



**Design of a PSO- Tuned Integral LQR Based Controller for Position and Sway
control of a Double Pendulum Crane System**

BY

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M. ENG. ELECTRICAL

**A DISSERTATION SUBMITTED TO THE DEPARTMENT OF ELECTRICAL
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(M.ENG) (CONTROL AND INSTRUMENTATIONS).**

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DECLARATION

I hereby declare that I carried out the work reported in this dissertation in the department of Electrical Engineering, Bayero University, Kano, under the supervision of Engr Dr. Mustapha Muhammad. I also solemnly declare that to the best of my knowledge, no part of this dissertation has been submitted here or elsewhere in a previous application for award of a degree. All sources of knowledge used have been duly acknowledged.

Ibrahim Abdulhamid Bako

SPS/16/MEE/00025

CERTIFICATION

This is to certify that the dissertation entitled “**Design of a PSO-Tuned LQR Based Controller for Position and Sway control of a Double Pendulum Crane System**” submitted by Ibrahim Abdulhamid Bako (SPS/16/MEE/00025) in the partial fulfillment of the requirement for the degree of Master (M.Eng) in Control and Instrumentation Engineering, to the Department of Electrical Engineering Bayero University Kano, is an Authentic work carried out by me under my supervisor. To the best of my knowledge and belief, all the materials presented in this thesis except where otherwise referenced is my own original work, and has not been published previously for the award of any other degree and diploma in any university.

Signature and date-----

Supervisor: Associate Professor Mustapha Muhammad

Signature and date-----

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APPROVAL

This dissertation has been examined and approved for the award of the degree of Master in Engineering (Electrical Control and Instrumentation).

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DEDICATION

This Research work is dedicated to the Family late Alh.Dahiru Lawal, my late Brother Abdussalam Abdulhamid Bako and my late Friend Rabi'u Musa. May their souls rest in perfect peace amen.

ABSTRACT

Double pendulum crane (DPC) is a type Gantry crane which is commonly used for point-to-point transportation, lowering and lifting of payload. The use of cable for hoisting of payload can lead to natural swaying which makes it difficult to perform alignment, fine positioning as well as detrimental to safe and efficient operation. For proper operation of DPC an effective control of the sway and position is essential. A mathematical model of the double pendulum crane system was derived from Euler-Lagrange equation. This research aimed at studying the complex nature of the non-linear under-actuated double pendulum crane system with the purpose of understanding the challenges in developing its control algorithm based on Linear Quadratic Regulator (LQR) that will not only minimized the sway and have a good position tracking but would also eliminate the use of many controllers. Hence LQR controller was adopted for this work which is an optimal state feedback controller that minimizes quadratic cost function J . The linearized model of the system was firstly obtained by using Taylor's expansion from the non-linear system and then the controllability of the system was tested. One of the important challenges in the design of LQR controller is the optimal choice of state and weighing matrices (Q and R) which play a vital role in determining the performance and optimality of the controller. Commonly, trial and error approach was employ for selecting the weighing matrix which not only burdens the design but also results in non-optimal response. Hence to choose the element of Q and R matrices optimally Particle Swarm Optimization (PSO) algorithm was formulated. The efficacy of the PSO-tuned LQR controller was tested on the DPC system via simulation. The simulation result shows that the proposed controller is able to track the trolley position relatively fast with minimum hook and payload swing angles hence, reduce the main problem of a double pendulum crane. From the performance comparison of the results, the proposed method has improved in terms of reducing the trolley position percentage overshoot, hook oscillation and payload oscillation with 65%, 89.8% and 88.6% respectively and also eliminating the use of many controllers.

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CHAPTER ONE

INTRODUCTION

1.1 Background and Motivation

Cranes are machine or device frequently used for the transportation of heavy loads (payload) and hazardous materials in industrial fields such as factories, construction sites, harbor, etc. usually cranes have very strong structures in order to lift heavy payloads as shown in fig 1. In factories, crane speed up the production processes by moving heavy materials to and from the factory as well as moving the products along production or assembly line. In building construction, cranes facilitate the transport of building materials to high and critical spots. Similarly on ships and in harbors crane save time and consequently money in making the process of loading and unloading ships fast and efficient Amjed A. et al[1]. Until recently, cranes were manually operated Amjed A. et al[1]. Although several types of cranes such as overhead crane, gantry crane, boom crane and tower crane show in fig. 2 are used for different industrial applications, but the problem is the same [2]: transporting the load from one point to another as fast as possible without causing any excessive swinging, but fast transportation cable-bound heavy load causes the excessive oscillations which threaten the operation safety. On the other hand, low operating speed causes the reduction of operating efficiency [1]. Therefore, the position of the crane needs to be controlled such that the swing of the load is suppressed.



Figure 1.1.1 Gantry Crane [18]



Figure 1.1.2 Double pendulum Gantry Crane [12]



Figure 1.1.3 Tower Crane [3]



Figure 1.1.4 Boom Crane [3]

The control objective of crane is to transport massive payloads to desired position accurately as well as suppress and eliminate the payload oscillation rapidly [4]. To increase transportation efficiency, and decrease payload oscillation various control method have been proposed. Most of the control method treats the payload oscillation as a single pendulum without considering a hook mass. However, in practical, there are cases where double pendulum is used [4]. In industrial environments, a fast and accurate positioning with minimum hook and payload oscillations are desirable for efficient and safe operating the crane system. Dynamic of a double pendulum crane is complicated and therefore control design is challenging.

The double pendulum crane system is an under-actuated non-linear system with one control input (trolley force) and these control variables: (trolley position, hook and payload oscillation angles). In recent years, numerous research involving double pendulum crane have been carried out, and since then many researchers investigated various control double pendulum crane such as input shaper [1]. Sliding mode [2] etc. In this research, an LQR base controller shall be proposed.

1.2 PROBLEM STATEMENT

Double Pendulum crane operations induces undesirable sway due to acceleration of an inertia payload [5]. After reaching the desired location, the payload still oscillates this may both be dangerous and inefficient, which may hinder the speed as well as safety of operations. During the operation, the load is free to swing in a pendulum-like motion. If the swing exceeds a proper limit, it must be damped or the operation must be stopped until the swing dies out. Either option consumes time, which reduces the facility availability as well as efficiency. LQR design involves the selection of some weighting factors for the cost

function to be minimized. However, there is no specific technique in choosing these values. The common approach to selecting these weighting matrices is via trial and error but this method could be time consuming, cumbersome and result in a non-optimized performance. Therefore, a more systematic approach in selecting these values is needed

1.3 Scope of The Study

The scope of study is limited to non-linear double-pendulum-crane system using PSO-Tuned LQR controller and comparing it with a PSO-PID base controlled double-pendulum-crane. The implementation of the control scheme is limited to simulation and also the tuning of the parameters of the LQR controller (Q and R) is done offline.

1.4 Aim

The aim is to design an LQR controller to control the position and sway angle of the double-pendulum-crane. To achieve this aim, the following objectives are set:

1. To develop an algorithm that will tune (optimize) the parameters of the LQR
2. To design the LQR controller for the double pendulum crane using the optimal parameters (Q and R).
3. To validate the performance of the controller by comparing with a PSO-Tuned PID controller for the double pendulum crane through simulation in MATLAB/SIMULINK

1.5 Methodology

The work is planned to be carried out in the following stages:

1. Development of MATLAB codes for PSO algorithm based on the linearized model of the DPC
2. Simulink implementation of the optimized LQR controller
3. Performance analysis and validation of the proposed controller with that of PSO-tuned PID controller via simulation.

1.6 Thesis Structure

A brief description for each chapter and the organization of this thesis is structured as follows;

Chapter two presents the previous works and discussions by individuals in different areas of gantry crane systems as well as double pendulum crane system.

Chapter three presents, the fundamentals of the dynamic model for the double pendulum crane system which is derived using Euler-Lagrange equation, linearizing the nonlinear model using Taylor's series and finally the controller design is also seen in this chapter.

Chapter four shows the simulation and results of both the PSO based LQR and PID control scheme for precise position tracking as well suppressing the sway of the double pendulum crane system.

Chapter five presents the conclusion, summary and suggestions for future work

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter looks into the literature review of some previous works on overhead gantry crane, double pendulum crane system and various control techniques. Dynamics of a double-pendulum crane is complicated and therefore control design is challenging. The double pendulum crane system is an under-actuated nonlinear system with one control input (trolley force) and three control variables (trolley position, hook and payload oscillation angles). Hence, it is very interesting and meaningful to be studied and explored on double-pendulum crane system due to real industrial applications

2.2 Previous Work

Low operating speeds causes the reduction of operating efficiency. So, position of the crane must be controlled such a way that the swing of the load is suppressed. For this purpose, numerous studies have been conducted for the position control of cranes as seen in [2]. In [6] a fuzzy logic anti-swing controller for 3-dimensional overhead-crane proposed control not only suppressed the load swing but also accurate control of the position. However the effect of the load mass was not considered in the tracking controller. The system responses are slow as compared with the optimal or feedback control.

On the other hand, Vinh, [7] developed an Anti-sway algorithm and its implementation for industrial overhead and gantry cranes. The theses mainly focus on developing an inexpensive, effective and convenient solution to minimize the effect of

load sway in industrial crane operation. This project encourages the replacement of manual controlling of the crane with computer-aided system to improve speed and reliability. Additional mechanical structure was installed to the crane to maximize the damping effect of crane's oscillation. Target of the project was to fulfill following requirements: Active sway control of a gantry crane using hybrid input shaping and PID control schemes, the solution combines both hardware and software to effectively reduce sway. Also, Tumari, et al[8] worked on investigations into the development of hybrid input-shaping and PID control schemes for active sway control of a gantry crane system. He applied positive input shaping technique that reduces the sway by creating a common signal that cancels its own vibration and used as a feed-forward control which is for controlling the sway angle of the pendulum, while the proportional integral derivative (PID) controller is used as a feedback control which is for controlling the crane position. The PID controller was tuned using Ziegler-Nichols method to get the best performance of the system. The hybrid input-shaping and PID control schemes guarantee a fast input tracking capability, precise payload positioning and very minimal sway motion. The modeling of gantry crane is used to simulate the system using MATLAB/SIMULINK software. The results of the response with the controllers were presented in time domains and frequency domains. The performances of control schemes were examined in terms of level of input tracking capability sway angle reduction and time response specification. On the other hand, [9] developed a new combination of nonlinear back stepping scheme with fuzzy system to control both position regulation and anti-swing control to avoid collision with other equipment. This study uses two different fuzzy systems: off-line and on-line to consider different concept. The back stepping design

procedure consists of two steps. In all steps to design back stepping controller, Lyapunov stability theorem was used to stabilize analysis and control design problems. Furthermore, the study develops a nonlinear back stepping scheme with different approaches such as a neuro fuzzy system and a fuzzy neural net to control position regulation and swing angle. By combining back stepping scheme and a neuro-fuzzy system or a fuzzy neural net, improved performance of control and system's stability was obtained through these controllers completely and in a short time.

In addition,[10] worked on Anti-sway control of a tower crane using inverse dynamics. The main purpose of the controller is input tracking of the tower crane to the desired position while maintaining a small swinging angle. This paper investigates the response of a tower crane using input shaper. Then the response of the input shaper is compared to a controller based on inverse dynamics. The implementation and results of both open loop input shaper and inverse dynamics have been presented for a tower crane model. Both the control schemes have been compared in terms of sway angle reduction. The open loop input shaper has been effective in suppressing the oscillations. Then the inverse dynamics controller was developed while taking into consideration all of the system dynamics, including the parameters that were neglected in the input shaping controller which are the mass of the payload, the mass of the trolley and the inertia of the jib. The inverse dynamics controller showed better results and a higher oscillation reduction than the open loop input shaper.

Lien et al,[11] in their paper developed a multivariable control scheme to reduce the load swing while the load is simultaneously hoisted (or lowered) and transferred. The control problem for the original linear time-varying system has been reduced to the

optimal control of a linear time- invariant system. Electronic anti-sway prevents the sway by manipulating the acceleration and deceleration of the trolley or gantry motion. Mathematical algorithms modify the velocity of the trolley or gantry so that the sway is compensated but the crane driver loses control of the travelling motions; they are not capable of preventing rotational sway, because the systems manipulate the acceleration/ deceleration of the trolley / gantry motions, not each individual hoist rope fall. Hydraulic sway damping with high reliability dissipates the sway energy of the load but do not prevent the sway from starting. Inclined auxiliary ropes damp the sway, since the head block is subject to very heavy mechanical shock loads and increase the weight of the lifted load and decrease the free space available for maintenance purposes.

[12] et al developed a Novel control method for overhead crane's load stability; the goal of the work is to design a control system that minimizes swinging of the load carried by the crane. A Lyapunov-based approach was taken in the research. A linear motor model Siemens 1FN3150 2WC0 (located in LUT's Laboratory of Intelligent Machines) was used with a metal structure attached to it. In this metal structure a pendulum is held and oscillates simulating the crane movement. The aim is to optimize the process making the movement and stabilization of the pendulum as fast and swiftly as possible. The PID controller parameters were tuned online for a pulse reference input. For this step the Lyapunov controller was ignored while only the position controller (PID) was being used. The procedure consisted of increasing the proportional gain until the first signs of overshoot appeared, then the integral and derivative gains was used to compensate it. In the interface used in Control Desk during the online PID tuning the PID gains could be changed with the different numerical inputs and position and position reference were

shown in a plotter. Angle and angle reference are also being plotted although not used. Everything was done to simulate the overhead crane load swinging problem for which an original approach was taken reducing the driver's role in the compensation of swinging problem allowing 100% manual control until the destination is reached and then the designed controller is enabled compensating the swinging automatically. The results were satisfactory concluding that implementing this system in real systems (real overhead cranes) could greatly improve productivity especially in lack of a skillful crane operator. [13] Proposed fuzzy system with LQR control considering the application of the control in real time. The result of the proposed control scheme shows a good position tracking and a minimum oscillation of the swing angle. However, it involves the use of many controllers which makes the system complex. [14] Change uses adaptive fuzzy controller which only used the position error and swing angle but also has a reduced fuzzy rule to control the crane. In the result of the paper, the trolley positions still keep large even when the trolley arrives at destination. As in Auwalu M. et al[15] used output-based command shaping for effectively payload sway control of a crane with hoisting. However, the position tracking of the trolley was not considered and also the control scheme was based on linearized model as well as an open loop control, which always has a problem of delay.

However Lingzhi Cao, et al[16] design a controller based on non-singular terminal sliding mode. His method has achieved a good speed performance, and was able to eliminate steady-state error as well as overshoot, but it draws high control current and also sway angle was neglected. Johua Vaughan, et al [17] used input shaper control method control a tower crane with double pendulum payload dynamic so as to tract the position

and also reduce the residual oscillation with robustness to error. But the resulting controller is open-loop which has the problem of delay and hence sensitive to disturbance. S. Pezeshki, et al [18] in their research, model-free adaptive controller (MFAC) using feedback linearization and adaptive fuzzy sliding mode controller (AFSMC) which used the trolley position and swing angle information to control the process. It has successfully regulated the crane in terms of disturbance (wind shift). Consequently it was assumed that the system is continuous time system but its output are sampled and used for the proposed controller. However, so, if the sample rate is chosen lower than the required value the controller cannot handled the closed-loop system very well and vice-versa. Hence it does not have good accuracy in computing the control signal. In other to avoid the open-loop disadvantages, Leila R. et al [19] conducted a research on a feedback control but they found out that the control performed poorly when implemented on a closed-loop form. The poor performance was attributed to limit cycle resulting from the oscillation of the control action around the switching surface also it was based on trial and error.

More so, a feedback with adaptive gains was employed by Jaroslaw S. et al [7] which was calculated based on the pole placement technique rather than adaptive. Research involving double pendulum crane system started in 1998 [20]. Since then, it has become an area of interest for which researchers investigated various control techniques for DPC and is becoming an attractive benchmark that involves delayed feedback control [21] decoupling control [22], wave-based control[23], passivity –based control [24] sliding mode control [25]. Alhazza et al[26] generated an acceleration profile for DPC system manoeuvres involving hosting using an iterative learning control. However, the

measurement of the payload oscillation was not discussed. Other work includes energy based control [27], super twisting, based control [28], online motion planning [29]. In[30][31] , and optimal trajectory planning was proposed.

Nevertheless, in [32] adaptive control method was proposed for the DPC system which successfully suppressed the maximum hook and play load oscillation, less maximum control force and lower transportation time. Nevertheless, all these control method involves rigorous mathematical analysis. Hazriq et al [33] proposed a PSO turned PID control method for the DPC system, this method successfully tracked the position but took longer time for the sway angles to settle. As well as the use of many controllers.

In this study, PSO based parameter optimization of a linear quadratic regulator (LQR) that is the Q and R which was designed for position and swing suppression of the DPC system is presented. In the best of my knowledge, this approach using PSO to tune the parameters of LQR has not been reported in literatures for double pendulum crane system

CHAPTER THREE

MATERIAL AND METHOD

3.1 Introduction

In this chapter, a general mathematical model of Double Pendulum Crane (DPC) system is presented. The system model is derived using the Euler-Lagrange method which is based on energy approach. The general model is simulated, validated and tested based on different reference input as well as different payload. The nonlinear mathematical model is derived to allow the study of the system dynamic with different input level and payload. The detailed modeling of the adopted model is presented in this chapter. The concept of PSO and LQR controller design are also presented in this chapter

3.2 General Mathematical Model of the Double Pendulum Crane (DPC) system

Using the Euler-Lagrange approach to derive the system mathematical model yields three set of differential equations. These equations are the equation of motion that describes the dynamic of the system.

As previously mentioned, the derivation is based on energy calculation of the physical system. Energy calculation is independent of vector representation; therefore this approach is more simplified when deriving a complex system when compared to Newton-Euler formulation which is based on forces of the system. One of the most imperative reasons of adopting the Euler-Lagrange approach is that it utilizes the generalized coordinate of the system to describe the degree of freedom in a general representation[34].

The crane consist of three independent generalized coordinates, the trolley position x , the hook angle θ_1 , the payload angle θ_2 as in the schematic diagram shown in fig 3.1 below

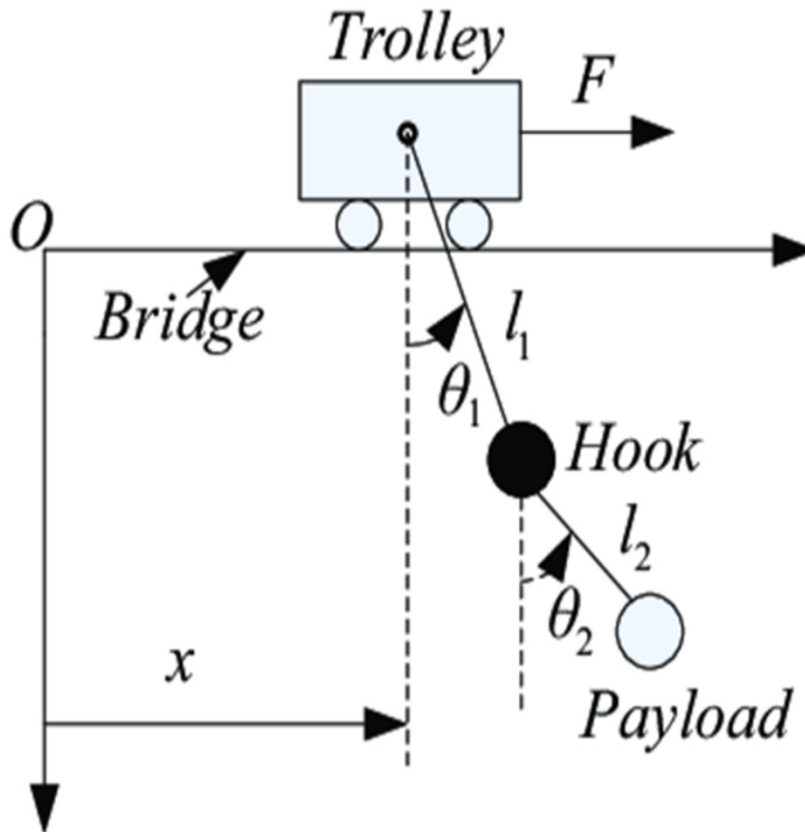


Figure 3.1.1 DPC Schematic diagram [33]

Table 3.1 Represent the system parameters [33]

<i>Parameters</i>	<i>Symbols</i>	<i>Values</i>	<i>Unit</i>
<i>Trolley mass</i>	<i>m</i>	6.5	<i>Kg</i>
<i>Mass of hook</i>	<i>m₁</i>	2.0	<i>Kg</i>
<i>Mass of payload</i>	<i>m₂</i>	0.6	<i>Kg</i>
<i>Cable length from trolley to hook</i>	<i>l₁</i>	0.53	<i>m</i>
<i>Cable length from hook to payload</i>	<i>l₂</i>	0.4	<i>m</i>
<i>Gravitational constant</i>	<i>g</i>	9.8	<i>ms⁻²</i>

Using the Lagrange formulation for the system, the following dynamic equation can be express for n degree of freedom (DOF) system.

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q} = T_i$$

(3.1)

And

$$L = K - P$$

(3.2)

Where

L = the Lagrangian function

Q = generalized force associated with a generalized coordinate q_i

q_i =generalized coordinate

$n=3$ DOF

K =total kinetic energy of the system

P = total potential energy of the system

3.3 System energy requirements

Assumptions made throughout the mathematical model derivation are as follows:

1. The bar that is connected between the cart and the hanged load is assumed to be rigid
and massless.
2. Friction force between the cart and the bridge is neglected.
3. The angle and angular velocity of load swing, also rectilinear position and velocity of the cart are measurable.
4. The load mass is concentrated at a point and the value of mass is exactly known.
5. The cart's mass and the length of the connecting bar are exactly known.
6. The hinged joint that connects the bar to the cart is frictionless.
7. The trolley and the load move in the x-y plane.

Therefore, Lagrangian technique considered the system energy, consisting of the kinetic and potential energy. The total energies can be expressed as:

$$K = k_1 + k_2 + k_2$$

(3.3)

$$P = p_1 + p_2 + p_2$$

(3.4)

Where

k_1 =kinetic energy of the cart

k_2 =kinetic energy of the hook

k_3 =kinetic energy of the payload

$p_1=0$, potential energy of the cart

p_2 =potential energy of the hook

p_3 =potential energy of the payload

For the cart with mass m , the kinetic energy is express as:

$$k_1 = \frac{1}{2}m\dot{x}^2$$

(3.5)

And the potential energy of the cart is:

$$p_1 = 0$$

(3.6)

For the Hook with mass m_1 , the Cartesian position coordinate from the schematic diagram is:

$$x_1 = x + l_1 \sin \theta_1 \quad (3.7)$$

$$y_1 = -l_1 \cos \theta_1 \quad (3.8)$$

Differentiating and taking the magnitude of their square gives the velocity of the Hook as:

$$V_2^2 = \dot{x}^2 + 2\dot{x}l_1\dot{\theta}_1 \cos \theta_1 + l_1^2\dot{\theta}_1^2 \quad (3.9)$$

Therefore the kinetic energy of the Hook is:

$$k_2 = \frac{1}{2}m_1[\dot{x}^2 + 2\dot{x}l_1\dot{\theta}_1 \cos \theta_1 + l_1^2\dot{\theta}_1^2] \quad (3.10)$$

The potential energy of the Hook is:

$$p_2 = m_1g[l_1(1 - \cos \theta_1)] \quad (3.11)$$

For the payload with mass m_2 , the Cartesian position coordinate from the schematic diagram is:

$$x_2 = x + l_1 \sin \theta_1 + l_2 \sin(\theta_1 - \theta_2) \quad (3.12) \quad y_2 = -l_1 \cos \theta_1 -$$

$$l_2 \cos(\theta_1 - \theta_2) \quad (3.13)$$

Differentiating and taking the magnitude of their square gives the velocity of the payload as:

$$V_3^2 = [\dot{x}^2 + 2\dot{x}l_1\dot{\theta}_1 \cos \theta_1 + 2\dot{x}l_2\dot{\theta}_2 \cos \theta_2 + l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos (\theta_1 - \theta_2)] \quad (3.14)$$

The kinetic energy of the payload is:

$$k_3 = \frac{1}{2} m_2 [\dot{x}^2 + 2\dot{x}l_1\dot{\theta}_1 \cos \theta_1 + 2\dot{x}l_2\dot{\theta}_2 \cos \theta_2 + l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos (\theta_1 - \theta_2)] \quad (3.15)$$

And the potential energy of the payload is:

$$p_3 = m_2 g [l_1 (1 - \cos \theta_1)] + m_2 g [l_2 (1 - \cos \theta_2)] \quad (3.16)$$

Substitute eq. (3.5), (3.10) and (3.15) into eq. (3.3) gives

$$K = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m_1 [\dot{x}^2 + 2\dot{x}l_1\dot{\theta}_1 \cos \theta_1 + l_1^2\dot{\theta}_1^2] + \frac{1}{2} m_2 [\dot{x}^2 + 2\dot{x}l_1\dot{\theta}_1 \cos \theta_1 + 2\dot{x}l_2\dot{\theta}_2 \cos \theta_2 + l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos (\theta_1 - \theta_2)] \quad (3.17)$$

Also substituting eq. (3.6), (3.11) and (3.16) into eq. (3.4) gives the total potential energy as:

$$P = m_1 g [l_1 (1 - \cos \theta_1)] + m_2 g [l_1 (1 - \cos \theta_1)] + m_2 g [l_2 (1 - \cos \theta_2)] \quad (3.18)$$

Putting eq. (3.17) and (3.18) into eq. (3.2) gives the Lagrangian function:

$$L = m \dot{x}^2 + \frac{1}{2} m_1 [\dot{x}^2 + 2\dot{x}l_1\dot{\theta}_1 \cos \theta_1 + l_1^2\dot{\theta}_1^2] + \frac{1}{2} m_2 [\dot{x}^2 + 2\dot{x}l_1\dot{\theta}_1 \cos \theta_1 + 2\dot{x}l_2\dot{\theta}_2 \cos \theta_2 + l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos (\theta_1 - \theta_2)] - [m_1 g [l_1 (1 - \cos \theta_1)] + m_2 g [l_1 (1 - \cos \theta_1)] + m_2 g [l_2 (1 - \cos \theta_2)]] \quad (3.19)$$

The dynamic equations are derived using the Lagrangian equation that is equation (3.1).

For the cart,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = F \quad (3.20)$$

Becomes:

$$\begin{aligned} (m + m_1 + m_2)\ddot{x} + (m_1 + m_2)l_1\ddot{x} \cos \theta_1 + m_2l_2\ddot{\theta}_2 \cos \theta_2 - (m_1 + m_2)l_1\dot{\theta}_1^2 \sin \theta_1 - \\ m_2l_2\dot{\theta}_2^2 \sin \theta_2 = F \end{aligned} \quad (3.21)$$

Considering the Hook, θ_1

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = 0 \quad (3.22)$$

Becomes:

$$\begin{aligned} (m_1 + m_2)l_1\ddot{x} \cos \theta_1 + (m_1 + m_2)l_1^2\ddot{\theta}_1 + m_2l_1l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2l_1l_2\dot{\theta}_2^2 \sin(\theta_1 - \\ \theta_2) + (m_1 + m_2)gl_1 \sin \theta_1 = 0 \end{aligned} \quad (3.23)$$

Now considering the payload, θ_2

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = 0 \quad (3.24)$$

Becomes:

$$\begin{aligned} m_2l_2\ddot{x} \cos \theta_2 + m_2l_1l_2\ddot{\theta}_1 \cos(\theta_1 - \theta_2) + m_2l_2^2\ddot{\theta}_2 - m_2l_1l_2 \sin(\theta_1 - \theta_2) \\ + m_2gl_2 \sin \theta_2 = 0 \end{aligned} \quad (3.25)$$

Equation (3.23)-(3.25) presents the general nonlinear model of the double pendulum crane system.

If the equations of motions are linearized around the equilibrium point by using Taylor series expansion with the following assumptions $\theta_1 = \theta_2 = 0$. Hence $\sin \theta_1 = \theta_1$ and $\sin \theta_2 = \theta_2$ and $\cos \theta_1 = \cos \theta_2 = 1$ and also $\cos(\theta_1 - \theta_2) = 1$ then the linear equations are;

$$\ddot{x} = -\frac{l_1(m_1+m_2)}{(m+m_1+m_2)}\ddot{\theta}_1 - \frac{m_2l_2}{(m+m_1+m_2)}\ddot{\theta}_2 + \frac{F}{(m+m_1+m_2)} \quad (3.26)$$

$$\ddot{\theta}_1 = -\frac{1}{l_1}\ddot{x} - \frac{m_2l_2}{l_1(m_1+m_2)}\ddot{\theta}_2 - \frac{g}{l_1}\theta_1 \quad (3.27)$$

$$\ddot{\theta}_2 = -\frac{1}{l_2}\ddot{x} - \frac{l_1}{l_2}\ddot{\theta}_1 - \frac{g}{l_2}\theta_2 \quad (3.28)$$

Using the parameters in Table 3.1, the state space description of the system can be obtained as

$$\dot{x} = Ax + BU \quad (3.29)$$

$$y = Cx \quad (3.30)$$

Where

$x = [x \ \dot{x} \ \theta_1 \ \dot{\theta}_1 \ \theta_2 \ \dot{\theta}_2]$ is the state variable

$$U = F$$

$$y = [[x \ \dot{x} \ \theta_1 \ \dot{\theta}_1 \ \theta_2 \ \dot{\theta}_2]]^T$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.9178 & 0 & 0.0013 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -31.4308 & 0 & 5.5461 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 31.8513 & 0 & -31.8531 & 0 \end{bmatrix}$$

$$, \quad B = \begin{bmatrix} 0 \\ 0.1539 \\ 0 \\ -0.2904 \\ 0 \\ 0 \end{bmatrix},$$

$$C = [1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0].$$

3.3 Simulink model of the Double pendulum Crane System

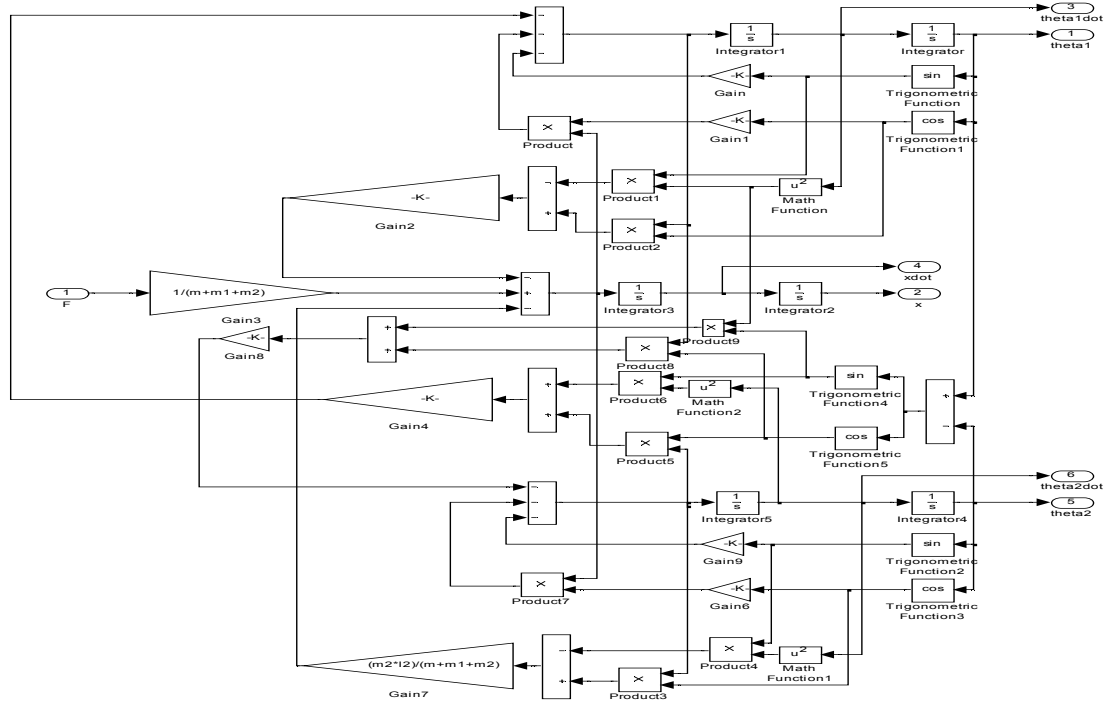


Figure 3.3.1 Simulink Model of Double Pendulum Crane system

3.4 Particle Swarms Optimization (PSO)

The idea on Particle Swarms Optimization (PSO) was first introduced by Russell Eberhart and James Kennedy in 1995 [35], and it was inspired by social behaviour of bird flocking and fish schooling[38]. Each individual within the swarm is represented by a particle in search space. The particle has one assigned vector, which determines the next movement of the particles called the velocity vector[36]. The PSO algorithm is initiated with position being randomly initialized, in which individuals (particles) move through a multidimensional search space [37]. Each of the particles places in the problem space a fitness value evaluated by an objective function to be optimized at its current location in every iteration. The particles in a local neighbourhood share memories of their best visited position (Russell Eberhart and James Kennedy, 1995).

Then, it will use these memories to adjust their own velocities and position according to eq. (3.31) and eq. (3.32)

$$V_i(t + 1) = \omega V_i(t) + C_1 r_1 (P_{best} - x_i(t)) + C_2 r_2 (g_{best} - x_i(t)) \quad (3.31)$$

$$x_i(t + 1) = x_i(t) + V_i(t + 1) \quad (3.32)$$

Hence position of the particles is concatenated to give the values of the Q and R used for the design of the LQR controller, the position is:

$$x_i = [q_{1m}, q_{2m}, \dots, q_{6m} \dots r_{1*n}] \quad (3.33)$$

Each particle remembers its previous best position which is used to obtain the best fitness position. The position is called the personal best position, P_{best} and the fitness value for this position is then stored [38]. The PSO algorithm also tracks the best position achieved by the individual in the swarm. This value is called the global best position, g_{best} . In this work, the PSO technique is used to determine the most optimum weighting values for Q and R for the controller to minimize the sway angles of the hook and payload as well as to track the position of the cart. Q and R are the entries that form a particle in the PSO algorithm and are chosen to be diagonal matrices with positive real elements. The fitness value used in this paper is calculated based on the quadratic cost function;

$$J = \int_0^{Tf} (x^T Q x + u^T R u) dt \quad (3.34)$$

. The PSO algorithm used in this thesis can be briefly discussed by the following steps and hence as shown the flow chart below

1: Initialize a population of ‘pop’ particles with random positions within the lower and upper bound of the problem space. Similarly initialize randomly pop’ velocities associated with the particles

2: Evaluate the optimization fitness functions J for the initial population.

3: Find the minimum fitness value for fitness functions J in step 2 and call it Jpbest and let the particle associated with it be Xpbest.

4: Initially set Jgbest equal to Jpbest.

5: Update the weight ω using the following constriction coefficient equation

$$\omega = \aleph, \aleph = \frac{2k}{2 - \varphi - \sqrt{\varphi^2 - 4\varphi}},$$

$$\varphi = \varphi_1 + \varphi_2, \varphi_1 = \varphi_2 = 2.05[39]$$

$$c_1 = \aleph * \varphi_1, c_2 = \aleph * \varphi_2$$

$$k = 1, 0 \leq k \leq 1$$

6: Update the velocity of each particle using (5)

7: Check V for the range [Vmax, Vmin]. Set it to the limiting values.

8: Update the position of each particle using (6) which gives the new population.

9: Repeat 7 for the new population.

10: Evaluate the optimization fitness functions J for new population.

11: Obtain J_{pbest} for fitness function J in step 10.

12: Compare the J_{pbest} obtained in step 11 with J_{gbest} . If J_{pbest} is better than J_{gbest} then set

J_{gbest} to J_{pbest}

13: Stop if convergence criteria are met, otherwise go to step 5. The stopping criteria are, good fitness value, reaching maximum number of iterations, or no further improvement in fitness [40]. The pseudo-code that runs the PSO algorithm is shown in appendix A below.

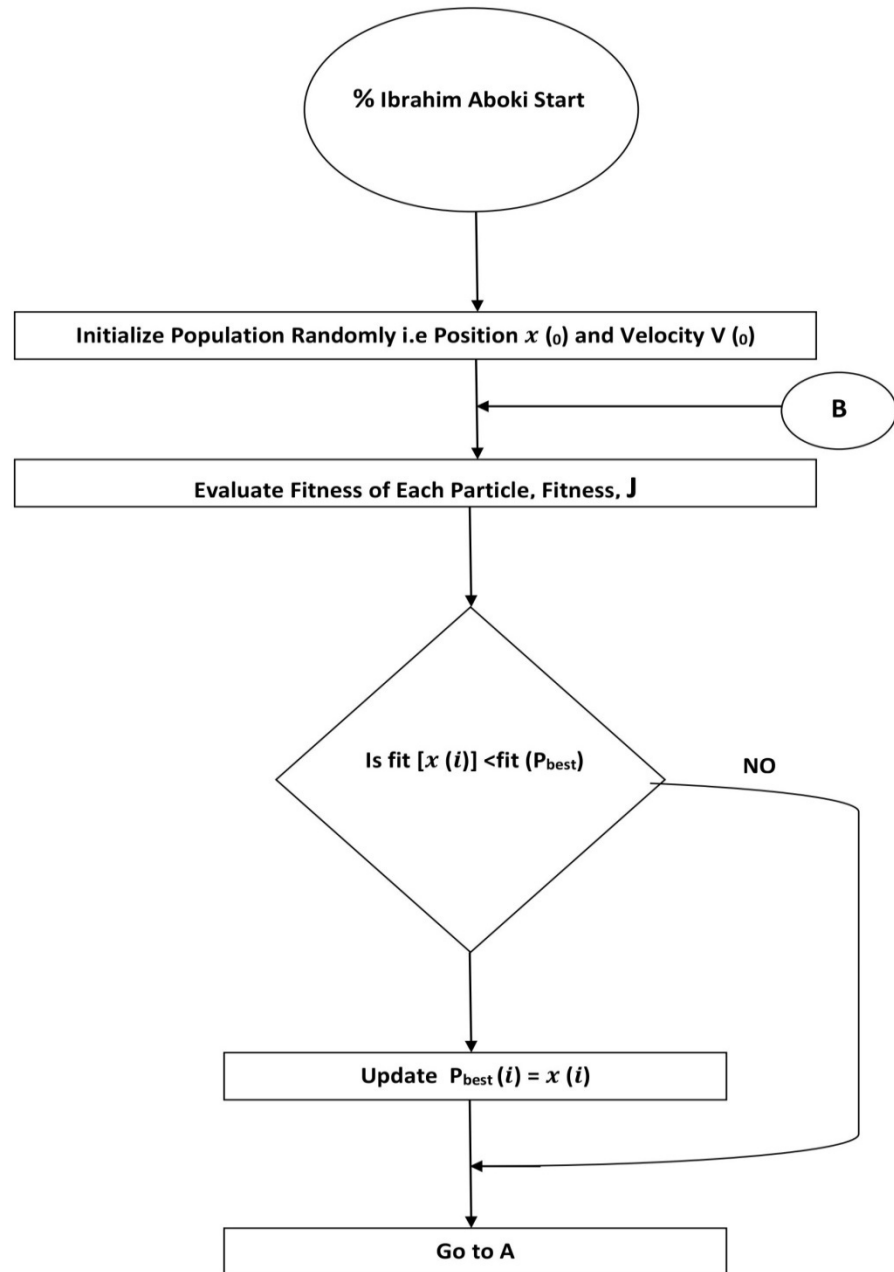


Figure 3.4.1 flow chart of the PSO algorithm

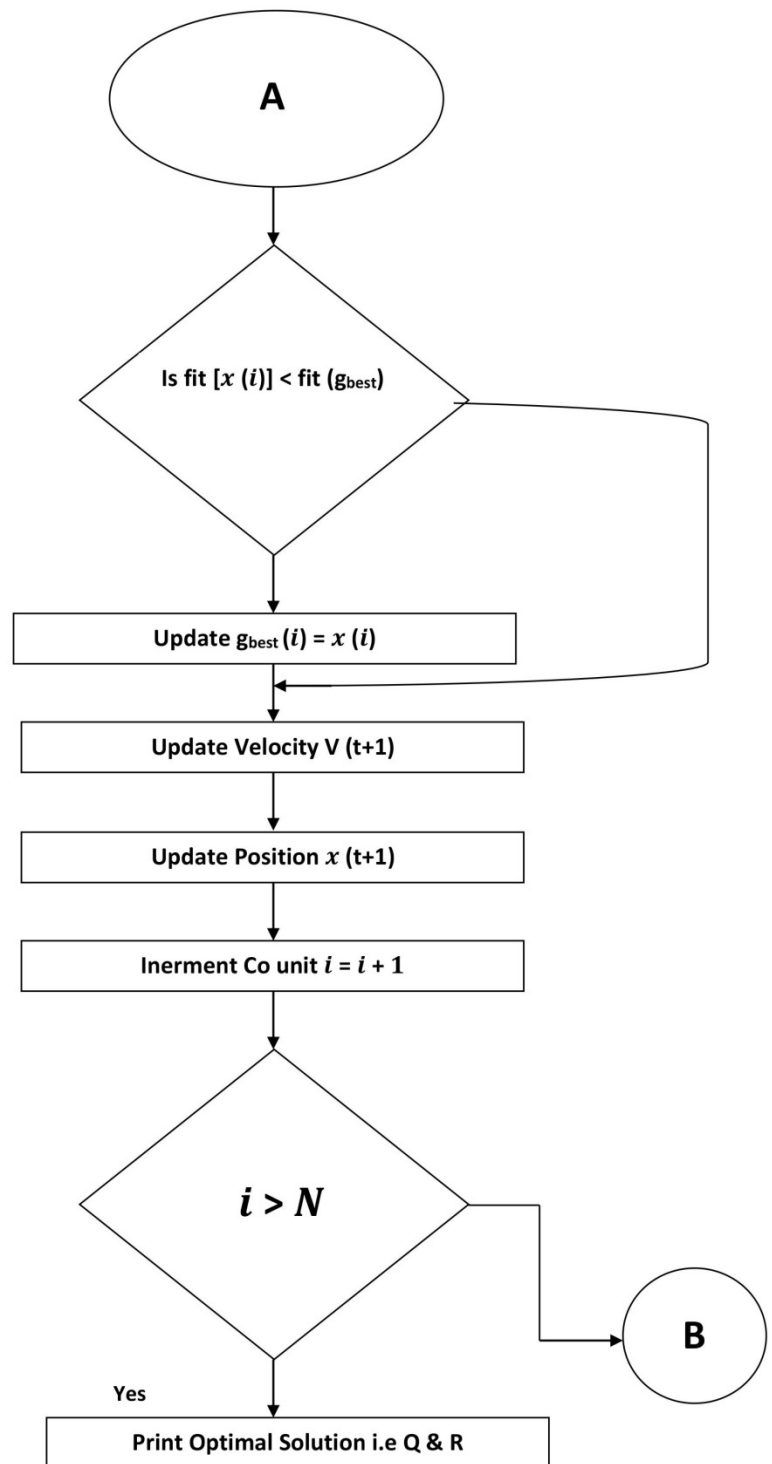


Figure 3.4.2 flow chart of the PSO algorithm

Table 3.2 Parameters use to run the PSO

<i>Parameters</i>	<i>Description</i>	<i>Value</i>
ω	<i>inertia weight</i>	0.7298
c_1, c_2	<i>accelaration coefficient</i>	1.4962
r_1, r_2	<i>random generated numbers</i>	0 – 1
N	<i>no of population</i>	100
i	<i>no of search space</i>	50

3.5 Controller Design

The section described the detail design of the control strategies for the stated feedback LQR controller for the DPC system

3.5.1 State feedback LQR controller

This type of controller uses linear model of a system in state space form $\dot{x} = Ax + Bu, y = Cx$, [41], an optimal LQR estimate the controller gain K. The aim of the controller is to minimize the cost function:

$$J = \int_0^{T_f} (x^T Q x + u^T R u) dt$$

(3.29)

Where Q and R must be positive- definite (or positive – semi definite) Hermiltian and real symmetric matrix [20] also the pair (A,B) are controllable [7].

The gain $K = R^{-1} B^T P$ of the control law $u = -kx(t)$ is obtained by solving the algebraic Riccati equation below

$$A^T P + PA + PBR^{-1}B^T P + Q = 0$$

(3.30)

The selection of the optimal values of Q and R was done by PSO and were found to be

$Q = \text{diag}(62.6696 \ 3.2370 \ 148.5614 \ 36.795 \ 76.3446 \ 35.6301)$, $R = 0.5$ and the controller gain $K = [k_o - k_i]$ is computed using LQR MATLAB command $K = \text{lqr}(A, B, Q, R)$ and the gain

$$k_o = [52.4306 \ 57.1565 \ -318.6918 \ -29.9244 \ 119.5803 \ -21.3701]$$

$$K_i = 20.2012$$

3.6 Command Tracking

LQR is a state feedback that is only concern with regulator design. In order to achieve good command tracking, it is necessary to introduced reference signal into the controller, and adds the integral action into the close loop called the state augmentation, (Cazzolato 2006). In this section the steady state tracking requirement for step reference input is address. There are two approaches viz;

- (i) Input gain method; this approach involves addition of input gain to the state feedback control law. But in this approach the knowledge of the open-loop of the system has to be known.
- (ii) Integral action; it is the inclusion of the integrator at the tracking error

In this research work, an integral command tracking is used and it is design as the follows:

Forming the augmented matrices A^* , B^* , C^* and Q^* as shown below;

$$A^* = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad (3.35)$$

$$B^* = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad (3.36)$$

$$C^* = [C \quad 0] \quad (3.37)$$

$$Q^* = \text{diag}(62.6696 \ 3.2370 \ 148.5614 \ 36.795 \ 76.3446 \ 35.6301 \ 1.0000),$$

$$R = 0.5$$

-Now adding the integral part yields a new control law $u = -k_o x + k_i \varepsilon$

$u = -[k_o \quad -k_i] \begin{bmatrix} x \\ \varepsilon \end{bmatrix}$, therefore the controller gain $K = [k_o \ -k_i]$ is computed using LQR

MATLAB command $K = \text{lqr}(A^*, B^*, Q^*, R)$ and the gain

$$k_o = [52.4306 \ 57.1565 \ -318.6918 \ -29.9244 \ 119.5803 \ -21.3701]$$

$$K_i = 20.2012$$

3.7 LQR control scheme

To reduce steady stated error integral action is added to the system full state feedback LQR with reference input and integral action is shown in the figure below the constant gain K_i is use as pre-compensator and it is obtained by matrix augmentation as shown eq. 3.31-3.33 above.

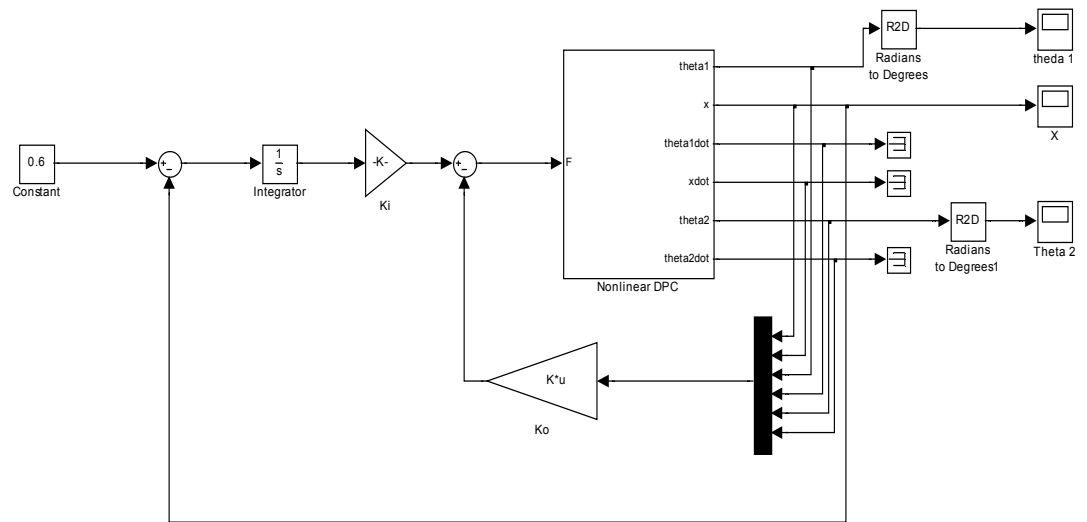


Figure 3.7.1 LQR control scheme for the DPC system

CHAPTER FOUR

SIMULATION RESULTS AND DISCUSSION

4.1 Introduction

In this chapter PSO-tuned LQR and PID controllers are simulated individually using MATLAB/SIMULINK and results are presented. The aim is to present the collections of results obtained with comparison. In this study, the results are analyzed base on some performance indices which provide a suitable benchmark for the comparison of the results. The robustness of the proposed controller is investigated due to reference input and payload variation and the result is compared with that from the PID control scheme.

4.2 Case I

For a reference step input of 0.6m, the position, hook and payload response are shown in the figure below:

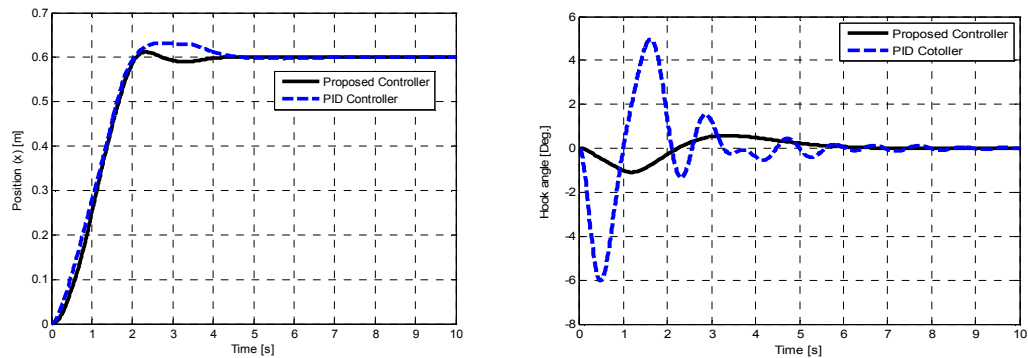


Figure 4.2.1: Trolley Position**Figure 4.2.2: Hook swing angle**

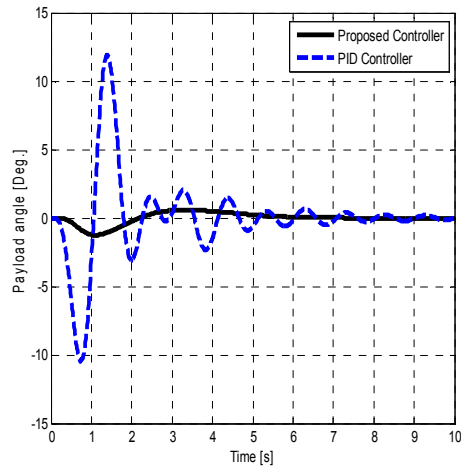


Figure 4.2.3 Payload swing angle

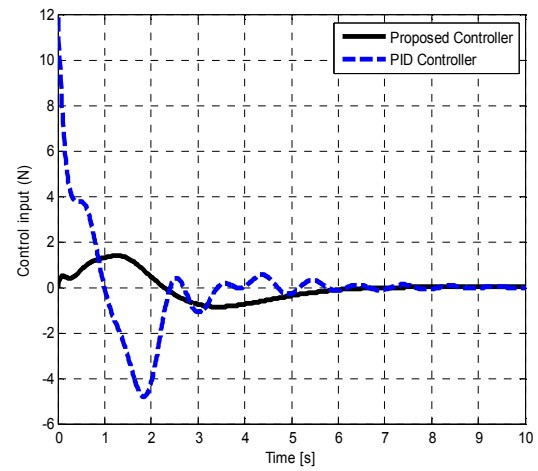


Figure 4.2.4 Control input signal

Table 4.1: Comparison of results for a step reference input of 0.6m

<i>Controller</i>	<i>Trolley Position</i>		<i>Hook Oscillation</i>		<i>Payload Oscillation</i>	
	$ST(s)$	$OS(\%)$	$ST(s)$	$\theta_{1max}(deg)$	$ST(s)$	$\theta_{2max}(deg)$
<i>Proposed</i>	2.3816	2.0465	5.6849	1.0853	5.5812	1.2470
<i>PID</i>	4.0032	5.3417	7.8155	6.0319	8.3495	11.9157

ST: Settling Time; OS: Overshoot

4.3 Case II

For a reference step input of 1.0m, the position, hook and payload response are shown in the figure below:

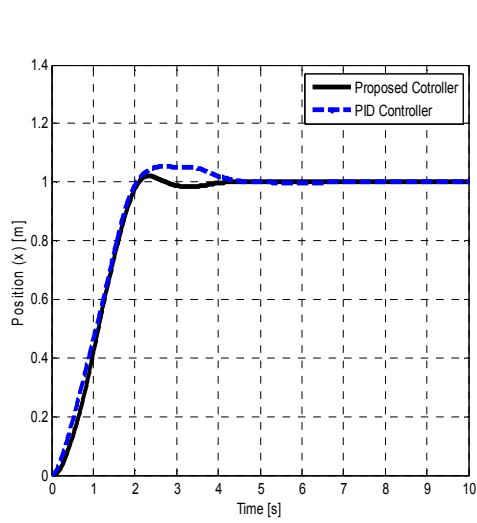


Figure 4.3.1: Trolley Position

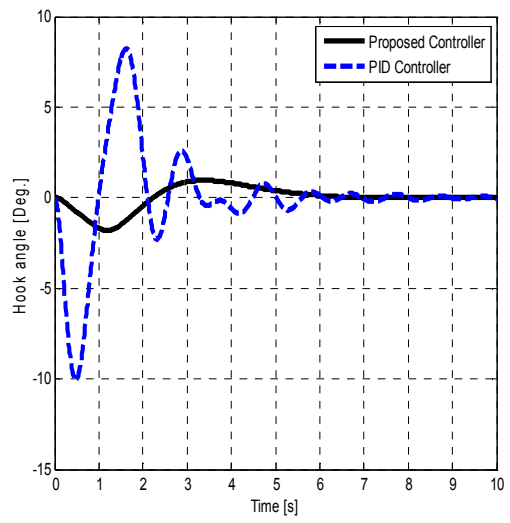


Figure 4.3.2: Hook swing angle

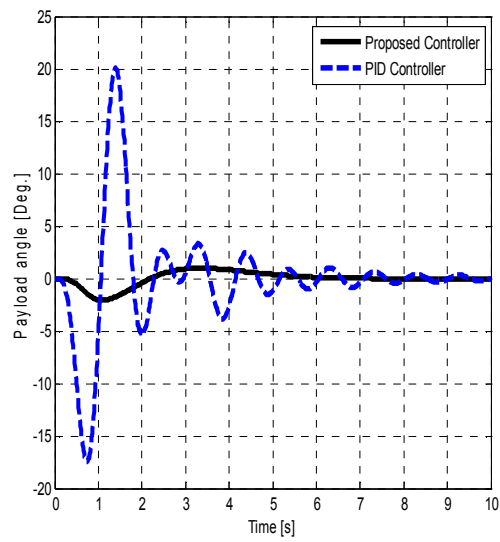


Figure 4.3.3: Payload swing angle

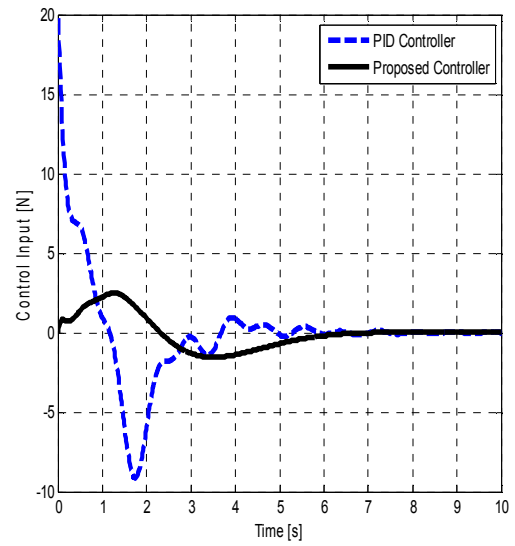


Figure 4.3.4: Control input signal

Table 4.2: Comparison of results for a step reference input of 1.0m

<i>Controller</i>	<i>Trolley Position</i>		<i>Hook Oscillation</i>		<i>Payload Oscillation</i>	
	$ST(s)$	$OS(\%)$	$ST(s)$	$\theta_{1max}(deg)$	$ST(s)$	$\theta_{2max}(deg)$
<i>Proposed</i>	2.2375	2.0608	5.6879	1.0853	5.5815	2.0782
<i>PID</i>	4.0038	5.3319	7.8187	10.0515	8.3715	20.0148

ST: Settling Time; OS: Overshoot

4.4 Case III

The robustness of the proposed controller is investigated due to payload variation and the result is compared with that from the PID control scheme. The simulation was carried out with the payload mass doubled. Figures 4.4.1-4.4.4 show the simulation results and Table 4.3 summarizes the simulation results under this condition

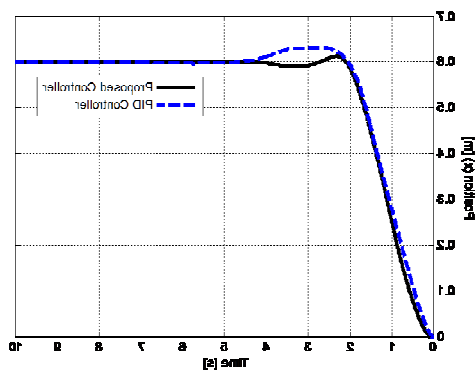


Figure 4.4.1: Trolley Position

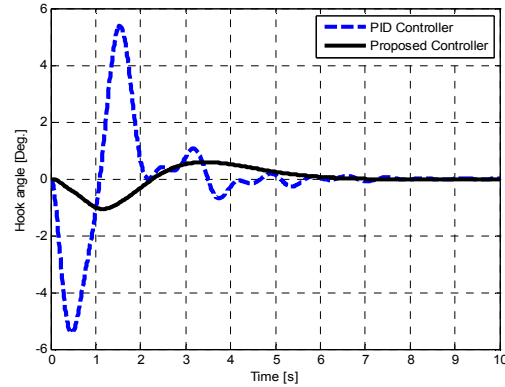


Figure 4.4.2 Hook swing angle

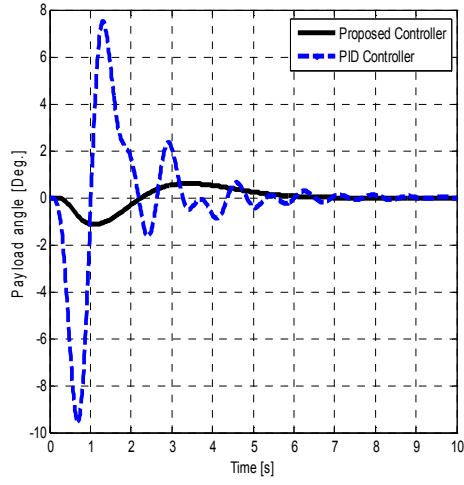


Figure 4.4.3: Payload swing angle

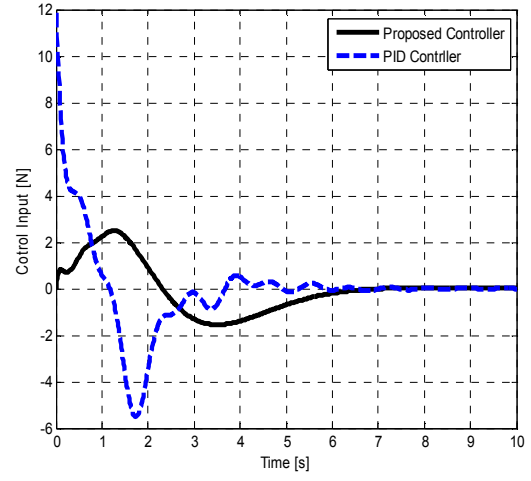


Figure 4.4.4 Control input signal

Table 4.3: Comparison of results for a step reference input of 0.6m and double the payload

<i>Controller</i>	<i>Trolley Position</i>		<i>Hook Oscillation</i>		<i>Payload Oscillation</i>	
	<i>ST(s)</i>	<i>OS(%)</i>	<i>ST(s)</i>	$\theta_{1max}(deg)$	<i>ST(s)</i>	$\theta_{2max}(deg)$
<i>Proposed</i>	3.6483	2.9169	5.5340	1.0650	5.9353	1.1492
<i>PID</i>	4.0032	5.3417	6.3852	5.3852	6.4139	9.5508

ST: Settling Time: OS: Overshoot

4.5 Case IV

For a conventional LQR controller with Q and R obtained by trial and error, for which $Q = \text{diag}(1200 \ 150 \ 100 \ 10 \ 50 \ 25 \ 1)$, $R = 0.5$, the tracking and regulator gain are $k_o = 180, k_i = (202.5311 \ 88.2343 \ -106.5751 \ 7.4003 \ 13.3735 \ -1.7977)$ and a reference step input of 0.6, the position, hook and payload response are shown in the figures below:

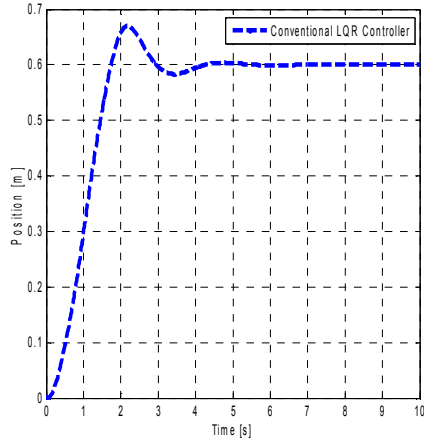


Figure 4.5.1: Trolley Position

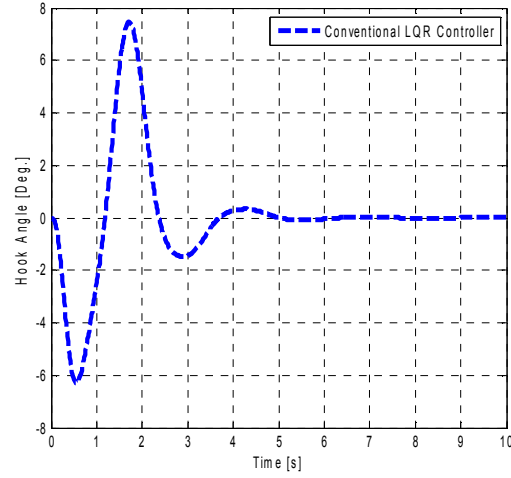


Figure 4.5.2: Hook swing angle

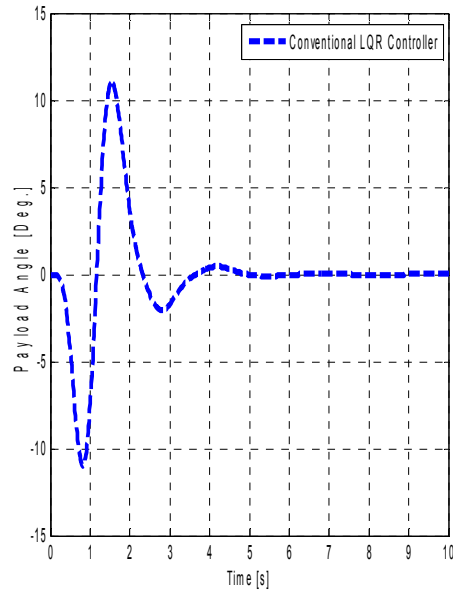


Figure 4.5.3: Payload swing angle

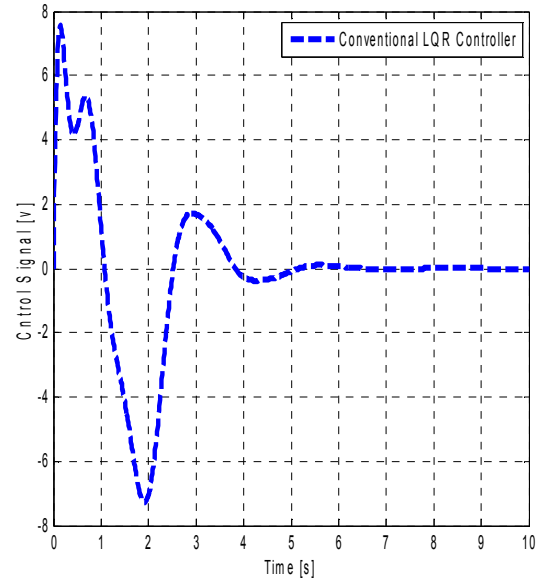


Figure 4.5.4: Control input signal

4.6 Case V

Fig.4.6.1-4.6.4 below show the results of the comparison between the PSO-Tuned LQR and Conventional LQR on the DPC system

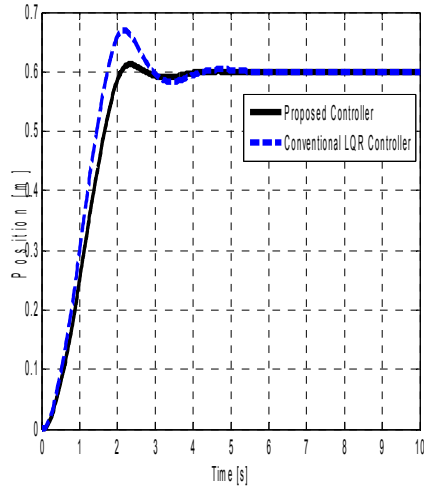


Figure 4.6.1: Trolley Position

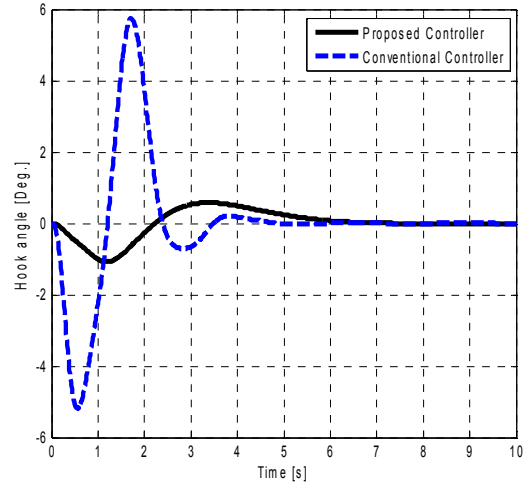


Figure 4.6.2: Hook swing angle

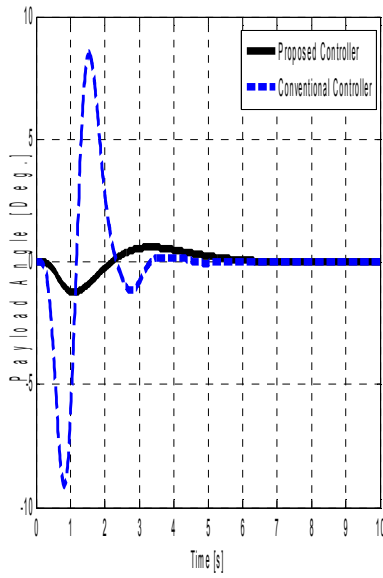


Figure 4.6.3: Payload swing angle

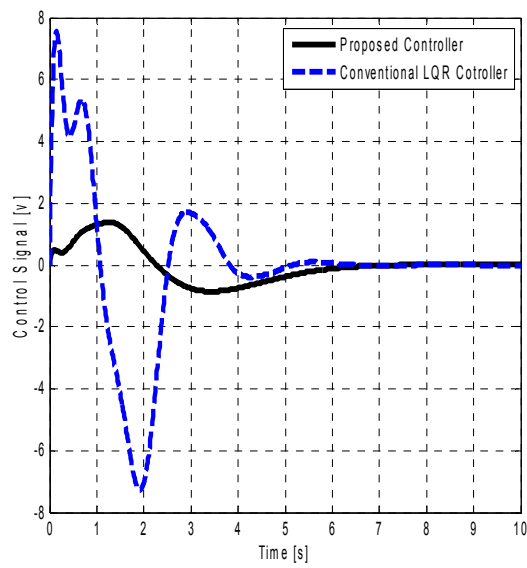


Figure 4.6.4: Control input signal

4.7 Summary of the Results Using Bar Chart

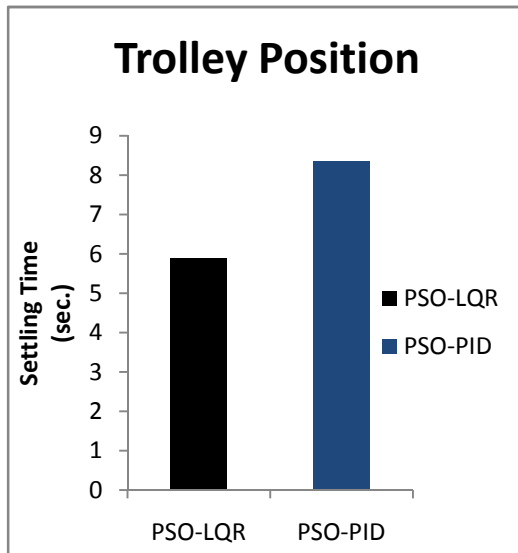


Figure 4.7.1 Trolley position settling Time

in Bar Chart

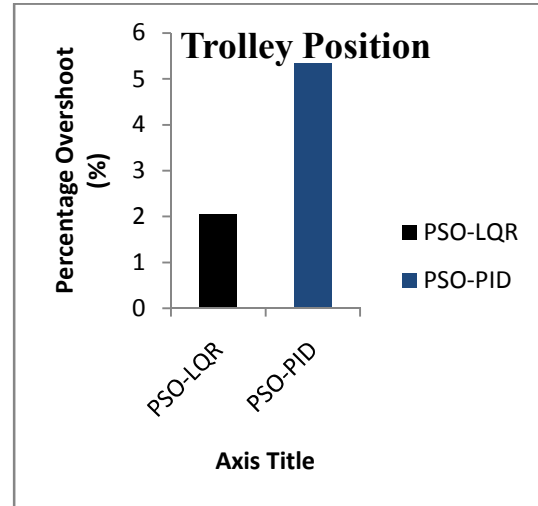


Figure 4.7.2 Trolley position overshoot

in Bar Chart

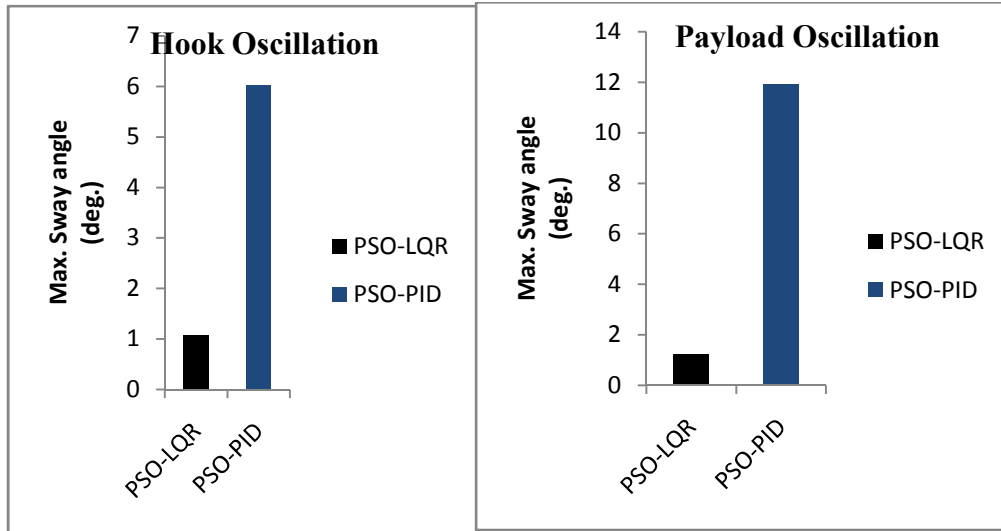


Figure 4.7.3 Hook Oscillation in Bar Chart Figure 4.7.4 Payload Oscillation Barchart

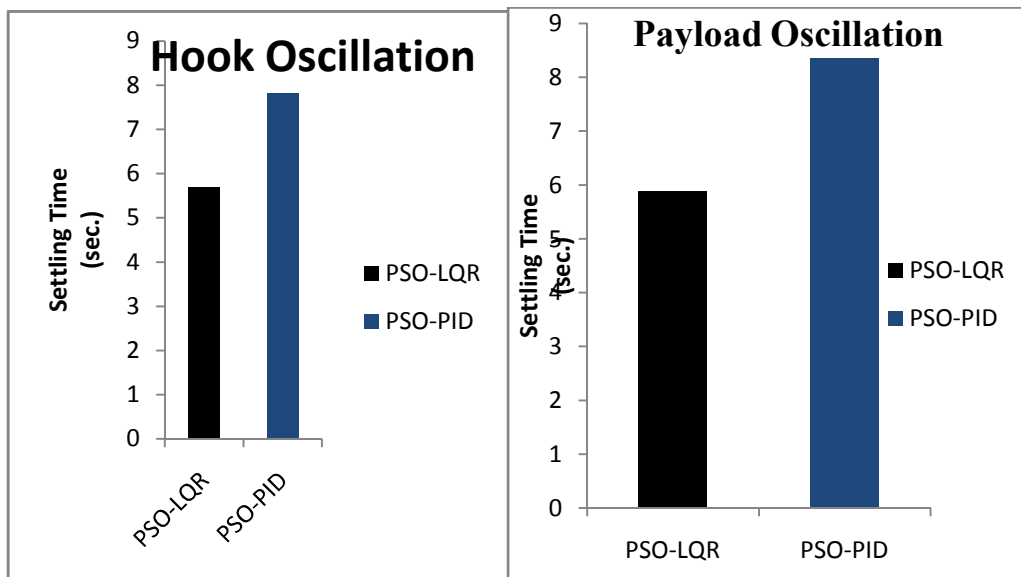


Figure 4.7.5 Hook Oscillation settling Time Figure 4.7.6 Payload Oscillation settling Time
in Bar Chart in Bar Chart

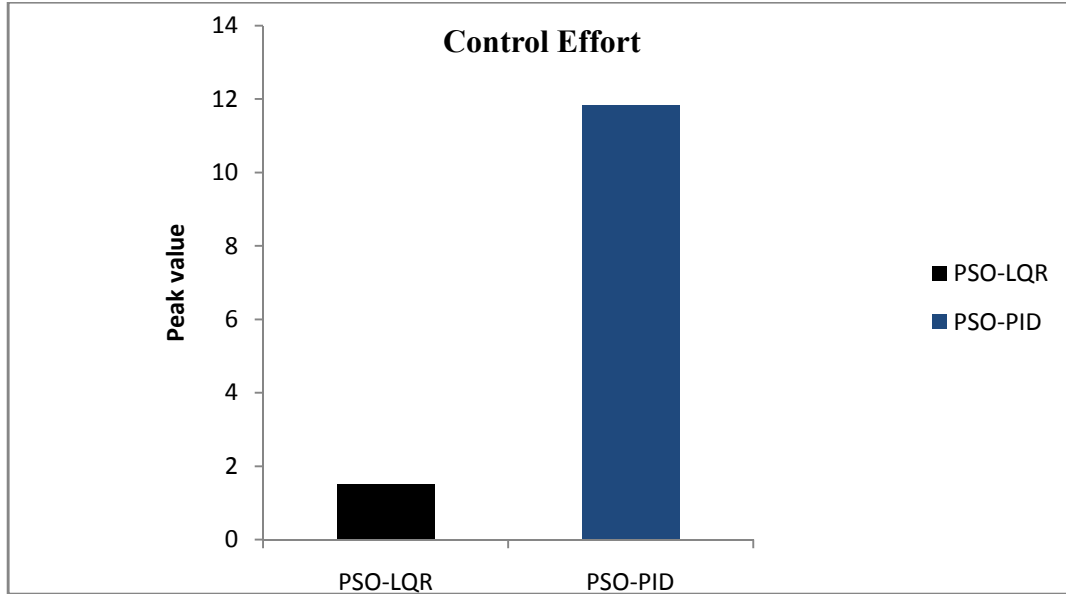


Figure 4.7.7 Control input signal

4.8 Discussion

For a step reference input of 0.6m, the proposed controller settled the trolley position in 2.3816 seconds with an overshoot of 2.0465% whereas the PID controller settled the trolley position in 4.0032 seconds with an overshoot of 5.3417% as shown in Figure 4.2.1 and Table 4.1. The proposed controller damped out the hook oscillation in 5.6849 seconds with a peak swing angle of 1.0853° whereas the PID controller damped out the hook oscillation in 7.8155 seconds with a peak swing angle of 6.0319° as shown in Figure 4.2.2 and Table 3. The proposed controller damped out the payload oscillation in 5.5812 seconds with a maximum swing angle of 1.2470° whereas the PID controller damped out the payload oscillation in 8.3495 seconds with a maximum swing angle of 11.9157° as shown in Figure 4.2.3 and Table 4.1. From Figure 4.2.4, it can be seen that the initial

control input required by the PID control scheme is high when compared to that required by the proposed controller.

Increasing the step reference input from 0.6m to 1.0m with the proposed controller, the trolley position settling time increases 2.3816 to 2.3875 seconds, the overshoot also increases from 2.0465% to 2.0608% as shown in Figure 4.3.1 and Table 4.1 and 4.2. While the PID controller settling time increases from 4.0032 seconds to 4.0038 seconds, and the overshoot decreases from 5.3417% to 5.3319% as shown in Figure 4.3.2 and Table 4.2. The proposed controller hook oscillation damping time increase from 5.6847 seconds to 5.6849 seconds with a peak swing angle of 1.0853^0 whereas the PID controller damping time of the hook oscillation increases from 7.8155 seconds to 7.8187 seconds with a peak swing angle of hook increasing from 6.0319^0 to 10.0515^0 as shown in Figure 4.3.3 and Table 4.2. The proposed controller damped out the payload oscillation in 5.5815 seconds with a maximum swing angle of 2.0782^0 as indicated in Figure 4.3.4 and Table 4.2. Whereas, using PID the oscillation damping time of the payload increases from 8.3495 seconds to 8.3715 seconds with the peak maximum angle increasing from 11.9157^0 to 20.0148^0 as shown in Figure 4.3.5 and Table 4.2 above.

Doubling the payload and a reference step input of 0.6m With the proposed controller, the trolley position settling time increases from 2.3816 seconds to 3.6438 seconds and the overshoot increases from 2.0465% to 2.9169% whereas, using the PID controller the settling time remains as 4.0032 seconds, but the overshoot increases from 5.3417% to 5.3672% as shown in Figure 4.4.1 and Table 4.3. With the proposed controller, the hook oscillation damping time reduces from 5.6849 seconds to 5.5240 seconds and maximum swing angle reduces from 1.0853^0 to 1.0650^0 whereas with the PID controller

oscillation damping time reduces from 5.3417 seconds to 5.3315 seconds and the peak swing angle reduces from 6.0319^0 to 5.3852^0 as shown in Figure 4.4.2 and Table 4.3. With the proposed controller, the payload oscillation damping time increases from 5.5812 seconds to 5.9353 seconds and the maximum swing angle reduces from 1.2470^0 to 1.1492^0 whereas with the PID controller the payload oscillation reduces from 8.3495 seconds to 6.4139 seconds and the maximum swing angle reduces from 11.9157^0 to 9.5508^0 as shown in Figure 4.4.3 and Table 4.3. Figure 4.4.4 shows the control input under this condition. However, Fig.4.6.1-4.6.4 show the results of the comparison between PSO-tuned LQR and that of the conventional LQR and it can be inferred that the proposed method not only performed better but also has a reduced values in terms of Q, R, k_o , and k_i . Also the results are presented pictorially using bar chart as shown in fig.4.7.1-4.7.7 above. It can be seen that the control effort required by the PID control scheme is higher than that required by the proposed controller.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

A PSO based LQR state feedback controller was proposed for the control of a double pendulum crane. The performance of the proposed controller was compared with that of a PID control scheme in terms of position tracking and robustness due to reference input and a payload variation via simulations. The simulations results showed that both controllers tracked the trolley position and swing angles. However, the proposed controller performed better than the PID control scheme in terms of reducing the maximum swing angles, fast hook and payload oscillations damping and reducing the required initial control input. In view of this, the PSO based LQR controller can be considered a better controller for some applications on a double pendulum crane system

as these applications give priority to accurate tracking of the reference input as well as robustness to payload variation. Nonetheless, the proposed controller could be a valuable control tools for the double pendulum crane system. Hence, the objectives of the dissertation are achieved.

5.2 Contributions

- I. A PSO algorithm was developed for tuning the parameters (that is the Q and R matrix) of the LQR controller using MATLAB/Simulink which solve the problem of trial and error method for obtaining the Q and R matrix of the LQR controller
- II. Testing the LQR controller for suppressing highly oscillatory system to dampens out the hook and payload sway
- III. Performance comparison between PSO-based LQR controller and PID was carried out on the double pendulum nonlinear system.

5.3 Recommendations

- I. Hoisting of the payload should be considered or incorporated in modeling the double pendulum crane system.
- II. Also it should be considered adding a disturbance
- III. Practical implementation of the PSO-based LQR controller on a practical system will provide a greater insight in the understanding of these controllers.
- IV. The tuning should also the carry out online

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Appendix A

M-file code for tuning the PSO

```
clc;
clear;
close all;

%%problem defination

Cost  $J = \int_0^{Tf} (x^T Q x + u^T R u) dt$ 

yact=[1;0;0;0;0;0];

nVar=6;

VarSize=[1 nVar];

VarMin=0;

VarMax=100;

%%Parameter of the PSO

MaxIt=1000;

npop=50;

t1=2.05;

t2=2.05;

t=t1+t2;

K=1;

Q=2*K/(abs(2-t-(sqrt(t^2-4*t))));

c1=Q*t1;
```

```

c2=Q*t1;

w=Q;

ShowIterinfo=true;

MaxVelocity=0.2*(VarMax-VarMin);

MinVelocity=-MaxVelocity;

%%Intialization

%the particle template

empty_particle.position=[];

empty_particle.velocity=[];

empty_particle.Cost=[];

empty_paticle.Best.position=[];

empty_particle.Best.Cost=[];

particle= repmat(empty_particle, npop, 1);

%Initialize Global Best

GlobalBest.Cost=inf;


%% Initialize population members

for i=1:npop

    %Generate random solution

    particle(i).Position=unifrnd(VarMin, VarMax, VarSize);

```

```

        %Initialize velocity

particle(i).Velocity=zeros(VarSize);

        %Evaluate

particle(i).Cost=(particle(i).Position);

        %Update the personal best

fvalue=10*(gbest(1)-1)^2+20*(gbest(2)-2)^2;

fff(run)=fvalue;

rgbest(run,:)=gbest


$$\varepsilon_{value} = 20 * (\max(os))^2 + 10 * (\max(sway\ angle))^2;$$


rpbest(run,:)= $\varepsilon_{value}$ 

particle(i).Best.Position=particle(i).Position;

particle(i).Best.Cost=particle(i).Cost;

%% Update Global Best

if particle(i).Best.Cost<GlobalBest.Cost;

    GlobalBest=particle(i).Best;

end

end

%Array to hold Best cost value one each iteration

BestCost=zeros(MaxIt,1);

```

```

%% main loop of PSO

for it=1:MaxIt

    for i=1:npop

        particle(i).Velocity=w*particle(i).Velocity...
            +c1*rand(VarSize).*(particle(i).Best.Position-
particle(i).Position)+c2*rand(VarSize).*(GlobalBest.Position-
particle(i).Position);

        %update position

        particle(i).Position=
particle(i).Position+particle(i).velocity;

        particle(i).Cost=(particle(i).Position);

        %Apply lower and upper Bound limits for position

        particle(i).Position=max(particle(i).Position,VarMin);
        particle(i).Position=min(particle(i).Position,VarMax);

        %Apply lower and upper Bound limits for velocity

        particle(i).velocity=max(particle(i).velocity,MinVelocity);
        particle(i).velocity=min(particle(i).velocity,MaxVelocity);

```

```
%update the personal Best
```

```
    if particle(i).Cost<particle(i).Best.Cost;
```

```
        particle(i).Best.position=particle(i).position;
```

```
        particle(i).Best.Cost=particle(i).Cost;
```

```
    end
```

```
%update the Global Best
```

```
    if particle(i).Best.Cost<GlobalBest.Cost;
```

```
        GlobalBest=particle(i).Best;
```

```
    end
```