

**GOLDEN RIEMANNIAN DYNAMICAL GRAVITO ELECTRIC FIELD
EQUATION MODEL FOR A STATIC HOMOGENEOUS SPHERICAL
CHARGED MASSIVE BODY**

BY

YUSUF EPHRAIM

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FACULTY OF NATURAL AND APPLIED SCIENCES
NASARAWA STATE UNIVERSITY, KEFFI
NIGERIA**

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DECLARATION

I hereby declare that this dissertation has been written by me and it is a report of my research work. It has not been presented in any previous application for M.Sc. All quotations are indicated and sources of information specifically acknowledged by means of references.

YUSUF, EPHRAIM

NSU/NAS/M.Sc/PHY/004/15/16.

CERTIFICATION

The dissertation Golden Riemannian Dynamical Gravito Electric Field Equation Model for a Static Homogeneous Spherical Charged Massive Body meets the regulations governing the award of M.Sc, of the School of Postgraduate Studies, Nasarawa State University, Keffi, and is approved for its contribution to knowledge.

Dr. W. L. Lumbi
Chairman, Supervisory Committee

Date

Prof. E.N. Chifu
Member, Supervisory Committee

Date

Dr. U. M. Gurku
Head of Department

Date

Dr. I. Umaru
Internal Examiner

Date

Prof. B.S. Jatau
Dean of Faculty

Date

Prof. D.D. Bakwa
External Examiner

Date

Prof. S.A.S. Aruwa
Dean, School of Postgraduate Studies

Date

DEDICATION

This work is dedicated to my niece late Deborah. M. Bulus.

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LIST OF SYMBOLS

∇^2 is the Laplacian operator

Φ is the electric scalar potential exterior and interior to the body

ρ is the density of electric charged.

ε is the permittivity

r is the mean distance from the centre of the sphere.

R is the radius of the static homogeneous spherical body.

Φ^+ is the golden Riemannian dynamical Gravito electric scalar potential
exterior to

the body

q is the electric charge of the particle

V_g is the gravitational field potential energy

V_e is the electric field potential energy

E is the mechanical energy

$x^0 = ct$ is the space time coordinate

∇_R^2 is the Riemannian Laplacian operator

$\Phi(x^0, r, \theta, \phi)$ is the complete generalized electric scalar potential exterior and

Interior to the body.

H is the magnetic field, which Maxwell called the "magnetic intensity".

J is the electric current density (with being the total current including displacement current).

D is the displacement field (called the "electric displacement" by Maxwell).

ρ is the free charge density (called the "quantity of free electricity" by Maxwell).

A is the magnetic potential (called the "angular impulse" by Maxwell).

φ is the electric potential (which Maxwell also called "electric potential").

σ is the electrical conductivity (Maxwell called the inverse of conductivity the "specific resistance", what is now called the resistivity).

ABSTRACT

The coupling of electromagnetism and gravitation approximately showed that both fields can be converted into each other. We find that every electromagnetic field is associated with a gravitomagnetic field, but gravitomagnetic fields can exist without electromagnetic ones in the case of neutral matter. In this research work the golden Riemannian Laplacian operator was used to construct a generalized Maxwell electric field equation for a static homogeneous spherical charged massive body. This field equation also known as the golden Riemannian dynamical gravito electric field equation was applied to a static homogeneous spherical charged of a massive body to obtain a golden Riemannian dynamical gravito electric scalar potential exterior and interior to the body. The golden Riemannian dynamical gravito electric scalar potential exterior to the body was substituted into the generalized Maxwell electric potential energy and also Maxwell generalized instantaneous Mechanical energy to obtain the golden Riemannian dynamical gravito electric potential energy as well as the mechanical energy respectively. Results shows that the generalized Maxwell's field equation, generalized electric scalar potential exterior and interior, generalized electric potential energy and generalized electric mechanical energy were augmented with additional correction terms which are not found in the well known Maxwellian field equation, electric scalar potential, electric potential energy and mechanical energy.

CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

Gravitation is the mysterious force that exists alongside electromagnetic and nuclear forces. It is the force that governs the motion of the moons, the planets and the galaxies in their various orbits (Harrison, 2000). Newton's dynamical theory of gravitation was published in the year 1686. According to Newton's dynamical theory all interactions in nature manifest through force. In his own words, "I deduced that the force which keep the planet on their orbit must be reciprocally as the square of their distances from centre about which the revolve" this theory was successful in explaining the gravitational phenomena on earth and the experimental fact of the solar system. One of the limitations of this theory is that, it failed to explain anomalous orbital precession of the planets as well as the gravitational shift by the sun (Michiel, 2001).

Einstein's geometrical theory of gravitation also known as general relativity was published in the year 1915. According to this theory, the course of gravity is not due to force rather is a manifestation of curvature of space and time. This theory offered an explanation of anomalous orbital precession as well as gravitational shift by sun in which Newton's theory could not account for (Holland, 1995). The limitations of this theory, is that, is physically unsound, mathematically difficult to solve and inconsistency with quantum mechanics (Kolb & Turner, 1990).

The theory of general relativity which describes gravitation can be linearized, unveiling a structure comparable to the Maxwell equations, which describe electromagnetism (Li, & Torr, 1991). This weak field approximation splits gravitation into components similar to

the electric and magnetic field. Although masses attract each other and equal electric charges repel, the comparable structure between both the gravitational and the electromagnetic field gives rise to a coupling between them because both fields originate from the same point source, the particle. In the case of the gravitational field, the source is the mass of the particle, whereas in the case of the electromagnetic field, the source is the charge of the particle. Moving the particle will both create a magnetic and gravitation using the analogy term, a gravitomagnetic field simultaneously. Hence, the induction of a gravitational field and a gravitomagnetic field using electromagnetic fields applied to material media must be possible. This fact is not commonly known in nowadays physics (Braginski *et al.*, 1977).

In 1864 Maxwell published his “dynamical theory of electric field,” which offered a unifying, mathematical explanation for electromagnetism. The equations are complex, but in general terms they described, a general relationship between electric field and electric charge. The equations predict the existence of electromagnetic wave which travel at the speed of light and consist of electric and magnetic field vibrating in harmonic in directions at right angle to each other (Stratton, 1941).

Maxwell’s equations failed to accurately predict and account for an increasing number of new and important experimental results. The Michelson-Morley experiment, the photo-electric effect and double slit interference all defied explanation by turn-of-the-century classical electrodynamics. The impetus behind the development of some of the foundational theories of modern Physics early in the twentieth century, including quantum mechanics and relativity theory, stems, in part, from the inability of Maxwell’s

classical formulation of electromagnetism to explain these experimental phenomena (Mohr, 2008).

1.2 Statement of the Problem

Maxwell's dynamical electric field equation was derived base on Euclidean Laplacian operator and towards the 20th century, it is well known that the Maxwell's dynamical electric field equation failed to accurately predict and account for an increasing number of new and important experimental results such as the Michelson-Morley experiment, the photo-electric effect and the double-slit interference (Mohr, 2008). On the basis of this, we shall show in this research work, one way of generalizing or extending Maxwell dynamical electric field theory using golden Riemannian Laplacian operator and the golden gravitation scalar potential in order to account for these observed phenomena.

1.3 Aim of the Study

The aim of this research work is to extend Maxwell dynamical electric field equation for a static homogeneous spherical charge massive body using a golden Riemannian geometry.

1.4 Objectives of the Study

In this research work, we used the golden Riemannian Laplacian operator to generalized Maxwell's electric field equation for a static homogeneous spherical charged massive body.

The following objectives will be achieved;

- (i). Construct the golden Riemannian dynamical gravito electric field equation.
- (ii). Obtain the golden Riemannian dynamical gravito electric scalar potential exterior and interior to the body.
- (iii). Obtain the golden Riemannian dynamical gravito electric potential energy.

(iv). Obtain the golden Riemannian dynamical gravito electric mechanical energy.

1.5 Significance of the Study

This research work will reveal some possible unknown extension or correction terms to Maxwell electric field equation, electric scalar potential exterior and interior to the body, electric potential energy and mechanical energy.

This research work can be apply to the motion of satellites, cameras in space, space craft and to give a better understanding to the behavior of charge body in space.

1.6 Scope of the Study

This research work is restricted only to the study of Maxwell's dynamical electric field theory for a static homogeneous spherical charged massive body.

CHAPTER TWO

LITERATURE REVIEW

2.1 The Universe

The universe is extremely large. Our sun is the star of the solar system alpha centauri which is the nearest star to the solar system is about $10^{13}km$ away. This is very close compared to the extent of our galaxy (called the Milky Way) which is about $10^{18}km$ wide. Also, the distance to the Hercules cluster of galaxies is about $10^{22}km$ away.

Also there is Andromeda galaxy (*M13*) the nearest large spiral to our own galaxy, and the star cluster, *M13*, which orbits within our galaxy.

The planets and their satellites are the smallest unit in the universe. According to the size of the units from the smallest to the largest, we have star systems (e.g solar system), galaxies, clusters and super clusters (Chifu & Maharaz, 2016).

2.2 The Solar System

The solar system consists of the sun and a large number of smaller objects gravitationally associated with it. These objects include the planets, their satellites, the comets, the asteroids, the meteoroids and an interplanetary medium of very sparse gas and microscopic solid particles (Chifu & Maharaz, 2016).

2.3 The Big Bang Model of the Universe

The big bang theory states the Universe started from a very hot, dense state, and expanded and cooled with time (Birkhoff, 1923). The theory, which is currently the best

in providing an account of the basic features of the Universe, makes three fundamental assumptions

These include:

- (i) The Universe is homogeneous and isotropic at large scale. By homogeneity, we mean that matter in the Universe is smeared out throughout space. This is particularly true at the largest scales; locally, matter can be seen in form of planets, stars and galaxies. By isotropic we mean that the Universe looks the same in every direction. Again, this is not true at small scales where we observe variation in the distribution of galaxies throughout space.
- (ii) The general theory of relativity is adequate for the study of the Universe at largest scales. This theory offers a diversity of possible geometries which determine whether the Universe is finite or infinite by allowing geometry evolve with time in the space-time geometry.
- (iii) The energy-momentum tensor of the Universe is well approximated to that of a perfect fluid which can be used to describe the dynamics of the Universe.

2.4 Kepler Theory

A brilliant young mathematician Johann Kepler (1571-1630). Kepler succeeded tycho as principal mathematician to the emperor and undertook to carry to completion a new set of astronomical tables based on elaborate observation. Kepler deduced his three laws of planetary motion as:

- (i) The planet moved round the sun in an orbit which are ellipses with the sun at one focus.
- (ii) The radius vector (from sun to planet) sweeps over equal areas in equal time.
- (iii) The square of the period of revolution of the planet around the sun are proportional to the cube of the (mean) radii of their respective orbit (Morse, 1953).

2.5 Gravitational Field

The gravitational field is a vector field that describes the gravitational force which would be applied on an object in any given point in space, per unit mass. It is actually equal to the gravitational acceleration at that point (Bruce, 1995).

It is a generalization of the vector form, which becomes particularly useful if more than two objects are involved (such as a rocket between the earth and moon). For two objects (e.g object two is a rocket, object one the earth), we simply write r instead of r_{12} and m instead of m_2 and define the gravitational $g(r)$ field as:

$$g(r) = -G \frac{m_1}{|r|^2} \hat{r} \tag{2.1}$$

where,

G is the gravitational constant.

r is the distance.

m is the mass.

This has the consequence that there exists a gravitational potential field $V(r)$ such that

$$g(r) = -\nabla V(r) \tag{2.2}$$

and ∇ is the gradient operator.

2.6 Newton's Theory of Universal Gravitation

Newton's law of universal gravitation state that a particle attracts every other particle in the universe using a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between there centers (Haus & Melcher, 1989). This is a general physical law derived from empirical observations by what Isaac Newton called inductive reasoning. It is a part of classical mechanics and was formulated in Newton's work philosophia Naturalis principia mathematica ("the principia"), first published on 15 July 1687 (Richard, 1978). Newton's law has since been superseded by Albert Einstein's theory of general relativity, but it is continues to use as an excellent approximation of the effects of gravity in most application. Mathematical expression of Newton law of gravitation is given as

$$F = \frac{GM_1M_2}{r^2} \tag{2.3}$$

where,

F is the force of attraction.

G is the gravitational constant.

M_1 and M_2 are the masses.

r is the mean distance.

2.7 Newton's Planetary Theory

Consider the sun as a homogeneous massive spherical body of radius R and rest mass M .

The Newton's field equation (Weinberg, 1972a; Howusu, 2013) is given by

$$\nabla^2 f = \begin{cases} 0; r > R \\ 4\pi G\rho; r < R \end{cases} \quad (2.4)$$

Hence in spherical polar coordinates (r, θ, ϕ) of S Newton's gravitational field equation for the sun (Howusu, 2013) are given by

$$\nabla^2 f = \left\{ \begin{array}{l} \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial}{\partial r} f(r, \theta, \phi) \right] \\ + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} \left[\sin \theta \frac{\partial}{\partial \theta} f(r, \theta, \phi) \right] \\ + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} f(r, \theta, \phi) \end{array} \right\} = \begin{cases} 0; r > R \\ 4\pi G\rho; r < R \end{cases} \quad (2.5)$$

where f is the Newton's gravitational scalar potential exterior and interior to the body, G is the universal gravitational constant, ρ is the density of rest mass, R is the radius of the earth and r is the radial distance.

By the symmetry of the distribution of mass it follows that the gravitational field equation will depend on only the radial coordinate r .

Equation (3.2) reduces to

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial}{\partial r} f^+(r) \right] = \begin{cases} 0; & r > R \\ 4\pi G\rho; & r < R \end{cases} \quad (2.6)$$

Therefore, since the body is homogeneous and spherical it possesses spherical symmetry and hence the solutions of (2.2), subject to the conditions of continuity, are given by

$$f^+(r) = -\frac{GM_0}{R} \quad (2.7)$$

Equation (2.7) is the well known Newton's gravitational scalar potential exterior to the body

$$f^-(r) = -\frac{3GM_0}{2R} + \frac{GM_0}{2R} \quad (2.8)$$

Equation (2.8) Is the well known Newton's gravitational scalar potential interior to the body.

In general, Newton's equation of motion with gravitational scalar potential are given by

$$\underline{a} = - \underline{\nabla} f^+(r) \quad (2.9)$$

where,

\underline{a} is the acceleration vector,

$\underline{\nabla}$ is del operator and

$f^+(r)$ is the gravitational scalar potential exterior to the body

Consequently for a planet (or comet or asteroid) of rest mass M_0 (regarded as a particle) in the gravitational field of the sun the equation of motion (Howusu,2013) is given in spherical coordinate as

$$\ddot{r} + r\dot{\theta}^2 - r\dot{\phi}^2 \sin^2 \theta = -\frac{k}{r^2} \quad (2.10)$$

$$\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2 \sin \theta \cos \theta = 0 \quad (2.11)$$

$$r\ddot{\phi} \sin \theta + 2\dot{r}\dot{\phi} \sin \theta + 2r\dot{\theta}\dot{\phi} \cos \theta = 0 \quad (2.12)$$

where, a dot denotes one - time differentiation.

Let us restrict it to the equatorial plane or for a planet whose motion is confined to the equatorial plane of sun, such as planet or comet or asteroid in the solar system.

Then

$$\theta \equiv \frac{\pi}{2}$$

Hence, Newton's dynamical equations of motion (2.10 – 2.12) reduces to

$$\ddot{r} - r\dot{\phi}^2 = -\frac{k}{r^2} \quad (2.13)$$

And the corresponding ϕ – component (2.12) reduces to

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} = 0 \quad (2.14)$$

The exact solution of the angular equation (2.14) is

$$\phi = -\frac{l}{r^2} \quad (2.15)$$

where, l is the constant of motion corresponding to the angular momentum per unit rest mass. The first integral of the radial equation (2.13) subject to (2.15) gives

$$\dot{r}^2 = 2k\left(\frac{1}{r} - \frac{1}{r_i}\right) - l^2\left(\frac{1}{r^2} - \frac{1}{r_i^2}\right) \quad (2.16)$$

where r_i is any apsidal distance. Equation (2.16) is exact Newton's radial speed of a planet. It therefore follows from (2.16) that

$$\ddot{r}(r) = -\frac{k}{r^2} + \frac{l^2}{r^3} \quad (2.17)$$

Equation (2.17) is exact Newton's radial acceleration of the planet, in terms of the radial coordinate. Finally the motion may be expressed in terms of the angular coordinate by the transformation.

$$u(\phi) = -\frac{1}{r(\phi)} \quad (2.18)$$

We have the planetary equation as

$$\frac{d^2u}{d\phi^2} + u = -\frac{k}{l^2} \quad (2.19)$$

This is Newton's planetary equation of motion.

2.8 Einstein's Theory of General Relativity

General theory of relativity (GTR) is the geometry theory of gravitation published by Albert Einstein in 1915, and the current description of gravitation in modern Physics. General relativity generalizes special relativity and Newton law of universal gravitation, providing a unified description of gravity as a geometry property of space and time or space-time. In particular, the curvature of space-time is directly related to the energy and momentum of whatever matter and radiation are presence. The relation is specified by the Einstein's field equation, a system of partial differential equation.

Einstein's theory has important astrophysical implication. For example, it imply the existence of black holes-regions of space in space and time are distorted in such a way that nothing, not even light, can escape as an end state for massive stars (Misner *et al.*, 1973).

In 1905 Einstein's started thinking about how to incorporate gravity into his new relativistic framework. In 1907, beginning with a simple though experiment involving an observer in free fall. The Einstein's field equation are non linear and very difficult to solve. Einstein used approximation methods in working out initial predictions of the

theory. But as early as 1916, the astrophysicist Karl Schwarzschild found the first non-trivial exact solution to the Einstein field equation, the Schwarzschild metric (Max, 1924). The Einstein's field equation is given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (2.20)$$

On the left hand side is the Einstein's tensors, a specific divergence free combination of the Ricci tensor $R_{\mu\nu}$ and the metric.

where, $G_{\mu\nu}$ is symmetric in particular

$$R = g^{\mu\nu}R_{\mu\nu} \quad (2.21)$$

Is the curvature scalar, The Ricci tensor itself is related to the more general Riemann curvature tensor as

$$R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu} \quad (2.22)$$

On the right hand side, $T_{\mu\nu}$ is the energy-momentum tensor and c speed of light. And the corresponding Einstein's geometrical equations of motion in gravitational fields are given tensorially (Howusu, 2010) as

$$\ddot{X}^\alpha + \Gamma_{\mu\nu}^\alpha \dot{X}^\mu \dot{X}^\nu = 0 \quad (2.23)$$

where x^ν are the coordinates of space-time and $\Gamma_{\mu\nu}^\alpha$ are the coefficients of affine connection pseudo tensor of the gravitational field, and each dot denotes one differentiation with respect to proper time. Therefore according to Einstein's Geometrical Theory of Gravitation, popularly known as General Relativity, the problem is to find the gravitational metric tensor field $g_{\mu\nu}$ of the coordinates of space-time.

Schwarzschild's metric is the solution of Einstein's gravitational field equations exterior to a static homogeneous spherical body (Howusu, 2009) given by

$$g_{11} = \left\{ 1 + \frac{2}{c^2} f(r) \right\}^{-1} \quad (2.24)$$

$$g_{22} = r^2 \quad (2.25)$$

$$g_{33} = r^2 \sin^2 \theta \quad (2.26)$$

$$g_{00} = -\left\{1 + \frac{2}{c^2}f(r)\right\} \quad (2.27)$$

$$g_{\mu\nu} = 0; \quad \textit{otherwise} \quad (2.28)$$

$$x^0 = ct \quad (2.29)$$

The corresponding contra-variant metric tensor for this gravitational field is given (Howusu, 2009) as

$$g^{00} = \left\{1 + \frac{2}{c^2}f(r)\right\}^{-1} \quad (2.30)$$

$$g^{11} = -\left\{1 + \frac{2}{c^2}f(r)\right\} \quad (2.31)$$

$$g^{22} = r^2 \quad (2.32)$$

$$g^{33} = -\{r^2 \sin^2 \theta\}^{-1} \quad (2.33)$$

$$g^{\mu\nu} = 0; \quad \textit{otherwise} \quad (2.34)$$

where,

$$f = -\frac{2GM}{c^2 r} \quad (2.35)$$

Einstein's equations of motion for a planet, comet or asteroid in spherical polar coordinates is given as

$$\ddot{t} + \frac{2k}{c^2 r^2} \left(1 - \frac{2k}{c^2 r}\right)^{-1} \dot{t} \dot{r} = 0 \quad (2.36)$$

$$\ddot{\theta} + \frac{2}{r} \dot{r} \dot{\theta} - \sin\theta \cos\theta \dot{\phi}^2 = 0 \quad (2.37)$$

$$\ddot{\phi} + \frac{2}{r} \dot{r} \dot{\phi} + 2 \cot\theta \dot{\theta} \dot{\phi} = 0 \quad (2.38)$$

$$\left\{ \begin{array}{l} \ddot{r} + \frac{k}{r} \left(1 - \frac{2k}{c^2 r}\right) \dot{t}^2 - \frac{k}{c^2 r^2} \left(1 - \frac{2k}{c^2 r}\right)^{-1} \dot{r}^2 \\ - r \left(1 - \frac{2k}{c^2 r}\right) \dot{\theta}^2 - r \left(1 - \frac{2k}{c^2 r}\right) \sin^2\theta \dot{\phi}^2 \end{array} \right\} = 0 \quad (2.39)$$

where a dot denotes one proper time differentiation. Let restrict the motion to the equatorial plane of the Sun,

$$\theta = \frac{\pi}{2} \quad (2.40)$$

The Schwarzschild's equation of motion becomes

$$\ddot{t} + \frac{k}{c^2 r^2} \left(1 - \frac{2k}{c^2 r}\right)^{-1} \dot{t} \dot{r} = 0 \quad (2.41)$$

$$\ddot{\phi} + \frac{2}{r} \dot{r} \dot{\phi} = 0 \quad (2.42)$$

$$\begin{aligned} \ddot{r} + \frac{k}{r} \left(1 - \frac{2k}{c^2 r}\right) \dot{t}^2 - \frac{k}{c^2 r^2} \left(1 - \frac{2k}{c^2 r}\right)^{-1} \dot{r}^2 \\ - r \left(1 - \frac{2k}{c^2 r}\right) \dot{\phi}^2 = 0 \end{aligned} \quad (2.43)$$

Under the condition that

$$t \rightarrow \tau ; r \rightarrow \infty \quad (2.44)$$

Equation (2.41) reduces to

$$\dot{t} = \left(1 - \frac{2k}{c^2 r}\right)^{-1} \quad (2.45)$$

Also equation (2.42) reduces to

$$\ddot{\phi} = \frac{l}{r^2} \quad (2.46)$$

where l is a constant corresponding to the angular momentum per unit mass of the planet.

Solving the radial equation (2.43) subject to (2.45) and (2.46) gives radial speed as

$$\dot{r}^2(r) = \left\{ \begin{array}{l} c^2 \left[1 - \left(1 - \frac{2k}{c^2 r_1}\right)^{-1} r \left(1 - \frac{2k}{c^2 r}\right) \right] \\ -l^2 \left(\frac{1}{r^2} - \frac{1}{r_1^2} \right) \left(1 - \frac{2k}{c^2 r}\right) \end{array} \right\} \quad (2.47)$$

where, r_1 is any apsidal distance.

Equation (2.46) is the Einstein's radial speed of the planet in terms of the radial

coordinate. It follows from (2.43), (2.45) (2.46) and (2.47) that

$$\ddot{r}(r) = \left\{ \frac{-k}{r^2} \left[\left(1 - \frac{2k}{c^2 r} \right)^{-1} - \frac{1^2}{c^2 r_1^2} + \frac{l^2}{r^3} - \frac{3kl^2}{c^2 r^4} \right] \right\} \quad (2.48)$$

Equation (2.48) is the Einstein's radial acceleration of the planet in terms of the radial coordinate.

By using the transformation in terms of the angular coordinate given as

$$u(\phi) = \frac{1}{r(\phi)} \quad (2.49)$$

$$\frac{d^2 u}{d\phi^2} + u - \frac{3k}{c^2} u^2 = \frac{k}{l^2} \quad (2.50)$$

Equation (2.42) is Einstein's planetary equation of motion.

2.9 Early History of Electric Charge

The first recorded observation of an electric effect was made in about 600 B.C by the Greek Mathematician Thales of Miletus (640-546 B.C). He discovered that a piece of amber which has been rubbed with wool, attracts small pieces of paper or pith. The word electric is derived from the Greek word electron, meaning amber.

The next significant advance in the study of electric effects did not occur until the sixteenth century, when William Gilbert (1544-1603), physician to Queen Elizabeth I,

discovered that different kinds of material can be charged by means of friction just as amber can.

In the early eighteenth century, Stephen Gray (1696-1736), discovered that charges can be transferred from one object to another when the objects are connected by a piece of metal. Gray thereby demonstrated that the agency responsible for electrification can be made to flow. The discovery that two different types of charges exist was made by Charles Francois Du Fay (1698-1739), a French chemist and physicist.

2.10 Coulomb's Law

In 1785, the French physicist, Charles Augustin de Coulomb (1736-1806), made the first direct measurements of the forces between charged objects. The results of the experiments performed by Coulomb are now called Coulomb's law (Kashimbila, 2001).

The law states that the mutual electrostatic force between two point charges q_1 and q_2 is directed along the line between the charges; the magnitude of the force is directly proportional to the product of the charges and is inversely proportional to the square of the distance between them.

$$f \propto \frac{q_1 q_2}{r^2} \quad (2.51)$$

$$f = k \frac{q_1 q_2}{r^2} \quad (2.52)$$

$$\text{where, } k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$$

2.11 The Electric Field

An electric field is a region where an electric charge experiences a force. The electric field produced by a stationary charge Q can be investigated by measuring the force on a test charge q_0 placed at various points in space (Kashimbila, 2001).

If q_0 is very small, the electric field intensity or the electric field strength E at point P due to Q is defined by the equation

$$E = \frac{F}{q_0} \tag{2.53}$$

i.e the magnitude of E is the force per unit charge and its direction is that of F . from coulomb's law, the magnitude of the force on q_0 due to the charge Q is

$$f = k \frac{q_1 q_2}{r^2} \tag{2.54}$$

Therefore, the force per unit charge i.e the electric field strength E , is

$$E = k \frac{Q}{r^2} \quad (2.55)$$

From equation (2.53), the units of electric field strength are Newton's per coulomb's N/C. equation (2.55) shows that E decreases with distance from the point charge according to an inverse square law.

2.12 Electromagnetism

Electromagnetic involves the macroscopic behavior of electric charges in vacuum and matter. This behavior can be accurately characterized by the Lorentz force law and Maxwell's equations, which were derived from experiments showing how forces on charges depend on the relative locations and motions of other charges nearby. Additional relevant laws of Physics include Newton's law, photon quantization, and the conservation relations for charge, energy, power, and momentum. Electromagnetic phenomena underlie most of the "electrical" in "electrical engineering" and are basic to a sound understanding of that discipline (Balanis, 1989).

Electrical engineering has delivered four "miracles" sets of phenomena that could each be considered true magic prior to their development. The first of these to impress humanity was the electrical phenomenon of lightning, often believed to be a tool of heaven, and the less powerful magnetic force that caused lodestones to point north. The explanation and application of these invisible forces during the eighteenth and nineteenth centuries vaulted electrical engineering to the forefront of commercial interest as motors, generators, electric lights, batteries, heaters, telephones, record players, and many other devices emerged. The second set of miracles delivered the ability to communicate

instantly without wires around the world, not only dots and dashes, but also voice, images, and data. Such capabilities had been commonplace in fairy tales, but were beyond human reach until Hertz demonstrated radio wave transmission in 1888, 15 years after Maxwell's predictions (Harrington, 1961). Marconi extended the technique to intercontinental distances. Third came electronics and photonics the ability to electrically manipulate individual electrons and atoms in vacuum and in matter so as to generate, amplify, manipulate, and detect electromagnetic signals (Ishimaru, 1991). During the twentieth century vacuum tubes, diodes, transistors, integrated circuits, lasers, and superconductors all vastly extended the capabilities and applications of electromagnetic. The fourth set of electrical phenomena involves cybernetics and informatics, the manipulation of electrical signals so complex that entirely new classes of functionality are obtained, such as optimum signal processing, computers, robotics, and artificial intelligence. This text focuses on the electromagnetic nature of the first three sets of phenomena and explores many of their most important applications (Smith, 1997).

2.13 Dynamical Theory of the Electromagnetic Field

The dynamical theory of the electromagnetic field is the third of James Clerk Maxwell's papers regarding electromagnetism, published in 1865. It is the paper in which the original set of four Maxwell's equations first appeared. The concept of displacement current, which he had introduced in his 1861 paper "On Physical Lines of Force", was utilized for the first time, to derive the electromagnetic wave equation (Elliott, 1993).

2.13.1 Maxwell's Dynamical Electric Theory

A Dynamical Theory of the Electromagnetic Field", which is entitled "General Equations of the Electromagnetic Field", Maxwell formulated twenty equations which were to become known as Maxwell's equations, until this term became applied instead to a set of

four vectorized equations selected in 1884 by Oliver Heaviside, which had all appeared in "On physical lines of force" (Jones, 1986). Heaviside's versions of Maxwell's equations are distinct by virtue of the fact that they are written in modern vector notation. They actually only contain one of the original eight equation "2.13" (Gauss's Law). Another of Heaviside's four equations is an amalgamation of Maxwell's law of total currents (equation "2.7") with Ampere's circuital law (equation "2.9"). This amalgamation, which Maxwell himself had actually originally made in "On Physical Lines of Force", is the one that modifies Ampere's Circuital Law to include Maxwell's displacement current (Tai, 1994).

Eighteen of the twenty original Maxwell's equations can be vectorized into 6 equations. Each vectorized equation represents 3 original equations in component form. Including the other two equations, in modern vector notation, they can form a set of eight equations (Jackson, 1975). They are listed below:

The law of total currents

$$J_{\text{tot}} = J + \frac{\partial D}{\partial t} \quad (2.56)$$

Definition of the magnetic potential

$$\mu H = \nabla \quad (2.57)$$

Ampere's circuital law

$$\nabla \times H = J_{\text{tot}} \quad (2.58)$$

The Lorentz force

$$\mathbf{E} = \mu \mathbf{V} \times \mathbf{H} - \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi \quad (2.59)$$

This force represents the effect of electric fields created by convection, induction, and by charges.

The electric elasticity equation

$$\mathbf{E} = \frac{1}{\epsilon} \mathbf{D} \quad (2.60)$$

Ohm's law

$$\mathbf{E} = \frac{1}{\sigma} \mathbf{J} \quad (2.61)$$

Gauss's law

$$\nabla \cdot \mathbf{D} = \rho \quad (2.62)$$

Equation of continuity of charge

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad (2.63)$$

where,

H is the magnetic field, which Maxwell called the "magnetic intensity".

J is the electric current density (with being the total current including displacement current).

D is the displacement field (called the "electric displacement" by Maxwell).

ρ is the free charge density (called the "quantity of free electricity" by Maxwell).

A is the magnetic potential (called the "angular impulse" by Maxwell).

E is the electric field (called the "electromotive force" by Maxwell, not to be confused with the scalar quantity that is now called electromotive force).

ϕ is the electric potential (which Maxwell also called "electric potential").

σ is the electrical conductivity (Maxwell called the inverse of conductivity the "specific resistance", what is now called the resistivity).

Maxwell did not consider completely general materials; his initial formulation used linear, isotropic, non dispersive permittivity ϵ and permeability μ , although he also discussed the possibility of anisotropic materials (Chew, 1990).

It is of particular interest to note that Maxwell includes $\mu \mathbf{v} \times \mathbf{H}$ a term in his expression for the "electromotive force" at equation "2.10", which corresponds to the magnetic force per unit charge on a moving conductor with velocity v . This means that equation "2.10" is effectively the Lorentz force. This equation first appeared at equation in "On Physical Lines of Force" quite some time before Lorentz thought of it. Nowadays, the Lorentz force sits alongside Maxwell's equations as an additional electromagnetic equation that is not included as part of the set.

When Maxwell derives the electromagnetic wave equation in his 1864 book, he uses equation "2.10" as opposed to using Faraday's law of electromagnetic induction as in

modern textbooks. Maxwell however drops the $\mu\nu \times H$ term from equation "2.10" when he is deriving the electromagnetic wave equation, and he considers the situation only from the rest frame (Kong, 1986).

2.13.2 Maxwell is the First to Propose that Light is an Electromagnetic Wave

In "A dynamical theory of the electromagnetic field", Maxwell utilized the correction to Ampere's Circuital Law that he had made in part III of "On physical lines of force" In part VI of his 1864 book "Electromagnetic theory of light" (Kunz & Luebbbers, 1993). Maxwell combined displacement current with some of the other equations of electromagnetism and obtained a wave equation with a speed equal to the speed of light. He commented, the agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws (Jin, 2002). Maxwell's derivation of the electromagnetic wave equation has been replaced in modern Physics by a much less cumbersome method which combines the corrected version of Ampere's Circuital Law with Faraday's law of electromagnetic induction. To obtain the electromagnetic wave equation in a vacuum using the modern method, we begin with the modern 'Heaviside' form of Maxwell's equations (Silvester & Ferrar, 1996). Using (SI units) in a vacuum, these equations are

$$\nabla \cdot E = 0 \tag{2.64}$$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \tag{2.65}$$

$$\nabla \cdot \mathbf{H} = 0 \quad (2.66)$$

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (2.67)$$

If we take the curl of the curl equations we obtain

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial}{\partial t} \nabla \times \mathbf{H} = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (2.68)$$

$$\nabla \times \nabla \times \mathbf{H} = \epsilon_0 \frac{\partial}{\partial t} \nabla \times \mathbf{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{H}}{\partial t^2} \quad (2.69)$$

If we note the vector identity

$$(\nabla \times (\nabla \times \mathbf{V})) = \nabla(\nabla \cdot \mathbf{V}) - \nabla^2 \mathbf{V} \quad (2.70)$$

Where \mathbf{V} is any vector function of space, we recover the wave equations

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = -c^2 \cdot \nabla^2 \mathbf{E} = 0 \quad (2.71)$$

$$\frac{\partial^2 \mathbf{H}}{\partial t^2} = -c^2 \cdot \nabla^2 \mathbf{H} = 0 \quad (2.72)$$

where,

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.99792488 \times 10^8 \text{ meters per second is the speed of light in}$$

free space

2.14 Divergence of \mathbf{E}

Consider a system in which the charge densities and current densities in the system are all in steady state prior to a time t_0 . This is equivalent to assuming that the material is initially undisturbed before applying perturbations when deriving the differential equations for sound and is done without the loss of generality (Volakis et al, 1998). Coulomb's law of electrostatics gives the electric field, \mathbf{E} , at a point of observation P located at a point $\mathbf{r}(x,y,z)$ and time t_0 that is produced by all the charge densities $\rho'(\mathbf{r}', t_0)$ located at the primed points $\mathbf{r}'(x',y',z')$

$$\underline{\mathbf{E}}(\mathbf{r}, t_0) = \frac{1}{4\pi\epsilon_0} \int_{\text{all space}} \frac{\hat{\eta}}{\eta^2} \rho'(\mathbf{r}', t_0) d\tau \quad (2.73)$$

where $\hat{\eta} = \mathbf{r} - \mathbf{r}'$ and $d\tau'$ denotes integrating over each of the primed spatial variables while the unprimed spatial variables remain constant. $\rho'(\mathbf{r}', t_0)$ is the charge density at \mathbf{r}' at time t_0 . \mathbf{E} is a function of the unprimed spatial variables x, y, z as well as time t . To calculate the change in the electric field with time at a point of observation, we assume that the information about changes can only propagate at a finite speed, so only those changes that occur within the finite volume can propagate quickly enough to change the observables (i.e. measurable quantities) at point P by the time $t_0 + \partial t$. A priori we make no assumptions about the shape of the volume or its extent (Silvester & Ferrar, 2006).

Hence we postulate that the partial time derivative of the E-field at the point of observation is:

$$\frac{\partial \underline{E}}{\partial t} = \lim_{V \rightarrow 0} \frac{1}{4\pi\epsilon_0} \int_V \frac{\hat{\eta}}{\eta^2} \frac{\partial \rho'}{\partial t} d\tau' \quad (2.74)$$

where the integration is only over an infinitesimal volume V that surrounds the point of observation P . As is standard convention, partial derivatives with respect to time are calculated assuming all spatial variables (i.e. primed and unprimed) are held constant. E is a function of the unprimed spatial variables x, y, z as well as time t . ρ' is a function of the primed spatial variables and time. For clarity we will omit including $\lim V \rightarrow 0$ in equations hereafter. The expression for the E-field at time t (where $t=t_0 + \partial t$) is then:

$$\begin{aligned} \underline{E}(\underline{r}, t) = & \frac{1}{4\pi\epsilon_0} \int_{\text{outside } V} \frac{\eta'}{\eta'^2} \rho'(\underline{r}', t_0) d\tau' + \frac{1}{4\pi\epsilon_0} \int_V \frac{\eta'}{\eta'^2} \rho'(\underline{r}', t_0) d\tau' \\ & + \left\{ \frac{1}{4\pi\epsilon_0} \frac{\eta'}{\eta'^2} \frac{\partial \rho'}{\partial t} d\tau' \right\} \partial t \end{aligned} \quad (2.75)$$

where $\partial \rho' / \partial t$ is calculated at $t=t_0$ and we have explicitly separated the contribution from the spatial integral over all space at t_0 into two parts, a volume integral outside V and a volume integral inside V . We state the definition of the del operator $\underline{\nabla}$

$$\underline{\nabla} = \hat{\mathbf{I}} \left(\frac{\partial}{\partial x} \right)_{y,z} + \hat{\mathbf{J}} \left(\frac{\partial}{\partial y} \right)_{x,z} + \hat{\mathbf{K}} \left(\frac{\partial}{\partial z} \right)_{x,y} \quad (2.76)$$

and note that for this operator, the variable t , and the primed variables x' , y' and z' , are considered constants. Equation (2.75) and Equation (2.76) lead to:

$$\underline{\nabla} \cdot \underline{\mathbf{E}} = \frac{1}{4\pi\epsilon_0} \int_v \underline{\nabla} \cdot \frac{\hat{\mathbf{n}}}{\eta^2} \left(\rho'(\underline{\mathbf{r}}', t_0) + \frac{\partial \rho'}{\partial t} \partial t \right) \partial \tau' \quad (2.77)$$

Since the divergence of the 1st integral in Equation (2.75) is zero. The term in curly brackets in Equation (2.77) only depends on primed variables and the time t , using the identity $\underline{\nabla} \cdot \frac{\hat{\mathbf{n}}}{\eta^2} = 4\pi\delta^3(\eta)$, we obtain the first of Maxwell's equations:

$$\underline{\nabla} \cdot \underline{\mathbf{E}} = \frac{\rho}{\epsilon_0} \quad (2.78)$$

where $\rho = \rho(\underline{\mathbf{r}}, t_0) + \frac{\partial \rho}{\partial t} \partial t$ is the charge density at the point of observation P at time t .

Equation (2.78) is the standard result from textbooks (that we call Maxwell's first equation). We note that because Equation (2.77), together with the other differential equations derived below for electromagnetism (i.e Maxwell's equations), have been derived for arbitrary changes during time ∂t , they therefore remain valid (on any length

scale and any frequency) for as long as Coulomb's law, the Biot-Savart law and the conservation of charge remain valid (Monk, 2003).

2.15 Theory of Propagation of Light

Cosmological observations are based on information obtained via electromagnetic radiation, and we need to understand the theory of light propagation through space-time in an expanding Universe (Belieski & Vendaguer, 2001). An important consequence of an expanding Universe is that light from distant sources suffers cosmological redshift (Ford & Roman, 1996).

2.16 Photoelectric Effect

The photoelectric effect is the process where electrons are ejected from a surface by the action of light (electromagnetic radiation). The process was discovered by Heinrich Hertz in 1887. Attempts to explain the effect by classical electromagnetic failed. In 1905, Albert Einstein presented an explanation based on the quantum concept of max Planck (Holand, 1995).

2.16.1 Einstein's Special Theory of Relativity

Explaining the photoelectric effect and Brownian motion forged new links between the microscopic structure of light and matter and observable properties. The last paper, describing the theory of relativity, really shook physicists' beliefs about the nature of the physical world and remains deeply surprising even today. In order to even try to make sense of it, we have to revisit the behavior of light. When you look at the clock in the front of your classroom to see how much longer your chemistry class could possibly last, what allows you to see the time is the light traveling from the clock to your eye, which

then registers the clock's image on your eye and your brain? The light has to travel from the clock to you, so the actual time you see is what the clock displayed a split second ago. If that isn't weird enough, the time that the front row sees is earlier than the time that the students in the back row will see because it takes a bit longer for the light to get to the back row. Even though these differences in time are too small for us to detect, technically you never get to see the actual time the clock displays, no matter how close you get to it.

Now let's pretend you are in a spaceship traveling at the speed of light away from that clock. The light from the clock at the current time will never reach you (since you and the light are traveling at the same speed and you left first), so as you travel, the clock will appear to have stopped at whatever time you left on your light speed journey. Your spaceship only has room for one, so your fellow students are stuck in the classroom with the clock. They watch the classroom clock keeping perfect time, getting closer to the end of class, even though to you it appears stopped. To further confuse you, if you look at the wrist watch that you are wearing in the spaceship, it continues to merrily tick away, time advancing, because it is traveling with you. This clock thing gets even more mysterious when you return to the classroom after your speed-of-light adventure. Even though your watch and the wall clock might have been in perfect synch before you took off, your watch now reads a time ahead of the wall clock. Einstein reasoned that time (and distance and matter and energy) are all relative to your frame of reference. Einstein started with the premise that in a vacuum the speed of light is a constant and that light moves at the fastest speed in the universe. This contradicted Newton's laws, which say that if you continue to accelerate, you will continue to get faster. Einstein's assumption that light is the universal speed limit, lead to some of the more bizarre revelations of relativity theory. His conclusions are well supported by 100 years of experimental evidence. He showed

that as you approach the speed of light, time and space are compressed such that our measuring devices (e.g., clocks and rulers) become distorted depending upon their frame of reference. Thus, time and space are relative states for the same object. Einstein went on to study and write about each of these three subjects more thoroughly. His special theory of relativity gave rise to a general theory of relativity, which incorporated other aspects of the behavior of the universe such as gravity and led to the prediction of black holes and the Big Bang theory of the origin of the universe (Einstein, 1905). His further work in the area of Brownian motion solidified atomic theory and the existence of molecules, and his explanations of the behavior of light and the photoelectric effect mechanical model of the atom. So 1905 was just a launch pad for young Albert Einstein, but what an amazing liftoff he had! Doris Kimbrough teaches chemistry at the University of Colorado-Denver. Her last article “More Than Blue” appeared in the April 2004 issue of Chem Matters. Faster than the speed of light! The blue glow of this reactor is due to Cherenkov radiation. This radiation is emitted when electrons pass through the reactor water faster than light (Light travels about 25% slower through water) (Bohm & Hiley, 2006).

2.16.2 Theory on Photoelectric Effect

In order to study the effect, a metal plate (the photo-sensitive surface) and a collecting electrode are placed in a vacuum. A variable source of potential difference is placed across electrodes. A sensitive current measuring device is placed in series with the collector. Light is allowed to strike the surface and the current at the collector is noted as a function of the potential difference (Auletta, 2000). Based on the experimental results, the following conclusions can be drawn.

- (i) If the frequency ν of the light is constant, the photoelectric current increases with increasing intensity of the light.
- (ii) The photoelectrons are emitted within less than 10^{-9} sec after the surface is illuminated by the light. The emission is essentially instantaneous with illumination.
- (ii) For a given photosensitive surface, the emission of the photoelectrons takes place only if different for different materials.
- (iv) The maximum kinetic energy, k_{max} , of the photoelectrons is independent of the intensity I of the incident light.
- (iv) The maximum kinetic energy, k_{max} , of the photoelectrons depends on the frequency of the incident light.
- (v) The relationship between k_{max} , and ν is linear.

Attempts to explain the effect by classical electromagnetic ran into trouble with several of these observed results. In particular there seemed to be no way to explain No. four. With very faint sources, classical theory would predict several days to collect enough light to cause electron ejection, clearly in disagreement with No. two.

The quantum hypothesis of electromagnetic radiation as proposed by planck would have the radiation in bundles of energy. Each bundle was called quantum or photon (Bacciagaluppi, 2006). The energy of each photon in a light source of frequency ν is $h\nu$.

When light strikes the photosensitive surface, an electrons absorbs one of these photons in order for the electron to escape from the surface, it uses some of the energy from the

photo. This energy w is known as the work function of the metal. The remainder of the energy from the photo goes into kinetic energy of the electron (Bohm & Hiley, 2006).

Thus we can write, $k_{max} = h\nu - w$, we see that for a given surface, the maximum kinetic energy depends on the frequency of the light and not on its intensity. If the ejected electrons reach a collecting plate, the resulting current can be detected. If an external, retarded potential, is placed between the metal surface and the collecting electrons, the current can be reduced. A sufficiently high potential will prevent even the fastest electrons from reaching the collector plate. In electricity, $u = qv$ where u is the potential energy and q is the charge and v is the voltage (Ballentine, 1970). Recall from the work energy theorem that we may write

$$w = \Delta kE = -\Delta u \quad (2.79)$$

In our case, the electron starts from rest and strikes the plate at zero potential relative to the first plate. So

$$\Delta kE = \frac{1}{2}mv^2 - 0 \quad (2.80)$$

And

$$-\Delta u = -(0 - qv) \quad (2.81)$$

An electron which just fails to reach the collector plate must have had an initial kinetic energy of

$$\frac{1}{2}mv^2 = eV \quad (2.82)$$

Where eV is the work done on a charge e which moves through the retarding potential differential v . the kinetic energy of the fastest electrons can therefore be established by finding the critical retarding potential necessary to reduce the current flow to zero.

Therefore

$$eV_{crit} = hv - w \quad (2.83)$$

Or

$$V_{crit} = \frac{h}{e}v - \frac{w}{e} \quad (2.84)$$

That is, h/e should be the slope of the graph of V_{crit} versus v . the v -intercept, W/e , is a constant which depends on the work function of the emitting surface and the contact potentials of the emitter and collector.

2.17 Michelson-Morley Experiment

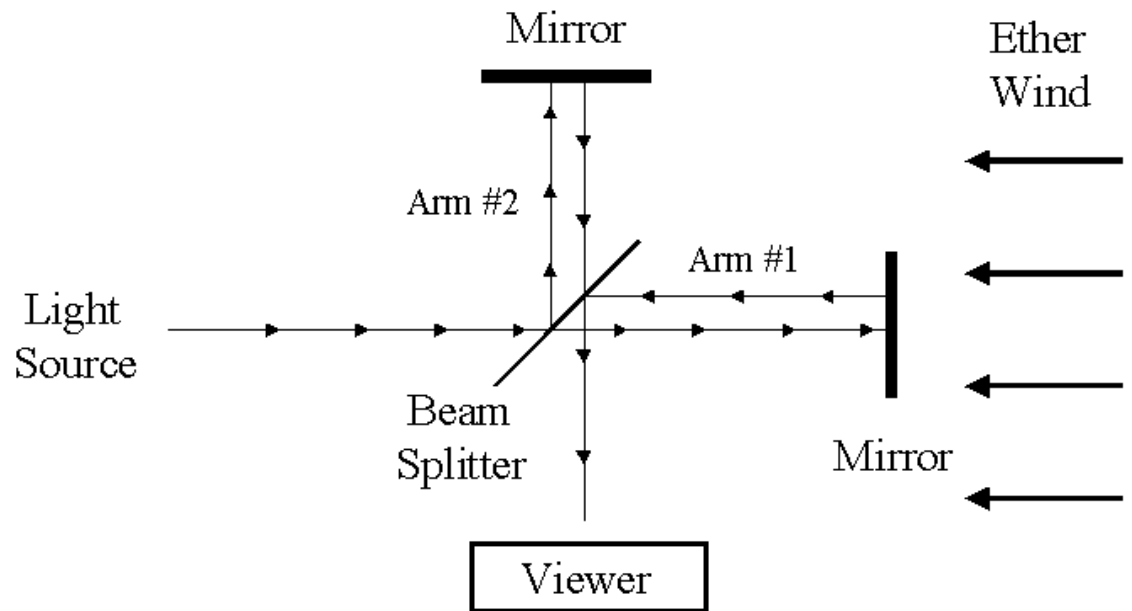
The wave theory of light was devised and perfected several decades before the electromagnetic nature of the waves became known. It was reasonable for the pioneers in optics to regard light waves as undulations in an all-pervading elastic medium called the ether, and their successful explanation of diffraction and interferences phenomena in terms of ether waves made of notion of the ether so familiar that its existence was accepted without question. Maxwell's development of the electromagnetic theory of light in 1864 and Hertz's experimental confirmation of it in 1887 deprived the ether of most of its properties, but nobody at the time seemed willing to discard the fundamental idea represented by the ether: that light propagates relative to some sort of universal frame of reference (Beiser, 1973).

Detecting the aether wind was the next challenge Michelson set himself after his triumph in measuring the speed of light so accurately. Naturally, something that allows solid bodies to pass through it freely is a little hard to get a grip on. But Michelson realized that, just as the speed of sound is relative to the air, so the speed of light must be relative to the aether. This must mean, if you could measure the speed of light accurately enough, you could measure the speed of light travelling upwind, and compare it with the speed of light travelling downwind, and the difference of the two measurements should be twice the wind speed. Unfortunately, it wasn't that easy. All the recent accurate measurements had used light travelling to a distant mirror and coming back, so if there was an aether

wind along the direction between the mirrors, it would have opposite effects on the two parts of the measurement, leaving a very small overall effect. There was no technically feasible way to do a one-way determination of the speed of light (Collin, 1991).

Sound waves need a medium through which to travel. In 1864 James Clerk Maxwell showed that light is an electromagnetic wave. Therefore it was assumed that there is ether which propagates light waves. This ether was assumed to be everywhere and unaffected by matter. This ether could be used to determine an absolute reference frame with the help of observing how light propagates through the ether (Felsen & Marcuvitz, 1994). The Michelson-Morley experiment (circa 1885) was performed to detect the Earth's motion through the ether as follows:

Figure 1: Diagram of Michelson-Morley Experiment



(Felsen & Marcuvitz, 1994)

The viewer will see the two beams of light which have traveled along different arms display some interference pattern. If the system is rotated, then the influence of the “ether wind” should change the time the beams of light take to travel along the arms and therefore should change the interference pattern. The experiment was performed at different times of the day and of the year (Born & Wolf, 1999). No change in the interference pattern was observed!

2.18 Double slit interference

This experiment is said to illustrate the essential mystery of quantum mechanics. This mystery is embodied in the apparent ability of a system to possess properties which, from a classical physics point-of-view, are mutually contradictory. We have already touched on one such instance, in which the same physical system can exhibit under different circumstances, either particle or wave-like properties, otherwise known as wave-particle duality. It is this that is examined in greater detail here. The experiment will be considered in three forms: performed with macroscopic particles, with waves, and with electrons. The first two experiments merely show what we expect to see based on our everyday experience. It is the third which displays the counterintuitive behavior of microscopic systems, a peculiar combination of particle and wave like behavior which cannot be understood in terms of the concepts of classical physics. The analysis of the two slit experiment presented below is more or less taken from Volume III of the Feynman Lectures in Physics (Feynmann et al, 1999).

2.18.1 An Experiment with Bullets

Imagine an experimental setup in which a machine gun is spraying bullets at a screen in which there are two narrow openings, or slits which may or may not be covered. Bullets

that pass through the openings will then strike a further screen, the detection or observation screen, behind the first, and the point of impact of the bullets on this screen are noted.

Suppose, in the first instance, that this experiment is carried out with only one slit opened, slit one say. A first point to note is that the bullets arrive in ‘lumps’, (assuming indestructible bullets), i.e. every bullet that leaves the gun arrives as a whole somewhere on the detection screen. Not surprisingly, what would be observed is the tendency for the bullets to strike the screen in a region somewhere immediately opposite the position of the open slit, but because the machine gun is firing erratically, we would expect that not all the bullets would strike the screen in exactly the same spot, but to strike the screen at random, though always in a region roughly opposite the opened slit (Tonomura et al, 1989).

2.18.2 An Experiment with Waves

Now repeat the experiment with waves. For definiteness, let us suppose that the waves are light waves of wavelength λ . The waves pass through the slits and then impinge on the screen where we measure the intensity of the waves as a function of position along the screen. First perform this experiment with one of the slits open, the other closed. The resultant intensity distribution is then a curve which peaks behind the position of the open slit, much like the curve obtained in the experiment using bullets. Call it $I_1(x)$, which we know is just the square of the amplitude of the wave incident at x which originated from slit one. If we deal only with the electric field, and let the amplitude of the wave at x at two. The word ‘amplitude’ is used here to represent the value of the wave at some point in time and space, and is not used to represent the maximum value of an oscillating wave. The time t be

$$E(x, t) = E(x)\exp(-i\omega t)$$

In complex notation, then the intensity of the wave at x will be

$$I_1(x) = |E_1(x, t)|^2 = E_1(x)^2.$$

Close this slit and open the other. Again we get a curve which peaks behind the position of the open slit. Call it $I_2(x)$. The result of directing waves at a screen when only one slit is open. The curves $I_1(x)$ and $I_2(x)$ give the intensities of the waves passing through slit one or two respectively and reaching the screen at x . (They are just the central peak of a single slit diffraction pattern) (Taylor, 1909).

2.18.3 An Experiment with Electrons

We now repeat the experiment for a third time, but in this case we use electrons. Here we imagine that there is a beam of electrons incident normally on a screen with the two slits, with all the electrons having the same energy E and momentum P . The screen is a fluorescent screen, so that the arrival of each electron is registered as a flash of light the signature of the arrival of a particle on the screen. It might be worthwhile pointing out that the experiment to be described here was not actually performed until the very recent past, and even then not quite in the way described here. Nevertheless, the conclusions reached are what would be expected on the basis of what is now known about quantum mechanics from a multitude of other experiments. Thus, this largely hypothetical experiment (otherwise known as a thought experiment or gedanken experiment) serves to illustrate the kind of behavior that quantum mechanics would produce, and in a way that can be used to establish the basic principles of the theory.

Let us suppose that the electron beam is made so weak that only one electron passes through the apparatus at a time. What we will observe on the screen will be individual point-flashes of light, and only one at a time as there is only one electron passing through

the apparatus at a time. In other words, the electrons are arriving at the screen in the manner of particles, i.e. arriving in lumps. If we close first slit 2 and observe the result we see a localization of flashes in a region directly opposite slit 1. We can count up the number of flashes in a region of size Δx to give the fraction of flashes that occur in the range x to $x + \Delta x$, as in the case of the bullets. As there, we will call the result $P_1(x)$. Now do the same with slit 1 closed and slit 2 opened. The result is a distribution described by the curve $p_2(x)$. These two curves give, as in the case of the bullets, the probabilities of the electrons striking the screen when one or the other of the two slits are open. But, as in the case of the bullets, this randomness is not to be seen as all that unexpected the electrons making their way from the source through the slits and then onto the screen would be expected to show evidence of some inconsistency in their behavior which could be put down to, for instance, slight variations in the energy and direction of propagation of each electron as it leaves the source. Now open both slits. What we notice now is that these flashes do not always occur at the same place in fact they appear to occur randomly across the screen. But there is a pattern to this randomness. If the experiment is allowed to continue for a sufficiently long period of time, what is found is that there is an accumulation of flashes in some regions of the screen, and very few, or none, at other parts of the screen (Dimitrova & Weis, 2008).

2.19 Review of Related Work

Chifu and Lumbi published a work in 2012 on the Relativistic dynamics in the vicinity of a uniformly charged sphere. Their work, provide a crucial link between gravitational and electric fields. It also introduces hitherto unknown corrections of all orders of c^{-2} to the expressions of instantaneous mechanical energy, spectral shift and equations of motion for test particles and photons in combined spherically symmetric gravitational and electric

field were obtained. Although the field equation, scalar potential, potential energy and mechanical energy in combined gravitational and electric field were not calculate (Chifu & Lumbi, 2012).

In 2004, Becker et al., published a work title “ On the classical coupling between gravity and electromagnetism”. In their work, the proposed system of an electron inside a uniformly charged shell increases the strength of the coupling significantly as compared to previously discussed physical systems. The charged shell would affect the free-fall acceleration of electrons by an amount of g . the electrostatic potential energy, the electric field and the electric force were calculated for a charge distribution within a uniformly charged shell. Though the electric scalar potential, electric potential energy and mechanical energy where not obtained in combined gravitational and electric fields (Becker, 2004).

In 2016, Andrey work on unified theory of force fields (electromagnetic and gravitational). In his work, the general equation of any strong electromagnetic or gravitational fields such as the gravitational field intensity, the gravitational charge, gravitational field and velocity field in electromagnetic field were obtain. Though the electric scalar potential, electric potential energy and mechanical energy where not obtained in combined gravitational and electric fields (Andrey, 2016).

In 2014, Mahdi, published a work title “ On the nature of electric charge”. In his work, he generalized interaction between charged particles to photon structure, and he also shown a way of redefining the gravitation and electromagnetic energy. His work is only restricted to virtual photon, because emission of a real photon would violet energy and momentum conservation where obtain. Though the electric scalar potential, electric

potential energy and mechanical energy were not obtained in combined gravitational and electric fields (Mahdi, 2014).

CHAPTER THREE

METHODOLOGY

3.1 Construction of Golden Riemannian Dynamical Gravito Electric Field Equation

It is well known that, Maxwell dynamical electric field equation (Howusu & Bakwa, 2007; Howusu, 2010; Haus & Melcher, 1989) is given as

$$\nabla^2 \Phi = \begin{cases} \frac{1}{\epsilon} \rho & ; \quad r < R \\ 0 & ; \quad r > R \end{cases} \quad (3.1)$$

where,

∇^2 is the Laplacian operator

Φ is the electric scalar potential exterior and interior to the body

ρ is the density of electric charged.

ϵ is the permittivity

r is the mean distance from the centre of the sphere.

R is the radius of the static homogeneous spherical body.

In the year 2009 Howusu published in his book title “Riemannian revolution in Physics and Mathematics” proposed that there are three different methods by which the Maxwell electric field equation can be extended or generalized namely;

- (i). The great Riemannian Laplacian operator.
- (ii). The golden Riemannian Laplacian operator.
- (iii). The super golden Riemannian Laplacian operator.

In other to achieve the first objective of this research work, we use the second method which is the golden Riemannian Laplacian operator which is given explicitly in spherical polar coordinates (Friedman, 1956; Howusu, 2010; Howusu, 2013) by

$$\begin{aligned}
\nabla_R^2 \Phi &= \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ \left(1 + \frac{2}{c^2} f \right) r^2 \frac{\partial}{\partial r} \Phi(x^0, r, \theta, \phi) \right\} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left\{ \sin \theta \frac{\partial}{\partial \theta} \Phi(x^0, r, \theta, \phi) \right\} \\
&+ \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \Phi(x^0, r, \theta, \phi) - \frac{\partial}{\partial x^0} \left\{ \left(1 + \frac{2}{c^2} f \right)^{-1} \frac{\partial}{\partial x^0} \Phi(x^0, r, \theta, \phi) \right\} \\
&= \begin{cases} \frac{1}{\varepsilon} \rho ; & r < R \\ 0 ; & r > R \end{cases} \quad (3.2)
\end{aligned}$$

where,

$x^0 = ct$ is the space time coordinate and $\Phi(x^0, r, \theta, \phi)$ is the complete generalized electric scalar potential exterior to the body.

3.2 The Golden Riemannian Dynamical Gravito Electric Scalar Potential Exterior and

Interior to the Body

We seek the exterior and interior solution of the generalized dynamical electric field equation (3.2) to obtain a generalized dynamical electric scalar potential (Φ) exterior and interior to the body by applying the following conditions:

- (i). continuity across all boundaries.
- (ii). continuity of normal derivative across all boundaries.
- (iii). imposing well known physical restriction between charge and density.

3.3 The Golden Riemannian Dynamical Gravito Electric Potential Energy

It is well known that Maxwell's dynamical electric field potential energy for a particle of non-zero rest mass in an electric field (Chifu & Lumbi, 2012; Howusu, 2012) is given as

$$V_e = q\Phi_e \tag{3.3}$$

where,

Φ_e is the electric scalar potential

V_e is the electric field potential energy

q is the electric charge of the particle

Consider the generalization of Maxwell electric field potential energy known as gravito electric potential energy given explicitly as

$$V_{ge} = q\Phi^+ \quad (3.4)$$

where,

Φ^+ is the golden Riemannian Gravito electric scalar potential exterior to the body

q is the electric charge of the particle

We use this equation to obtain a golden Riemannian Dynamical gravito electric potential energy to all order of c^{-2} for particles of non-zero rest masses.

3.4 The Golden Riemannian Dynamical Gravito Electric Mechanical Energy

It is well known that the instantaneous mechanical energy for test particle in combined gravitational and electric field (Chifu & Lumbi, 2012; Howusu, 2010) is given as

$$E = T + V_g + V_e \quad (3.5)$$

where,

T is the total relativistic kinetic energy

V_g is the gravitational field potential energy

V_e is the electric field potential energy

E is the mechanical energy

Consider the generalization of Maxwell mechanical energy known as gravito mechanical energy given explicitly as

$$E = T + V_{ge} \quad (3.6)$$

where,

V_{ge} is the Gravito electric field potential energy

We use this equation to obtain the golden Riemannian gravito mechanical energy to all order of c^{-2} for particle of non-zero rest masses.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Construction of Golden Riemannian Dynamical Gravito Electric Field Equation

Consider a static homogeneous spherical massive body of radius R and rest mass M carrying a uniform distribution of total electric charge Q . The well known Maxwell's dynamical electric field equation (Haus & Melcher, 1989; Weinberg, 1972a; Howusu, 2010) is given by

$$\nabla^2\Phi = \begin{cases} \frac{1}{\epsilon}\rho & ; r < R \\ 0 & ; r > R \end{cases} \quad (4.1)$$

where,

∇^2 is the Laplacian operator

Φ is the electric scalar potential exterior and interior to the body

ρ is the density of electric charged.

ϵ is the permittivity

r is the mean distance from the centre of the sphere.

R is the radius of the static homogeneous spherical body.

The Riemannian Laplacian operator (Friedman, 1956; Howusu, 2010) is given by

$$\begin{aligned}
\nabla_R^2(x^0, r, \theta, \phi; f) &= \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ \left(1 + \frac{2}{c^2} f\right) r^2 \frac{\partial}{\partial r} \right\} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left\{ \sin \theta \frac{\partial}{\partial \theta} \right\} \\
&+ \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} - \frac{\partial}{\partial x^0} \left\{ \left(1 + \frac{2}{c^2} f\right)^{-1} \frac{\partial}{\partial x^0} \right\} \\
&= \begin{cases} \frac{1}{\varepsilon} \rho ; & r < R \\ 0 ; & r > R \end{cases} \quad (4.2)
\end{aligned}$$

The general electric field equation (Howusu, 2010; Haus & Melcher, 1989) is given by

$$\nabla_R^2(x^0, r, \theta, \phi; f) = \frac{1}{\varepsilon} \rho(x^0, r, \theta, \phi; f) \quad (4.3)$$

Or more explicitly in spherical polar coordinate

$$\begin{aligned}
\nabla_R^2 \Phi(x^0, r, \theta, \phi) &= \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ \left(1 + \frac{2}{c^2} f\right) r^2 \frac{\partial}{\partial r} \Phi(x^0, r, \theta, \phi) \right\} \\
&+ \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left\{ \sin \theta \frac{\partial}{\partial \theta} \Phi(x^0, r, \theta, \phi) \right\} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \Phi(x^0, r, \theta, \phi) \\
&- \frac{\partial}{\partial x^0} \left\{ \left(1 + \frac{2}{c^2} f\right)^{-1} \frac{\partial}{\partial x^0} \Phi(x^0, r, \theta, \phi) \right\} = \begin{cases} \frac{1}{\varepsilon} \rho ; & r < R \\ 0 ; & r > R \end{cases} \quad (4.4)
\end{aligned}$$

where,

$x^0 = ct$ is the space time coordinate

$\nabla_{\mathbb{R}}^2$ is the Riemannian Laplacian operator

$\Phi(x^0, r, \theta, \phi)$ is the complete generalized electric scalar potential exterior and interior to the body.

The electric scalar potential field function is a function of only the radial coordinate r .

Therefore

$$\frac{\partial \Phi}{\partial \theta} = 0 \tag{4.5}$$

$$\frac{\partial \Phi}{\partial \phi} = 0 \tag{4.6}$$

$$\frac{\partial \Phi}{\partial x^0} = 0 \tag{4.7}$$

Equation (4.4) reduces to

$$\begin{aligned}\nabla_R^2 \Phi(r) &= \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ \left(1 + \frac{2}{c^2} f^+ \right) r^2 \frac{\partial}{\partial r} \Phi(x^0, r, \theta, \phi) \right\} \\ &= \begin{cases} \frac{1}{\varepsilon} \rho ; & r < R \\ 0 ; & r > R \end{cases}\end{aligned}\quad (4.8)$$

Consider a generalization of Maxwell electric field equation for a static homogeneous spherical charged massive body using the golden gravitational scalar potential exterior to the body (Lumbi et., 2016) given as

$$f^+(r) = \frac{-k}{r} \left\{ 1 - \frac{3k}{5c^2 R} \right\} + \frac{2k^2}{c^2 r^2} \quad (4.9)$$

Substituting equation (4.9) into equation (4.8) and differentiating we obtain equation which gives

$$\begin{aligned}\left\{ \frac{2k}{c^2 r} - \frac{6k^2}{5c^4 r^2 R} - \frac{8k^2}{c^4 r^3} \right\} \Phi' + \frac{2}{r} \left\{ 1 - \frac{2k}{c^2 r} + \frac{6k^2}{5c^4 r R} + \frac{4k^2}{c^4 r^2} \right\} \Phi' \\ + \left\{ 1 - \frac{2k}{c^2 r} + \frac{6k^2}{5c^4 r R} + \frac{4k^2}{c^4 r^2} \right\} \Phi'' = \begin{cases} \frac{1}{\varepsilon} \rho ; & r < R \\ 0 ; & r > R \end{cases}\end{aligned}\quad (4.10)$$

Or more explicitly

$$\begin{aligned} \frac{2}{r}\Phi' + \Phi'' - \frac{2k}{c^2r}\Phi'' - \frac{2k}{c^2r^2}\Phi' + \frac{6k^2}{5c^4r^2R}\Phi' + \frac{6k^2}{5c^4rR}\Phi'' + \frac{4k^2}{c^4r^2}\Phi'' \\ = \begin{cases} \frac{1}{\epsilon}\rho ; & r < R \\ 0 ; & r > R \end{cases} \end{aligned} \quad (4.11)$$

Equation (4.11) is the golden Riemannian dynamical gravito electric field equation for static homogeneous spherical charge massive bodies. This equation,

- (i). Reduces to c^0 to the corresponding pure Maxwell dynamical electric field equation and to the order of c^{-2} , it contains unknown Post-Maxwell's additional correction terms which are uncovered for theoretical development and application.
- (ii). The golden Riemannian dynamical gravito electric field equation contains $1 + \frac{2}{c^2}f$ which are not found in the existing well known Maxwell's dynamical electric field equations hence it implies that it will predict gravito-electric phenomena.
- (iii). The golden Riemannian dynamical gravito electric field equation can be used to generates electromagnetic field.
- (iv). Contains the time component which is not found in the well known Maxwell's dynamical electric field equation and thus implying that it will predict the existence of gravitational waves
- (v) Indicates that it is possible to unify all forces in nature.

4.2 The Golden Riemannian Dynamical Gravito Electric Scalar Potential Exterior and Interior to the Body

Let us seek the solution of the exterior field equation (4.11) as

$$\Phi^+(r) = \frac{A_1}{r} + \frac{A_2}{r^2} + \dots \quad (4.12)$$

Differentiating equation (4.12) with respect to r yields (4.13) and (4.14)

$$\Phi' = -\frac{A_1}{r^2} - \frac{2A_2}{r^3} + \dots \quad (4.13)$$

$$\Phi'' = \frac{2A_1}{r^3} + \frac{6A_2}{r^4} + \dots \quad (4.14)$$

Where A_1 and A_2 are arbitrary constants.

Substituting equation (4.13) and (4.14) into the golden Riemannian dynamical gravito electric field equation (4.11) and equating coefficients of $\frac{1}{r^3}$, $\frac{1}{r^4}$, $\frac{1}{r^5}$ and $\frac{1}{r^6}$ on both sides, we obtain equations which gives

$$A_1 = 0 = \text{Arbitrary constant} \quad (4.15)$$

$$A_2 = \frac{A_1 k}{c^2} \left(1 - \frac{3k}{5c^2 R} \right) \quad (4.16)$$

or

$$A_2 = \frac{A_1 k}{c^2} \left(1 + \frac{3k}{5c^2 R} \right) \quad (4.17)$$

Hence, the general solution of the exterior field equation (4.12) is given by

$$\Phi^+(r) = \frac{A_1}{r} + \frac{A_1 k}{c^2 r^2} \left(1 - \frac{3k}{5c^2 R} \right) + \dots \quad (4.18)$$

or

$$\Phi^+(r) = \frac{A_1}{r} + \frac{A_1 k}{c^2 r^2} \left(1 + \frac{3k}{5c^2 R} \right) + \dots \quad (4.19)$$

Let us seek the solution of the interior field equation (4.11) as

$$\Phi^-(r) = \Phi_c^- + \Phi_p^- \quad (4.20)$$

where,

Φ_c is the complementary solution

Φ_p is the particular solution

The complementary solution of the interior field equation

$$\Phi_c^-(r) = B_0 \tag{4.21}$$

where,

B_0 is an arbitrary constant

Seeking the particular solution the interior field equation in the form

$$\Phi_p^-(r) = D_2 r^2 + D_4 r^4 + \dots \tag{4.22}$$

Differentiating equation (4.22) with respect to r yields (4.23) and (4.24)

$$\Phi' = 2D_2 r + 4D_4 r^3 + \dots \tag{4.23}$$

$$\Phi'' = 2D_2 + 12D_4 r^2 + \dots \tag{4.24}$$

where, D_2 and D_4 are arbitrary constants

Substituting equation (4.23) and (4.24) into the golden Riemannian dynamical gravito electric field equation (4.11) and equating coefficients of $r^0, \frac{1}{r}, \frac{1}{r^2}, r$ and r^2 on both sides, we obtain equations which gives

$$D_2 = \frac{1}{6\epsilon} \rho \quad (4.25)$$

$$D_4 = 0 \quad (4.26)$$

Using the well known physical relationship between charge and density for a sphere of radius R equation (4.25) become

$$D_2 = \frac{Q}{8\pi\epsilon R^3} \quad (4.27)$$

where, Q is the total charge on the sphere

The general solution of the interior field equation (4.20) is given by

$$\Phi^-(r) = B_0 + D_2 r^2 + D_4 r^4 + \dots \quad (4.28)$$

By the condition of continuity of electric scalar potential function across boundaries,

$$\Phi^+(r) = \Phi^-(r)$$

$$B_0 + D_2 r^2 + D_4 r^4 + \dots = \frac{A_1}{r} + \frac{A_2}{r^2} + \dots \quad (4.29)$$

By the condition of continuity of normal derivatives of electric scalar potential function across all boundaries,

$$\left(\frac{\partial \Phi^+}{\partial r} \right)_{r=R} = \left(\frac{\partial \Phi^-}{\partial r} \right)_{r=R}$$

$$2D_2 R + 4D_4 R^3 + \dots = -\frac{A_1}{R^2} - \frac{A_2}{R^3} + \dots \quad (4.30)$$

but

$$A_2 = \frac{A_1 k}{c^2} \left(1 - \frac{3k}{5c^2 R} \right)$$

$$2D_2R + 4D_4R^3 = -\frac{A_1}{R^2} - \frac{A_1k}{c^2} \left(1 - \frac{3k}{5c^2R}\right) \quad (4.31)$$

Solving equation (4.31) for A_1 gives

$$A_1 = -2D_2R^3 + \frac{4kR^2}{c^2} D_2 \left(1 - \frac{3k}{5c^2R}\right) \quad (4.32)$$

Substituting the value of D_2 into equation (4.32), we obtain

$$A_1 = -\frac{Q}{4\pi\epsilon} + \frac{kQ}{2\pi\epsilon c^2 R} \left(1 - \frac{3k}{5c^2R}\right) \quad (4.33)$$

also

$$A_2 = \frac{A_1k}{c^2} \left(1 - \frac{3k}{5c^2R}\right) \quad (4.34)$$

$$A_2 = \frac{k}{c^2} \left(1 - \frac{3k}{5c^2R}\right) \left[-\frac{Q}{4\pi\epsilon} + \frac{kQ}{2\pi\epsilon c^2 R} \left(1 - \frac{3k}{5c^2R}\right)\right] \quad (4.35)$$

Hence the exterior field equation is given by

$$\begin{aligned}\Phi^+(r) &= -\frac{Q}{4\pi\epsilon r} + \frac{kQ}{2\pi\epsilon c^2 r R} \left(1 - \frac{3k}{5c^2 R}\right) \\ &+ \frac{k}{c^2 r^2} \left(1 - \frac{3k}{5c^2 R}\right) \left[-\frac{Q}{4\pi\epsilon} + \frac{kQ}{2\pi\epsilon c^2 R} \left(1 - \frac{3k}{5c^2 R}\right)\right]\end{aligned}\quad (4.36)$$

Equation (4.36) reduces to all order of c^{-2} as

$$\Phi^+(r) = -\frac{Q}{4\pi\epsilon r} + \frac{kQ}{2\pi\epsilon c^2 r R} - \frac{kQ}{4\pi\epsilon c^2 r^2} + \dots \quad (4.37)$$

Solving the equation (4.29) for B_0 to the order of c^{-2} gives

$$B_0 = \frac{A_1}{R} + \frac{A_2}{R^2} - D_2 R^2 - D_4 R^4 \quad (4.38)$$

Substituting the value of A_1, A_2 and D_2 into equation (4.38), we get

$$\begin{aligned}B_0 &= -\frac{Q}{4\pi\epsilon R} + \frac{kQ}{2\pi\epsilon c^2 R^2} \left(1 - \frac{3k}{5c^2 R}\right) \\ &+ \frac{k}{c^2 R^2} \left(1 - \frac{3k}{5c^2 R}\right) \left[-\frac{Q}{4\pi\epsilon} + \frac{kQ}{2\pi\epsilon c^2 R} \left(1 - \frac{3k}{5c^2 R}\right)\right] - \frac{Q}{8\pi\epsilon R}\end{aligned}\quad (4.39)$$

Equation (4.39) reduces to all order of c^{-2} as

$$B_0 = -\frac{3Q}{8\pi\epsilon R} + \frac{kQ}{4\pi\epsilon c^2 R^2} \quad (4.40)$$

Hence the interior field equation is given by

$$\Phi^-(r) = -\frac{3Q}{8\pi\epsilon R} + \frac{Q}{8\pi\epsilon R^3} r^2 + \frac{kQ}{4\pi\epsilon c^2 R^2} + \dots \quad (4.41)$$

Equation (4.37) and (4.41) are the golden Riemannian dynamical gravito electric scalar potential exterior and interior to the body for static homogeneous spherical charge massive bodies. This equation,

(i). Reduces to c^0 to the corresponding pure Maxwell's electric scalar potential and consist of Post-Maxwell additional terms of all order of c^{-2} which are uncovered for theoretical development and application.

The consequences of the golden Riemannian dynamical gravito electric scalar potentials are:

(i) They can be applied to the dynamical relativistic equation of motion for particles of non-zero rest masses in combined gravitational and electric fields to describe the motion of satellites and behavior of charged bodies in space.

- (ii) They can be applied into spectral shift for particles of nonzero rest masses in combined gravitational electric fields to describe the motion of signals wave.
- (iii) They can be applied into the relativistic quantum mechanical wave equation for particles of non zero rest masses in combined gravitational and electric fields to describe the state of a system.
- (iv) They can be substituted into any other electric equation of motion to obtain corresponding results and the physical effects.

4.3 The Golden Riemannian Dynamical Gravito Electric Potential Energy

Consider Maxwell's theory of electromagnetism, the electric potential energy for a particle of non-zero rest mass in an electric field (Chifu & Lumbi, 2012; Howusu, 2012) is given by

$$V_e = q\Phi_e \tag{4.42}$$

where,

Φ_e is the electric scalar potential

V_e is the electric field potential energy

q is the electric charge of the particle

The golden Riemannian dynamical gravito electric scalar potential in the region of the body define in this field as

$$\Phi^+(r) = -\frac{Q}{4\pi\epsilon r} + \frac{kQ}{2\pi\epsilon c^2 r R} - \frac{kQ}{4\pi\epsilon c^2 r^2} + \dots \quad (4.43)$$

Consider the generalization of Maxwell electric field potential energy known as gravito electric potential energy given explicitly as

$$V_{ge} = q\Phi^+ \quad (4.44)$$

where,

Φ^+ is the golden Riemannian Gravito electric scalar potential exterior to the body

q is the electric charge of the particle

Substituting equation (4.43) into (4.44) we obtain

$$V_{ge} = -\frac{qQ}{4\pi\epsilon r} + \frac{kqQ}{2\pi\epsilon c^2 r R} - \frac{kqQ}{4\pi\epsilon c^2 r^2} + \dots \quad (4.45)$$

Equation (4.45) is the golden Riemannian dynamical gravito electric potential energy for static homogeneous spherical charge massive body to all order of c^{-2} for particle of non-zero rest masses. This equation,

- (i) Reduces to the limit c^0 to the corresponding pure Maxwell potential energy equation and contains post-Maxwell additional terms of all order c^{-2} which are uncovered for experimental investigation and theoretical development and applications.
- (ii). Can be used to determine the gravito electric potential between two massive bodies.
- (iii) Can be used to show that every electromagnetic field is coupled to a gravito electric/gravito-magnetic field and that induction between both fields is possible.
- (iv). Can be used to investigate how energy propagate in a gravitational field (gravitational poynting vector).

4.4 The Golden Riemannian Dynamical Gravito Electric Mechanical Energy

Consider the instantaneous mechanical energy for test particles in combined gravitational and electric fields (Howusu, 2010; Chifu & Lumbi, 2012) is defined by

$$E = T + V_g + V_e \quad (4.46)$$

where,

T is the total relativistic kinetic energy

V_g is the gravitational field potential energy

V_e is the electric field potential energy

E is the mechanical energy

The golden Riemannian dynamical in combined gravitational and electric potential energy in the region of the body define in this field as

$$V_{ge} = -\frac{qQ}{4\pi\epsilon r} + \frac{kqQ}{2\pi\epsilon c^2 r R} - \frac{kqQ}{4\pi\epsilon c^2 r^2} + \dots \quad (4.47)$$

Consider the generalization of Maxwell mechanical energy known as gravito mechanical energy given explicitly as

$$E = T + V_{ge} \quad (4.48)$$

where,

V_{ge} is the Gravito electric field potential energy.

The total relativistic kinetic energy T is given as

$$T = \left[\left(1 - \frac{u^2}{c^2} \right)^{-\frac{1}{2}} - 1 \right] m_0 c^2 \quad (4.49)$$

Substituting equation (4.47) and (4.49) into (4.48) we obtain

$$E = -\frac{qQ}{4\pi\epsilon r} + m_0 c^2 \left[\left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} - 1 \right] + \frac{kqQ}{2\pi\epsilon c^2 r R} - \frac{kqQ}{4\pi\epsilon c^2 r^2} + \dots \quad (4.50)$$

Equation (4.50) is the golden Riemannian dynamical gravito electric mechanical energy for static homogeneous spherical charge massive body to all order of c^{-2} for particle of non-zero rest masses. This equation,

- (i) Reduces to the limit c^0 to the corresponding pure instantaneous mechanical energy equation and contain post-Maxwell additional terms of all order c^{-2} which are uncovered for experimental investigation and theoretical development.
- (ii) Can be applied to a generalized spectral shift for photons in combined gravitational and electric fields to determine the change in the frequency of light wave observe from a relative motion of a source.
- (iii). Can be used to send signals in space to source information through the study of cosmological bodies.
- (iv). Contains relativistic total kinetic energy which are not found in the well known existence mechanical energy to determine the mechanical motion and interaction of bodies in space.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

In this research work we generalized Maxwell's electric field equation using Riemannian Laplacian operator and golden gravitational scalar potential called golden Riemannian dynamical gravito electric field equation and this field equation were applied to static homogeneous spherical charged massive bodies to obtain golden Riemannian dynamical gravito electric scalar potential exterior and interior to the bodies.

The golden Riemannian dynamical gravito electric scalar potential exterior to the body where substituted into a generalized Maxwell's electric potential energy and mechanical energy for particles of non zero rest masses in an electric field to obtained golden Riemannian dynamical gravito electric potential energy and golden Riemannian dynamical gravito electric mechanical energy. These equations reduce to the limit c^0 to the corresponding pure Maxwell terms and contains additional terms of all order of c^{-2} which are not found in the existing Maxwell's dynamical theory.

5.2 Conclusion

The results obtained in this research work, can be used to explain the problem of some observed experimental phenomena particularly, the Michelson-Morley experiment, Photo-electric effect and double slit interferences which was not predicted by Maxwell's electric theory of electromagnetism.

The presented coupling of electromagnetism and gravitation in the weak field approximately showed that both fields can be converted into each other. We find that every electromagnetic field is associated with a gravitomagnetic field, but gravitomagnetic fields can exist without electromagnetic ones in the case of neutral matter.

Electromagnetism is associated with available wide ranges of frequencies, currents and potentials and is therefore more "engineerable" than the handling of heavy masses. Hence, using electromagnetic fields to induce non-Newtonian gravitational fields is an issue closely related to material sciences, cryogenic technology and extremely high frequency problems.

Our proposed coupling concept between gravitation and electromagnetism will help to understand the similarity of both force fields despite their different mathematical description in general relativity and Maxwell theory. This is necessary along the goal of physicist to unify all forces in nature.

5.3 Recommendations

5.3.1 Gravitation is the study of nature which gives better understanding of the universe.

Most of the bodies in the universe are governed by the gravitational force. Electromagnetic involves the macroscopic behavior of electric charge in vacuum and matter. This behavior can be accurately characterized by the Lorentz force law and Maxwell's equation. This research work will give us better understanding of the behavior of charge bodies in space. Therefore it is recommended to be applied to the motion of the planets or satellites in the solar system as well as to the motion of

test particles in combined gravitational and electric fields of a static homogenous spherical charge massive bodies.

In this research work, we have assumed static homogenous spherical charge massive bodies to obtain our results. As an extension to this research, we suggest the following:

5.3.2 The work can be extended to include the non homogenous, non static and non spherical charge massive bodies.

5.3.3 This entire work can be repeated for all the coordinate systems in Physics and Mathematics in nature such as the spherical polar coordinates, the cartessian, cylindrical, spheriodal, prolate, oblate, conical e.t.c

5.3.4 The work can be extended to obtain the relativistic quantum mechanical wave equation for particles of non zero rest masses in combined gravitational and electric fields of static, non static and non homogenous massive bodies.

5.4 Limitation of the Study

This research work is only limited to the study of the electric field of static homogeneous spherical charge massive body. It can also be extended to include non-static and non-homogeneous bodies.

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