}}\cdot B\left(\frac{\psi_{i,j-1,k+1}-\psi_{i,j,k+1}}{V_T}\right)\cdot\frac{a_{i-1}+a_i}{2b_{j-1}}$$

$$+n_{i-1,j,k+1}\cdot D_{n_{i-1/2,j,k}}\cdot B\left(\frac{\psi_{i-1,j,k+1}-\psi_{i,j,k+1}}{V_T}\right)\cdot\frac{b_{j-1}+b_j}{2a_{i-1}}$$

$$-n_{i,j,k+1}\cdot\Bigg[D_{n_{i,j-1/2,k}}\cdot B\left(\frac{\psi_{i,j,k+1}-\psi_{i,j-1,k+1}}{V_T}\right)\cdot\frac{a_{i-1}+a_i}{2b_{j-1}}$$

$$+D_{n_{i-1/2,j,k}}.B\left(\frac{\psi_{i,j,k+1}-\psi_{i-1,j,k+1}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_{i-1}}$$

$$+D_{n_{i+1/2,j,k}}.B\left(\frac{\psi_{i,j,k+1}-\psi_{i+1,j,k+1}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_i}$$

$$+D_{n_{i,j+1/2,k}}.B\left(\frac{\psi_{i,j,k+1}-\psi_{i,j+1,k+1}}{V_T}\right).\frac{a_{i-1}+a_i}{2b_j}$$

$$+\frac{1}{\Delta t}.\frac{a_{i-1}+a_i}{2}.\frac{b_{j-1}+b_j}{2}\Bigg]$$

$$+n_{i+1,j,k+1}.D_{n_{i+1/2,j,k}}.B\left(\frac{\psi_{i+1,j,k+1}-\psi_{i,j,k+1}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_i}$$

$$+n_{i,j+1,k+1}.D_{n_{i,j+1/2,k}}.B\left(\frac{\psi_{i,j+1,k+1}-\psi_{i,j,k+1}}{V_T}\right).\frac{a_{i-1}+a_i}{2b_j}$$

$$=\left[G_{i,j,k}-\frac{n_{i,j,k}}{\Delta t}\right].\frac{a_{i-1}+a_i}{2}.\frac{b_{j-1}+b_j}{2} \qquad \text{(A.3.1)}$$

For holes:

$$p_{i,j-1,k+1}.D_{p_{i,j-1/2,k}}.B\left(\frac{\psi_{i,j,k+1}-\psi_{i,j-1,k+1}}{V_T}\right).\frac{a_{i-1}+a_i}{2b_{j-1}}$$

$$+p_{i-1,j,k+1}.D_{p_{i-1/2,j,k}}.B\left(\frac{\psi_{i,j,k+1}-\psi_{i-1,j,k+1}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_{i-1}}$$

$$-p_{i,j,k+1}.\Bigg[D_{p_{i,j-1/2,k}}.B\left(\frac{\psi_{i,j-1,k+1}-\psi_{i,j,k+1}}{V_T}\right).\frac{a_{i-1}+a_i}{2b_{j-1}}$$

$$+D_{p_{i-1/2,j,k}}.B\left(\frac{\psi_{i-1,j,k+1}-\psi_{i,j,k+1}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_{i-1}}$$

$$+D_{p_{i+1/2,j,k}}.B\left(\frac{\psi_{i+1,j,k+1}-\psi_{i,j,k+1}}{V_T}\right).\frac{b_{j-1}+b_j}{2a_i}$$

$$+D_{p_{i,j+1/2,k}}.B\left(\frac{\psi_{i,j+1,k+1}-\psi_{i,j,k+1}}{V_T}\right).\frac{a_{i-1}+a_i}{2b_j}$$

$$+\frac{1}{\Delta t}.\frac{a_{i-1}+a_i}{2}.\frac{b_{j-1}+b_j}{2}$$

$$+p_{i+1,j,k+1} \cdot D_{p_{i+1/2,j,k}} \cdot B\left(\frac{\psi_{i,j,k+1} - \psi_{i+1,j,k+1}}{V_t}\right) \cdot \frac{b_{j-1} + b_j}{2a_i}$$

$$+p_{i,j+1,k+1} \cdot D_{p_{i,j+1/2,k}} \cdot B\left(\frac{\psi_{i,j,k+1} - \psi_{i,j+1,k+1}}{V_t}\right) \cdot \frac{a_{i-1} + a_i}{2b_j}$$

$$= \left(G_{i,j,k} - \frac{n_{i,j,k}}{\Delta t}\right) \cdot \frac{a_{i-1} + a_i}{2} \cdot \frac{b_{j-1} + b_j}{2} \qquad (A.11)$$

where k is the time step index.

References

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