SEASONAL AUTOREGRESSIVE INTEGRATED MOVING AVERAGE MODELLING AND FORECASTING OF INFLATION RATES IN NIGERIA

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NSU/MSC/STA/0021/17/18

M.Sc. STATISTICS

DECEMBER, 2019

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A DISSERTATION SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES, NASARAWA STATE UNIVERSITY, KEFFI IN PARTIAL FUFFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF DEGREE OF MASTERS IN SCIENCE (M.Sc.) IN STATISTICS

DEPARTMENT OF STATISTICS,

FACULTY OF NATURAL AND APPLIED SCIENCES,

NASARAWA STATE UNIVERSITY, KEFFI,

NIGERIA

DECLARATION

I hereby declare that the dissertation has been written by me and it is a report of my research work. It has not been presented in any previous application for Masters of Science (M.Sc.) Degree in Statistics. All quotations are indicated and sources of information specifically acknowledged by means of references.

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ODUNUKWE, ADAORA DARLINGTINA
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CERTIFICATION

The dissertation (Seasonal Autoregressive Integrated Moving Average Modelling and Forecasting of Inflation Rates in Nigeria) meets the regulations governing the award of Masters of Science (M.Sc.) Degree in Statistics, of the school of Postgraduate Studies, Nasarawa State University, Keffi and is approved for its contribution to knowledge.

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DEDICATION

This dissertation is dedicated to Almighty God who has been the source of my strength and who made it possible for me to carry out this research work. To him glory be forever.

ACKNOWLEDGEMENTS

My sincere appreciation goes to Almighty God for his Guidance and protection throughout my Academic program.

A word of thanks to my project Supervisor, Dr. Mrs M.O. Adehi for her kindness and effort in putting me through.

I thank the Head of Department, Dr. N. O. Nweze and my able Lecturers, Dr. Adenomon, Mr. Ibrahim for their contributions and knowledge they impacted in me. throughout the period of my study. May God bless all your efforts.

Special thanks and appreciation to my mother, Mrs Chinwe Odunukwe for her love and encouragement throughout the period of my study.

Furthermore, a word of Thanks to Prof. C.M. Ojinnaka, Mr. Philip C. Umeakuike, Mr Ifeoluwa David and my siblings for their constant support.

To my collegues, Mr. Rotimi Ojo, Mr. Omale Yohanna, Mr. Emeka John, Esther, Elizabeth Olanrewaju and others I didn’t mention, I say ‘Thank you’ and I wish you all success.

In a special way, I remember Late Tina Okon, you were part of my success story, I appreciate all your efforts and kindness throughout this period. Continue to RIP.

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ABSTRACT

Nigeria faces a macroeconomic problem of inflation for a long period of time. The problem somehow slows down the economic growth in this country. As we all know, inflation is one of the major economic challenges facing most countries in the world especially those in Africa including Nigeria. Therefore, forecasting inflation rates in Nigeria becomes very important for the government to design economic strategies or effective monetary policies to combat any unexpected high inflation in the country. This study utilizes seasonal autoregressive integrated moving average model (SARIMA) to forecast inflation rates in Nigeria. Using monthly inflation data from January 1999 to December 2018, we discover that SARIMA $(1,1,1) \times (0,0,1)_{12}$ can represent very well the data behaviour of inflation rate in Nigeria. Based on the selected model i.e. SARIMA $(1,1,1) \times (0,0,1)_{12}$ we obtained at twelve (12) month forecast of inflation rates in Nigeria outside the sample period (i.e. from January 2019 to December 2019). The forecast results show a decreasing pattern and the accuracy of forecasts measure showed that the Mean Absolute Percentage Error (MAPE) is 3.56% and Theil's inequality coefficient (U-statistic) is 0.018.

CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

It is believed that Nigeria is a developing country. Out of all the micro and macro economy variables which affect the growth of the economy, the effect of inflation is likely to be significant since inflation is created locally in the case of Nigeria (i.e. there is general rise in price of goods in Nigeria at the devaluation of naira) while we import inflation due to over reliance on imported goods from inflation affected countries.

Inflation is the rate at which the general level of prices for goods and services is rising and, consequently, the purchasing power of currency is falling. As prices rise, a single unit of currency loses value as it buys fewer goods and services. This loss of purchasing power impacts the general cost of living for the common public which ultimately leads to a deceleration in economic growth. The consensus view among economists is that sustained inflation occurs when a nation's money supply growth outpaces economic growth. To combat this, a country's appropriate monetary authority, like the central bank, is expected to take the necessary measures to keep inflation within permissible limits and keep the economy running smoothly.

Inflation is a problem in all facets of life and in all economic entities. Government of any nation is saddled with the responsibility of ensuring that her plans and programmes are not frustrated by unpredictable and galloping prices. Every firm desires a stable macro-economic environment that is devoid of unrepentant price change that can bring about reliable forecast and planning. An individual also strives that he is not worse off by unexpected price increase. All these bring home the need to explore the study of inflation so as to form a timeless and dependable model of its tendency (Taiwo, 2011).

Sloman and Kevin (2007) explain that inflation may be either demand pull inflation or cost push inflation. Demand pull inflation is caused by persistent rises in aggregate demand thus the firms responding by raising prices and partly by increasing output. Cost push inflation is associated with persistent increase in the costs experienced by firms. Firms respond by raising prices and passing the costs on to the consumer and partly cutting back on production. Hendry (2006) agrees that inflation is the resultant of many excess demands and supplies in the economy.

Tucker (2007) observed that there are many measures of inflation, because there are many different price indices relating to different sectors of the economy. Two widely known indices for which inflation rates are reported in many countries are the Consumer Price Index (CPI) which measures prices that affect typical consumers, and the Gross Domestic Product (GDP) deflator, which measures prices of locally-produced goods and services.

Price stability is one of the main objectives of every government as it is an important economic indicator that governments, politicians, economists and other stakeholders use as basis of argument when debating on the state of the economy (Suleman and Sarpong, 2012). In recent years, rising inflation has become one of the major economic challenges facing most countries in the world, especially developing countries such as Nigeria. David (2001) described inflation as a major focus of economic policy worldwide. This is rightly so as inflation is the frequently used economic indicator of the performance of a country's economy due to the fact that it has a direct effect on the state of the economy. Webster (2000) defined inflation as the persistent increase in the level of consumer prices or a persistent decline in the purchasing power of money. Hall (1982) also expresses inflation as a situation where the demand for goods and services exceeds their supply in the economy.

According to www.investopedia.com/terms/i/inflation.asp The Central Bank in any country is empowered to perform duties that will ensure soundness of the financial and monetary system. In order to achieve the monetary stability, it is always confronted with the challenge of choosing the right strategy to apply in order to meet the envisaged end. Among the most popular strategies are exchange rate targeting, monetary targeting, Nominal GDP targeting and inflation targeting.

1.2. Statement of Problem

Inflation has been a problem facing many countries of the world especially unindustrialized countries. It started during the early 60s, which results to the incorporation of economic policies as measures to reduce the effect of inflation in the societies. Most of these measures taken by developing countries to check the problem of inflation are in the form of the use of central bank instruments of credit control. This is aimed at reducing the volume of money in circulation and sustaining it to ensure low cost of living. It is believed that Nigeria is a developing country. Out of all the micro and macro economy variables which affect the growth of the economy, the effect of inflation is likely to be significant since inflation is created locally in the case of Nigeria while we import inflation due to over reliance on imported goods from inflation affected countries.

Over the years as a growing update, there is general increase in the price of goods in Nigeria as well as devaluation of naira in the last four years, making it difficult to depend on old model in forecasting possible inflation rate for each month. This motivated this research to study the structure and pattern of inflation rates in Nigeria and fit appropriate model using Box Jenkins approach. This is meant to develop a model that will accurately forecast the Nigeria inflation rate in subsequent future

1.3 Aim and Objectives of the Study

The aim of this research is to Model and forecast inflation rates in Nigeria using SARIMA model.

The specific objectives are to:

1. Test for stationarity on the series using the Augmented Dickey Fuller test.
2. Investigate the pattern of inflation rates in Nigeria
3. Fit a model for Nigeria inflation rates and obtain a 12 month forecast.

1.4 Significance of the Study

The research work is of high significance as it attempts to provide awareness and understanding about Nigeria Inflation rates, and government and policy makers may find the results very useful.

It is believed that a study of this nature will expose the suffering of masses, policy makers, corporate bodies etc. through its findings, for formulating of most effective plan on how they can cope with inflation and better life for every citizen. This research will be of immense importance for students in statistics as a template for further research work,assist the planning of unit of government through the provision of more efficient information on the effectiveness of their anti-inflationary policies.

1.5 Scope of Study

Monthly inflation series from January 1999- December 2018 will be used for this research work and the data for this work are inflation rate the series is obtained from the Central Bank of Nigeria (CBN) and the National Bureau of Statistics (NBS)

1.6 Definition of Operational Terms and Acronyms

- **ARIMA-** Autoregressive Integrated Moving Average.
- **Forecasting-** Forecasting is the process of making predictions of the future based on past and present data and most commonly by analysis of trends.
- **Inflation Rate-** Inflation rate is the percentage increase in general level of prices over a period.
- **Model-** A statistical model is a mathematical model that embodies a set of statistical assumptions concerning the generation of sample data (and similar data from a larger population.
- **SARIMA-** Seasonal Autoregressive Integrated Moving Average.

CHAPTER TWO

LITERATURE REVIEW

2.1 Inflation and the Nigerian Economy

Inflation is one of the seemingly intractable problems facing the Nigerian economy. Inflation has been considered to be a direct result of the policies of the country's governments.

Fakiyesi (1996) mentioned that the Nigerian economy registered low rates of inflation in the years immediately after independence. Nwuba and Adeagbo (2007) established that since the end of the Nigerian civil war in 1970, especially following the introduction of the Structural Adjustment Programme (SAP) in 1986, urban housing construction costs and house rents have been rising at uncomfortable rates. This situation caused serious urban housing problems, resulting in multiplicity of slum settlements and shanty towns.

According to Anyanwu (1993) the oil boom of the 1970s brought with it fundamental changes in the Nigerian economy, the structure of policy incentives and controls encouraged import-oriented production and consumption pattern with little incentives for non-oil exports. Again, the competiveness of the agricultural sector in the world market was eroded by overvalued naira exchange rate, inadequate pricing policies, rural-urban migration and neglect, arising from 'too much oil syndrome'. However, prior to 1986, precisely from the mid-1981, when the world oil market began to collapse (due to glut), Nigeria witnessed a traumatic economic crisis. The resultant fall in oil exports and prices were reflected in foreign exchange receipts and government revenues. Consequently, the external resources fell sharply, foreign debts mounted in the face of rising imports, government deficits widened and efforts at containing the adverse development created some other serious problems such as economic depression, unemployment, persistent balance of payment deficits, rising prices/inflation. According to Anunobi (1997) these inflationary trends had rippling effects on the Nigerian economy.

2.2 Review of Empirical Studies

In recent times, a number of inflation studies have been carried out with data from different African countries. The results obtained seem to be similar in most of these studies. Whereas in few cases monetary aggregates are important for movements in prices, other variables such as the exchange rate, foreign prices and interest rate measures, seem to be more significant in explaining movements in inflation.

London (1989), analyzing the experience of 23 African countries for the period 1974-85, found considerable support for the role of exchange rate adjustments, as well as monetary growth, in explaining inflationary developments. In this study, both cross-section and time series regressions indicate that models of inflation based solely on monetary expansion and real income growth (which is related negatively to the inflation rate) leave sizable portions of the inflationary process unexplained. Moreover, the addition of a bilateral exchange rate variable appears to significantly relate to the rate of domestic inflation during the period 1980-85.

Chhibber et al (1989), looking specifically at Zimbabwe, where data on wage costs and interest rates were available, identified unit labor costs and interest rates in addition to the exchange rate, foreign prices, monetary growth, and real income growth, as factors explaining inflation. Tegene (1989), using Granger and Pierce causality tests, found evidence of causation running from monetary expansion to inflation in six African countries.

Agenor (1989), examining trends in inflation in four African countries, has identified an important role for parallel market exchange rates, as compared to official rates, and monetary expansion in explaining both the decomposition of the forecast error variance of inflation rates and in estimating the response of the inflation rate to unit shocks in each of these variables. Except for Tegene and Agenor, the studies thus far have all employed traditional econometric techniques, which involve estimating structural economic models and are therefore subject to the usual problems of invalid restrictions and specification error.

An empirical analysis of causes of inflation in Nigeria by Asogu (1991) indicated that real output, money supply, domestic food prices, exchange rate and net exports were the major determinants of inflation in Nigeria. Moser (1995) and Fakiyesi (1996) studied Nigeria's headline inflation using both the long-run and the dynamics error correction model and autoregressive distributed lag approaches, respectively. Their results confirmed that the basic findings of Asogu (1991) and agro-climatic conditions were the major factors influencing inflation in Nigeria. Also, using the framework of error correction mechanism, Olubusoye and Oyaromade (2008) found that the lagged CPI, expected inflation, petroleum prices and real exchange rate significantly propagate the dynamics of inflationary process in Nigeria. More recently, Adebiyi et al (2010) examined the different types of inflation forecasting models including ARIMA and showed that ARIMA models were modestly successful in explaining inflation dynamics in Nigeria.

Peng (2007) tested the fisher effect in China and found that inflation rate and rate of interest are cointegrated in the long run, but he did not find any significant relationship among these variables in the short run. He further suggested that monetary policy must be conducted independently according to the changes in the market situation with no political interference in its implementation. Obi et al. (2009) inspected the authenticity of Fisher effect in Nigeria by applying cointegration and error correction technique. They regressed lagged inflation rate, money supply and overall fiscal deficit on nominal interest rate and found partial Fisher effect in Nigeria. They suggested that government should curb inflation by investing in productive activities e.g. by improving infrastructure to boost the economic activity of the country instead of increasing money supply.

Gul and Ekinci (2006) found long-run relationship between nominal rate of interest and inflation in Turkey by applying Johansen cointegrating technique; furthermore, they found unidirectional relationship which runs from interest rate to inflation rate. Holod (2000) examined the relationship between money supply, exchange rate and price level in Ukraine by using vector autoregressive approach and suggested that exchange rate should serve as an important instrument for targeting inflation and had weak evidence that money supply shocks affect the level of prices in Ukraine.

Pufnik and Kunovac (2006) provided a method of forecasting the Croatia's CPI by using univariate seasonal ARIMA models and forecasting future values of the variables from past behavior of the series. Their paper attempts to examine whether separate modeling and aggregating of the sub-indices improves the final forecast of the all items index. The analysis suggests that given a somewhat longer time horizon (three to twelve months), the most precise forecasts of all items CPI developments are obtained by first forecasting the index's components and then aggregating them to obtain the all items index.

Alnaa and Ferdinand (2011) used ARIMA approach to estimate inflation in Ghana using monthly data from June 2000 to December 2010. They found that ARIMA (6, 1, 6) is best for forecasting inflation in Ghana. Also, Suleman and Sarpong (2012) employed an empirical approach to modeling monthly CPI data in Ghana using the seasonal ARIMA model. Their result showed that ARIMA $(3,1,3)(2,1,1)_{[12]}$model was appropriate for modeling Ghana's inflation rate. Diagnostic test of the model residuals with the ARCH LM test and Durbin Watson test indicates the absence of autocorrelations and ARCH effect in the residuals. The forecast results inferred that Ghana was likely to experience single digit inflation values in 2012.

Meyler et al (1998) outlined the practical steps which need to be undertaken to use ARIMA time series models for forecasting Irish inflation. They considered two alternative approaches to the issue of identifying ARIMA models – the Box Jenkins approach and the objective penalty function methods. The approach they adopted is 'unashamedly' one of model mining with the aim of optimizing forecast performance.

Abledu and Agbodah (2012) found ARIMA (1, 1, 0) to be the best model in an attempt to analyze and forecast the macroeconomic impact of oil price fluctuations in Ghana using annual data from 2000 to 2011. Suleman and Sarpong (2012) concluded that the ARIMA (3, 1, 3) × $(2,1,1)_{[12]}$ best represent the behaviour of inflation rates. Alnaa and Ahiakpor (2011) found ARIMA (6, 1, 6) to be the best fitted model for forecasting inflation in Ghana.

Hafer and Kutan (1997) explored the importance of the commercial bill spread and argue that the conclusion on its significance was a result of wrong stationarity assumptions about the money variables. They found that by carefully modeling the data, money variables were still useful beyond the 1980s. Black et al. (2000) performed forecasts of inflation and output, using the same data. They estimated an AR (1) model as a base model and calculated its mean absolute percentage error. They then forecast inflation, adding one variable at a time to the AR (1) model and compared the Mean Absolute Percentage Errors (MAPEs) of the different models. They found that money improves the forecasts of inflation. Studies testing the importance of monetary aggregates have also been carried out using Australian data. One such study is that by Orden and Fisher (1993). They obtained variance decompositions and found that money shocks contributed up to 30% of variations of prices.

Kairala (2011) used seasonal ARIMA and exponential seasonal smoothing method and winter models to scrutinize the forecast revenue of Nepal which pursues an erratic movement along time- there were over-estimation of revenue followed by under-estimation. He pointed out that SARIMA model was superior to any other forecasting models. He also argued that existing models of revenue forecasting in Nepal were constructed on the basis of growth rate; resulting in frequent higher discrepancies in the estimation. In an attempt Fullerton (1991) applied univariate ARIMA model integrated with a composite method of sales tax revenue to forecast using quarterly revenue data. This study suggested that a composite model based on univariate ARIMA projections of Idaho retail sales tax receipts provided better forecasts than single models. He also posited that given any existing efficient institutional capacity, forecast error appears because of some undesired occurrences in both external and internal factors intrinsic to tax system.

Barnichon and Peiris (2007) explored the sources of inflation in Sub-Saharan Africa by examining the relationship between inflation, the output gap, and the real money gap. Using heterogeneous panel cointegration estimation techniques, he estimated cointegrating vectors for the production function and the real money demand function to recover the structural output and money gaps for seventeen African countries. The central finding is that both gaps contain significant information regarding the evolution of inflation, albeit with a larger role played by the money gap.

Adnan and Ullah (2007) analyzed the impact of volatility in government borrowing from central bank on domestic inflation in Pakistan. The paper utilized Generalized Auto Regressive Conditional Hetroskedasticity model to estimate volatility in government borrowing using monthly data from July 1992 to July 1992 to June 2007. The empirical results, based on auto regressive distributed lag with bound testing technique suggest that domestic inflation in Pakistan is related with volatility in government borrowing from central bank in the long run. Furthermore, error correction model (ECM) estimates show that in the short run, inflation is also affected by volatility in government borrowing.

Akdogan et.al (2012) produced short-term forecasts for inflation in Turkey, using a large number of econometric models such as the univariate ARIMA models, decomposition based models, a Phillips curve motivated time varying parameter model, a suit of VAR and Bayesian VAR models and dynamic factor models. Their result suggests that the models which incorporate more economic information outperformed the random walk model at least up to two quarters ahead. Mordi et al (2012) developed a short-term inflation forecasting framework to serve as a tool for analyzing inflation risks in Nigeria. Their framework follows mostly a structural time series model for each

CPI component constructed at a certain level of disaggregation. Short-term forecasts of the all items CPI were made as a weighted sum of the twelve CPI components forecasts. Thereafter, the all items CPI and the twelve CPI components were used to calculate short-term inflation forecasts. The framework is intended to serve as a tool for analyzing inflation risks with the aid of fan charts and given its disaggregated nature, appears informative and capable of improving the credibility of the policy makers.

Akhter (2013) forecasted the short-term inflation rate of Bangladesh using the monthly CPI from January 2000 to December 2012. The paper employed the seasonal ARIMA models proposed by Box et al (1994). Because of the presence of structural break in the CPI, the study truncates the series and using data from September 2009 to December 2012 fitted the seasonal ARIMA $(1,1,1)(1,0,1)_{[12]}$ model. The forecast result suggests an increasing pattern and high rates of inflation over the forecast period of 2013.

Omane-Adjepong et al (2013) examined the most appropriate forecasting method for Ghana's inflation. The monthly dataset used was divided into two sets, with the first set used for modeling and forecasting, while the second set was used as test. Seasonal ARIMA and Holt-Winters approaches are used to obtain short-term out of sample forecast. From the results, they concluded that an out of sample forecast from an estimated seasonal ARIMA$(2,1,2)(0,0,1)_{[12]}$model far supersedes any of the HoltWinters' approach with respect to forecast accuracy. Following several researches and papers discussed above, we as well wish to discuss and analyze Nigeria inflation rate using Time series analysis (Seasonal autoregressive integrated moving average i.e. SARIMA).

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Research Design

In this study, secondary data will be used and this data will be obtained from the Centeral Bank of Nigeria (CBN) and National Bureau of Statistics (NBS).

3.2 Population, Sample and Sampling Techniques

This study was based on secondary data obtained from Centeral Bank of Nigeria (CBN) and National Bureau of Statistics (NBS) from the period of January 1999- December 2018.

3.3 Method of Data Collection

The relevant data needed for this work is monthly data on inflation rate (1999-2018). These data were obtained from the Central Bank of Nigeria (CBN) and the National Bureau of Statistics (NBS).

3.4 Technique for Data Analysis and Model Specification

3.4.1 Technique for Data Analysis

For the Seasonal Autoregressive Integrated moving Average modeling we will be using two Statistical packages i.e SAS and Eviews because Eviews is an econometric software for data analysis and this study deals with inflation which is an economic factor while SAS will be mostly used for data visualization in this study.

3.4.2 Model Specification

In this study, a univariate Box-Jenkins Methods (Box et al., 1994) in particular, a seasonal autoregressive integrated moving average (ARIMA) model is used, since most of the economic series are non-stationary in nature.

3.4.2.1 Time Plot

The first step in the analysis of time series is usually to plot the data and obtain simple descriptive measures of the main property of the series via a visual inspection of the time series plot. This may reveal one or more of the following characteristics: seasonality, trends either in the mean level or the variance of the series, long- term cycles, and so on. If any such patterns are present, then these are signs of non-stationarity.

3.4.2.2 Correlogram

One way to characterize a series with respect to its dependence over time is to plot its sample autocorrelation function (SACF). The sample partial autocorrelation function denoted by SPACF which is similar to the SACF and can be described as the correlation between $x_t$andx_{t-s}(observations of the time series recorded at two moments in time units apart) after controlling for the common linear effects of the intermediate lags. Both functions are used in Box-Jenkins modeling as correlogram to reveal important information regarding the order of the autoregressive (AR) and moving average (MA) factors present in the generating process of the given time series as well as to assess stationarity.

3.4.2.3 Unit Root Test

For a univariate time series, the unit root test is frequently employed for testing stationarity. The test first poses the null hypothesis that the given time series has a unit root, which means that the time series is non-stationary and tests if the null hypothesis is to be statistically rejected in favour of the alternative hypothesis that the given time series is stationary. To detect whether a given series has non stationarity, let's assume that the relationship between current value (in time t) and last value (in time $t-1$) in the time series is as follows;

$$x_t = \emptyset x_{t-1} + w_t \quad (1)$$

where,

$x_t$is an observation value at time t, $w_t$is a white noise process. This model is a first order autoregressive process. The time series $x_t$converges, as $t \to \infty$, to a stationary time series if $|\phi| < 1$. If $|\phi| = 1$ or > 1, the series x_t is not stationary and the variance of $x_t$is time dependent (Diebold et al., 2010). In other words, the series has a unit root. The unit root test subsequently tests the following one-sided hypothesis

H_0: $\phi = 1$ (has a unit root).

H_1: $\phi < 1$ (has root outside the unit circle).

The name, unit root, comes from the fact that the coefficient of $x_{t-1}$is unity, if the time series is non-stationary, and the unit root test, as the name suggests, tests if ϕ is unity or not. If $x_{t-1}$is subtracted from the right and left sides of the above equation, we get:

$$\nabla x_t = (\emptyset - 1)x_{t-1} + w_t \quad (2)$$

This equation is expressed as a first order difference equation. If ϕ is taken as one in the equation (1), the effect of unit root can be removed from the actual series that has non stationarity via a first differencing. The tests above are valid only if w_t is a white noise. In particular, w_t is assumed not to be auto correlated, but would be so if there was autocorrelation in the dependent variable of the regression $\{x_t\}$ which has not been modeled. If this is the case, the test would be oversized, meaning that the true size of the test (the proportion of times a correct null hypothesis is incorrectly rejected) would be higher than the nominal size used (e.g. 5%). The solution is to augment the test using *p* lags of the dependent variable.

3.4.2.3.1 The Augmented Dickey-Fuller (ADF) Test

The presence of trends and unit roots can be detected from the slowly decaying autocorrelation function (ACF) in a univariate process, thus indicating non-stationary, but this has very little power to detect the process of trend or unit root.

Consider the AR (1) series

$$x_t = \phi x_{t-1} + w_t \tag{3}$$

Since we know that if $-1 < \phi < 1$ then x_t is stationary

If $\phi = 1, x_t$ is not stationary

Hence the unit root hypothesis is

$$H_0: \phi = 1 \quad vs$$

$$H_1: \phi < 1 \tag{4}$$

Subtract x_{t-1} from (3) we have

$$x_t - x_{t-1} = \phi x_{t-1} - x_{t-1} + w_t$$

$$\Delta x_t = (\phi - 1)x_{t-1} + w_t \tag{5}$$

Let $\delta = \phi - 1, hence$

$$\Delta x_t = \delta x_{t-1} + w_t \tag{6}$$

Thus, testing for $\phi = 1$ is tantamount to testing for $\delta = 0$.

The Augmented Dickey Fuller (ADF) test involves checking through and testing the three sets of models:

$$(i) \Delta x_t = (\lambda - 1)x_{t-1} + \sum_{j=1}^{t} \beta_j \, \Delta x_{t-j} + w_t \tag{7}$$

$$(ii)\Delta x_t = \alpha + (\lambda - 1)x_{t-1} + \sum_{j=1}^{t} \beta_j \, \Delta x_{t-j} + w_t \quad (8)$$

$$(iii)\Delta x_t = \alpha + \delta_t + (\lambda - 1)x_{t-1} + \sum_{j=1}^{t} \beta_j \, \Delta x_{t-j} + w_t \quad (9)$$

Where equation (7) is a pure random walk model (A time series that has a unit root is known as random walk and a random walk is an example of a non-stationary time series). Equation (8)contains an intercept or drift term and equation(9) contains both the drift and linear time trend.

Unit root test involves one or more of the above equations and the associated standard errors and comparing the test statistic with the appropriate values in the Dickey Fuller table.

In practice, most economic time series are non-stationary. A univariate process that is non-stationary (contain trend) can be made stationary by differencing and the resulting series can be modeled using univariate Box-Jenkins methodology.

3.4.2.4. Time Domain Approach

This approach focuses on modeling some future values of the series as function of the current and the past. In this respect, the time-domain approach of univariate time series continues to be an important topic. An intrinsic feature of the time-domain approach is that, typically, adjacent points in time are correlated and that future values are related to past and present values. Autoregressive integrated moving average (ARIMA) modeling is one of the most widely implemented methods for analyzing univariate time series data (Box and Jenkins, 1976). In order to understand the modeling procedure, it is useful to briefly introduce the following basic models.

3.4.2.4.1 Autoregressive (AR) Models

Autoregressive models are based on the idea that the current value of the series x_t can be explained as a function of p past values, $x_{t-1}, x_{t-2}, \dots, x_{t-p}$ where p determines the number of steps into the past needed to forecast the current value. An autoregressive model of order *p*, abbreviated AR (p) can be written as:

$$x_t = \emptyset_1 x_{t-1} + \emptyset_2 x_{t-2} + \cdots + \emptyset_p x_{t-p} + e_t \qquad (10)$$

where x_t is stationary series, $\emptyset_1, \emptyset_2, \dots, \emptyset_p$ are the parameters of the AR ($\emptyset_0 \neq 0$), ($\emptyset_p \neq 0$). and the e_t are typically assumed to be uncorrelated (0, σ^2). Unless otherwise stated, we assume that e_t is a Gaussian white noise series with mean zero and variance (σ_e^2). The highest order p is referred to as the order of the model.

The model in lag operators takes the following form:

$$\left(1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_p B^p\right) x_t = e_t \qquad (11)$$

where the lag (backshift) operator B is defined as: $B^p x_t = x_{t-p},\ p = 0,1,2,\dots.$

More concisely we can express the model as:

$$\emptyset(B) x_t = e_t \qquad (12)$$

The autoregressive operator $\emptyset(B)$ is defined to be

$$\emptyset(B) = 1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_p B^p \qquad (13)$$

The values of $\emptyset$ which make the process stationary are such that the roots of $\emptyset(B) = 0$ lie outside the unit circle in the complex plane (Chatfield, 1991). If all roots of $\emptyset(B)$ are larger than one in absolute value, then the process is a stationary process satisfying the autoregressive equation and can be represented as:

$$x_t = \sum_{j=0}^{\infty} \psi_j e_{t-j} \tag{14}$$

The coefficients ψ_j converge to zero, such that $\sum_{j=0}^{\infty} |\psi_j| < \infty$. If some roots are "exactly" one in modulus, no stationary solution exists.

A plot of the ACF of a stationary AR (p) model show a mixture of damping sine and cosine patterns and exponential decays depending on the nature of its characteristic roots.

Another characteristics feature of AR (p) models is that the partial autocorrelation function defined as $PACF_j = corr.\left(x_t, x_{t-j} \,/\, x_{t-1}, x_{t-2}, \dots, x_{t-j+1}\right)$ becomes "exactly" zero for values larger than p (Tsay, 2005).

3.4.2.4.2 Moving average (MA) Models

As an alternative to the autoregressive representation in which the $x_t$on the left-hand side of the equation are assumed to be combined linearly, the moving average model of order q, abbreviated as MA(q), assumes the white noise e_t on the right-hand side of the defining equation are combined linearly to form the observed data.

A series x_t is said to follow a moving average process of order q or simply MA (q) process if

$$x_t = w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2} + \cdots + \theta_q w_{t-q} \tag{15}$$

where $\theta_1, \theta_2, \dots, \theta_q$ are the MA parameters. MA(q) models immediately define stationary, every MA process of finite order is stationary (Diebold et al., 2006). In order to preserve a unique representation, usually the requirement is imposed that all roots of $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q = 0$ are greater than one in absolute value. If all roots of $\theta(B) = 0$ lie outside the unit circle, the MA process has an autoregressive representation of generally infinite order $\sum_{j=0}^{\infty} \psi_j x_{t-j} = w_t$with $\sum_{j=0}^{\infty} |\psi_j| < \infty$ MA process as with an infinite order autoregressive representation are said to be invertible.

A characteristics feature of MA(q) is that their ACF, ρ_j becomes statistically insignificant after $j = q$. The property of the ACF should be reflected in the correlogram, which should cut off after q. The PACF converges to zero geometrically.

3.4.2.4.3 Autoregressive Moving Average (ARMA)

We now proceed with the general development of autoregressive, moving average, and mixed autoregressive moving average (ARMA), models for stationary time series. In most case, it is best to develop a mixed autoregressive moving average model when building a stochastic model to represent a stationary time series. The order of an ARMA model is expressed in terms of both *p* and *q*. The model parameters relate to what happens in period *t* to both the past values and the random errors that occurred in past time periods. A general ARMA model can be written as follow:

$$x_t = \emptyset_1 x_{t-1} + \emptyset_2 x_{t-2} + \cdots + \emptyset_p x_{t-p} + w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2} + \cdots + \theta_q w_{t-q} \quad (16)$$

Equation (16) of the time series model will be simplified by a backward shift operator *B* to obtain

$$\emptyset(B) x_t = \theta(B) w_t \quad (17)$$

The ARMA model is stable, that is it has a stationary solution if all roots of $\emptyset(B) = 0$ are larger than one in absolute value. The representation is unique if all roots of $\emptyset(B) = 0$ lie outside the unit circle where $\emptyset(B) = 0$ and $\theta(B) = 0$ donot have common roots. Stable ARMA models always have an infinite order MA representation. If all roots of $\emptyset(B)$ are larger than one in absolute value, it has an infinite order AR representation. The process is invertible only when the roots of $\theta(B)$ lie outside the unit circle .Furthermore, a process is said to be causal when the roots of $\emptyset(B)$lie outside the unit circle. To have $ARMA(p, q)$model, both ACF and PACF should show a pattern of decaying to zero. The autocorrelation of an $ARMA(p, q)$ process is determined at greater lags by the AR (*p)* part of the process as the effect of the MA part dies out. Thus, eventually the ACF consists of mixed damped exponentials and sine terms. Similarly, the partial autocorrelation of an $ARMA(p, q)$ process is determined at greater lags by the MA (q) part of the process. Thus, eventually the partial autocorrelation function will also consist of a mixture of damped exponentials and sine waves.

3.4.2.4.4 Autoregressive Integrated Moving Averages (ARIMA) Models

Autoregressive integrated moving average (ARIMA) models are specific subset of univariate modeling, in which a time series is expressed in terms of past values of itself (the autoregressive component) plus current and lagged values of a white noise error term (the moving average component). ARIMA models are univariate models that consist of an autoregressive polynomial, an order of integration(d) and a moving average polynomial.

A process (x_t) is said to be an autoregressive integrated moving average process, denoted by $ARMA(p, d, q)$if it can be written as:

$$\emptyset(B)\nabla^d x_t = \theta(B)w_t \tag{18}$$

where$\nabla^d = (1 - B)^d$ with $\nabla^d x_t$and d^{th}consecutive differencing (Vandale,1983)

If $E(\nabla^d x_t) = \mu$, we write the model as;

$$\emptyset(B)\nabla^d x_t = \alpha + \emptyset(B)w_t \tag{19}$$

Where α is a parameter related to the mean of the process$\{x_t\}$, by $\alpha = \mu\{1 - \emptyset_1 - \cdots - \emptyset_p\}$ and this process is called a white noise process, that is, a sequence of uncorrelated random variables from a fixed distribution (often Gaussian) with constant mean $E\{x_t\} = \mu$ usually assumed to be "zero" and constant variance. If d = 0, it is called $ARIMA(p,q)$ model while when d = 0 and q = 0, it is referred to as autoregressive of order p model and denoted by AR *(p)*. When p = 0 and d = 0, it is called Moving Average of order q model, and is denoted by MA (q).

3.4.2.4.5 Seasonal ARIMA (SARIMA)

Here we introduce several modifications made to the ARIMA model to account for seasonal and non-stationary behaviour. Often, the dependence on the past tends to occur most strongly at multiples of some underlying seasonal lags. Seasonal ARIMA (SARIMA) is used when the time series exhibits a seasonal variation. Natural phenomena such as temperature and rainfall have strong components corresponding to seasons. Hence, the natural variability of many physical, biological, and economic processes tends to match with seasonal fluctuations. Because of this, it is appropriate to introduce autoregressive and moving average polynomials that identify with seasonal lags. The resulting pure seasonal autoregressive moving average model, say, $ARMA(P,Q)s$, then takes the form (Shumay and Stoffer, 2010):

$$\Phi_P(B^s)x_t = \Theta_Q(B^s)w_t \qquad (20)$$

with the following definition of the operators

$$\Phi_P(B^s) = 1 - \Phi_{1s}B^s - \Phi_{2s}B^{2s} - \cdots - \Phi_{ps}B^{ps} and \Theta_Q(B^s)$$
$$= 1 + \Phi_{1s}B^s + \Phi_{2s}B^{2s} + \cdots$$
$$+ \Phi_{Qs}B^{Qs} \qquad (21)$$

are the seasonal autoregressive operator and the seasonal moving average operator of orders P and Q, respectively, with seasonal period S. Analogous to the properties of non-seasonal ARMA models, the pure seasonal $ARMA(P,Q)s$is causal only when the roots of $\Phi_P(B^s)$ lie outside the unit circle, and it is invertible only when the roots of $\Theta_Q(B^s)$ lie outside the unit circle. In general, we can combine the seasonal and non-seasonal operators into a multiplicative seasonal autoregressive moving average model, denoted by $ARMA(p,q) \times (P,Q)s$ and write as the overall model as:

$$\Phi_P(B^s)\emptyset(B)x_t = \Theta_Q(B^s)\theta(B)w_t \quad (22)$$

A seasonal autoregressive notation (P) and a seasonal moving average notation (Q) will form the multiplicative seasonal autoregressive integrated moving average model, denoted by ARIMA (p, d, q)*(P, D, Q)s, of Box and Jenkins (1976) and is given by:

$$\Phi_P(B^s)\emptyset(B)\nabla_S^D x_t = \alpha + \Theta_Q(B^s)\theta(B)w_t \quad (23)$$

where w_t is the usual Gaussian white noise process. The ordinary autoregressive and moving average components are represented by polynomials $\emptyset(B)$and $\theta(B)$ of orders p and q, respectively and the seasonal autoregressive and moving average components by $\Phi_P(B^s)$ and $\Theta_Q(B^s)$ of orders P and Q. The ordinary and seasonal difference components can be written as$\nabla^d = (1-B)^d \; and \; \nabla_S^D = (1-B^S)^D$.

3.4.2.5 Building ARIMA Models

To identify a perfect ARIMA model for a particular time series data, Box and Jenkins (1976) proposed a methodology that consists of four phases: i) Model identification; ii) Estimation of model parameters; iii) Diagnostic checking for the identified model and iv) Application of the model (i.e. forecasting).

3.4.2.5.1 Model Identification

The purpose of the identification stage is to determine the differencing required to achieve stationarity and the order of both the seasonal and the non- seasonal AR and MA operators for the residual series. There are a number of identification methods proposed in the literature. The autocorrelations function (ACF) and the partial autocorrelation functions (PACF) are the two most useful tools in any attempt at time series model identification (Granger and Newbold, 1986).

3.4.2.5.2 Autocorrelation Function (ACF)

The sample ACF (γ_k) measures the amount of linear dependence between observations in a time series that are separated by a lag k. To use the ACF in model identification, estimateγ_k and then plot γ_k series against lag k up to a maximum lag of about five times the seasonality interval and this should be less than to one fourth of the series under study (Hipel et al., 1977). To identify the number of non-seasonal and seasonal autoregressive, and non-seasonal and seasonal moving average parameters, we examine the ACF based on the theoretical pattern for the identified parameters using Tables 1 and 2 as summarized (Shumway and Stoffer, 2010)

Table 1: Behaviour of the ACF and PACF for ARMA Models

	AR(P)	MA(Q)	ARMA(P,Q)
ACF	Tails off	Cuts off after lags Q	Tails off
PACF	Cuts off after lags P	Tails off	Tails off

Table 2: Behaviour of the ACF and PACF for Pure SARMA Models

	AR(P)s	MA(Q)s	ARMA(P,Q)s
ACF	Tails off at lags Ks, K=1, 2 ,…	Cuts off after lags Qs	Tails off at lags Ks
PACF	Cuts off after lags Ps	Tails off at lags ks	Tails off at lag Ps

Where the values at non seasonal lags $h \neq Ks, for\ K = 1,2, \ldots$ are zero. When the process is $SARIMA(0, d, q) \times (0, D, Q)s$model, γ_k truncates and is not significantly different from zero after $lag\ q + sQ$. If γ_k spikes out at lags that are multiples of s, this implies the presence of a seasonal autoregressive component. The failure of the autocorrelation function to truncate at other lags may imply that a non seasonal autoregressive term is required. The autocorrelation of order $k$is simply the correlation between $x_k$and x_{t-k}, that is,

$$\rho_k = \frac{E[(x_t - \bar{x})(x_{t-k} - \bar{x})]}{E[x_t - \bar{x}]^2} \tag{24}$$

In practice, one never knows the true autocorrelations and partial autocorrelations and at the identification stage, one has to rely on the sample autocorrelation and partial autocorrelation functions imitating the behaviour of the corresponding parent quantities. True autocorrelations (γ_k) can be estimated by:

$$\gamma_k = \frac{\frac{1}{n}\sum_{t=k+1}^{n}(x_t - \bar{x})(x_{t-k} - \bar{x})}{\frac{1}{n}\sum_{t=1}^{n}[x_t - \bar{x}]^2} \tag{25}$$

Where$\bar{x}$ is the sample mean of the x_t.

3.2.4.5.3 Partial Autocorrelation Function (PACF)

Partial autocorrelation function can also be used for determining the possible order of seasonal autoregressive, non-seasonal autoregressive, moving average and seasonal moving average that should be incorporated in the model by the help of Table 3.1 and 3.2 above. When the process is a pure $SARIMA(p,d,0) \times (P,D,0)_{12}$ model, γ_{KK} cuts off and is not significantly different from zero after $lagp + SP$. If γ_{KK} damps out at lags that are multiples of s, this suggests the incorporation of a seasonal moving average component in to the model. The failure of the partial autocorrelation function to truncate at other lags may imply that a non-seasonal MA term is required (Hipel et al., 1977**).** To obtain an estimate for partial autocorrelations (ρ_{kk})at $lagk$, we can employ successive autoregressive estimation procedure. The first step is to model the x_t series by finite autoregressive models of order K given by (Box and Jenkins, 1976):

$$x_t = \rho_0 + \sum_{k=1}^{K} \rho_{kk}\, x_{t-k} \tag{26}$$

Where ρ_{kk} is the k^{th} autoregressive coefficient and $k = 1,2,\dots,K$. Estimate of these coefficients by ordinary least squares or maximum likelihood estimation method gives the k^{th} sample partial autocorrelation (Hipel et al., 1977).

3.4.2.6 Parameter Estimation

After choosing the most appropriate model (step (i) above), the model parameters are estimated by using several estimation procedures. The estimation-stage results will be used to check: (i) parameter estimates, (ii) the appropriateness of coefficient estimates which includes the statistical significance of estimated coefficient and standard error and correlation matrix. In maximum likelihood methods, the likelihood function is maximized in order to obtain the parameter estimates. The likelihood of a set of data is the probability of obtaining that particular set of data, given its distribution. The philosophy behind maximum likelihood estimates is to find a set of parameters which maximize the likelihood of observing the data to which the model is being fitted. The linear optimization algorithm is used to maximize the likelihood function with respect to the parameter space (Shumway and Stofferr, 2010). In Time series analysis, there may be several adequate models that can be used to represent a given data set, and hence, numerous criteria for model comparison have been introduced in the literature. One of them is based on the so-called information criteria. The idea is to balance the risks of under fitting (selecting an order smaller than the true order) and over fitting (selecting an order larger than the true order).

Akaike (1978) introduced a criterion called Akaike Information Criterion (AIC) in the literature. The AIC is a mathematical selection criterion of model building. When there are several competing models to choose from, select the model that gives the minimum of the AIC defined by (Shumay and Stoffar, 2010):

$$AIC = log\hat{\delta}_k^2 + \frac{n+2k}{n} \qquad (27)$$

where $\hat{\delta}_k^2 = \frac{SSE_k}{n}$denotes the maximum likelihood estimator for the error variance and k is the number of seasonal and non-seasonal autoregressive and moving average parameters to be estimated in the model, that is, according to Wei (1990), k = p + q + P+ Q + 1 and *n* is the number of observations. The optimal order of the model is chosen by the value of *k*, which is a function of *p* and *q*, P and Q so that the value of k yielding the minimum AIC specifies the best model. Wei (1990) expressed the need to select the model that has fulfilled all the diagnostic checks and has as few parameters as possible in terms of parsimony. Schwartz (1978) suggested a Bayesian criterion called Schwartz's Bayesian Criterion (SBC) having the form:

$$SBC = log\hat{\delta}_k^2 + \frac{klogn}{n} \quad (28)$$

The (SBC) can also measure the parsimony of model building. A model that has the smallest SBC value among the competing models fit to time series is preferred. The optimal order of the model is chosen by the value of *k*, which is a function of *p* and *q*, P and Q so that the value of k yielding the minimum SBC specifies the best model as AIC.

3.4.2.7 Diagnostic Checking

After fitting a provisional time series model, we can assess its adequacy in various ways. The usual approach is to extract from the data, a sequence of residuals to correspond to the underlying, last unobservable, white noise sequence, and to check that the statistical properties of these residuals are indeed consistent with white noise. Most diagnostic tests deal with the residual assumptions in order to determine whether the residuals from fitted model are independent, have a constant variance, and are normally distributed. Several diagnostic statistics and plots of the residuals can be used to examine the goodness of fit of the tentative model to the historical data. The first approaches that can be used to evaluate the adequacy of a model are the plot of the errors over time, which can be written (Shumay and Stoffer, 2010):

$$w_t = (x_t - x_t^{t-1}) \Big/ \sqrt{p_t^{t-1}} \quad (29)$$

where$x_t - x_t^{t-1} is$ the one-step-ahead prediction of (x_t) based on the fitted model and p_t^{t-1} is the estimated one-step-ahead error variance. If visual inspections of the errors reveal that they are randomly distributed over time, then we have a good model.

The autocorrelation function (ACF) of the series can also be used to examine whether the residual of the fitted model is white noise or not. If the ACF is significantly different from zero, this implies that there is dependence between observations (Janacek and Swift, 1993; Ferguson et al., 2000). There are different applications related to the Residual ACF (RACF) for the independence of residuals. The first one is the correlogram drawn by plotting $\gamma_k(w)$ against lag k.

$$\gamma_{ak} = \frac{\sum_{t=k+1}^{n} w_t w_{t-k}}{\sum_{t=1}^{n} w_t^2} \tag{30}$$

Under the assumption that residual follows a white noise process the standard errors of these γ_{ak} are approximately equal to $1/\sqrt{T}$. Thus, under the null hypothesis that residual follows a white noise process, roughly 95% of the autocorrelation coefficient (γ_{ak}) should fall within the range$\pm 1.96/\sqrt{T}$. If more than 5% of the coefficient fall outside of this range, then most likely residual does not follow a white noise process (Lehmann and Rode, 2001). There are many statistical tests used for diagnostic checking of randomness. The Ljung- Box Q statistic, Turning point and Runs tests can be used for the diagnostic checking of residuals for independence.

3.4.2.7.1 Ljung-Box Q (LBQ) Statistic

The Ljung-Box Q or $Q(r)$ statistic can be employed to check independence of residual instead of visual inspection of the sample autocorrelations. A test of hypothesis can be done for the model adequacy by choosing a level of significance and then comparing the value of calculated chi-square with the tabulated chi-Square critical value. A useful test in these concepts is the portmanteau lack of fit test. This uses the entire residual sample with null hypothesis that $H_0: \rho_1 = \rho_2 = \cdots = \rho_k = 0$.

The test statistic is calculated by the following equation (Ljung and Box, 1998):

$$Q(r) = n'(n'+2)\sum_{j=1}^{k}\frac{\mu_j^2}{n-j} \tag{31}$$

where $n' = (n - d), n$ is the number of observations in the original time series, μ_j^2 is the sample autocorrelation of the residuals at lag j and d is the degree of non- seasonal differencing used to transform the original time series values into stationary time series values and k is a sufficiently large integer.

This test statistic is the modified Q- statistic originally proposed by Box and Pierce (1970). Under the null hypothesis of model adequacy, Ljung and Box (1978) show that the Q- statistic approximately follows the distribution where m is the number of parameters estimated in the model. If a model is correctly specified, residuals should be uncorrelated and $Q(r)$ should be small (p- value should be large).

3..4.2.8 Forecasting

The last step in time series modeling is forecasting. There are two kinds of forecasts: sample period forecasts and post-sample period forecasts. The former will be used to develop confidence in the model and the latter will be used to generate genuine desired forecasts. In forecasting, the goal is to predict future values of a time series, $x_{t+m}, m = 1,2, \ldots$ based on the data collected to the present, $x = \{x_t, x_{t-1}, \ldots, x_1\}$.

3.4.2.8.1 Forecasting Accuracy Measures

Once forecasts are made they can be evaluated if the actual values of the series to forecast are observed. There are some measurements of the accuracy of forecasts. These are Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute percentage Error (MAPE).

$$MAE = \frac{1}{h+s}\sum_{t=s}^{h+s}\left(\hat{X}_t - X_t\right)^2 \tag{33}$$

the root mean square forecast error (RMSE) is defined as:

$$RMSE = \sqrt{\frac{1}{h+s}\sum_{t=s}^{h+s}\left(\hat{X}_t - X_t\right)^2} \tag{34}$$

and the mean absolute percentage forecast error MAPE is given as,

$$MAPE = \frac{100}{h+s}\sum_{t=s}^{h+s}\left|\frac{\hat{X}_t - X_t}{\hat{X}_t}\right| \tag{35}$$

$where\ t = s, 1+s, \ldots, h+s$.The actual and predicted values for corresponding t values are denoted by $\hat{X}_t\ and\ X_t$respectively. The smaller the values of *RMSE* and *MAPE*, the better the forecasting performance of the model.

CHAPTER FOUR

DATA PRESENTATION AND ANALYSIS

In this chapter, the Nigerian inflation series between January, 1999 and December, 2018 was analyzed. The seasonal autoregressive integrated moving average (SARIMA) technique was adopted for the analysis. Two Statistical packages i.e. Statistical Analysis Software (SAS), and E-views9 were used to analyse the data.

4.1 Time Plot

Nigerian Inflation Rate January,1991- December,2018

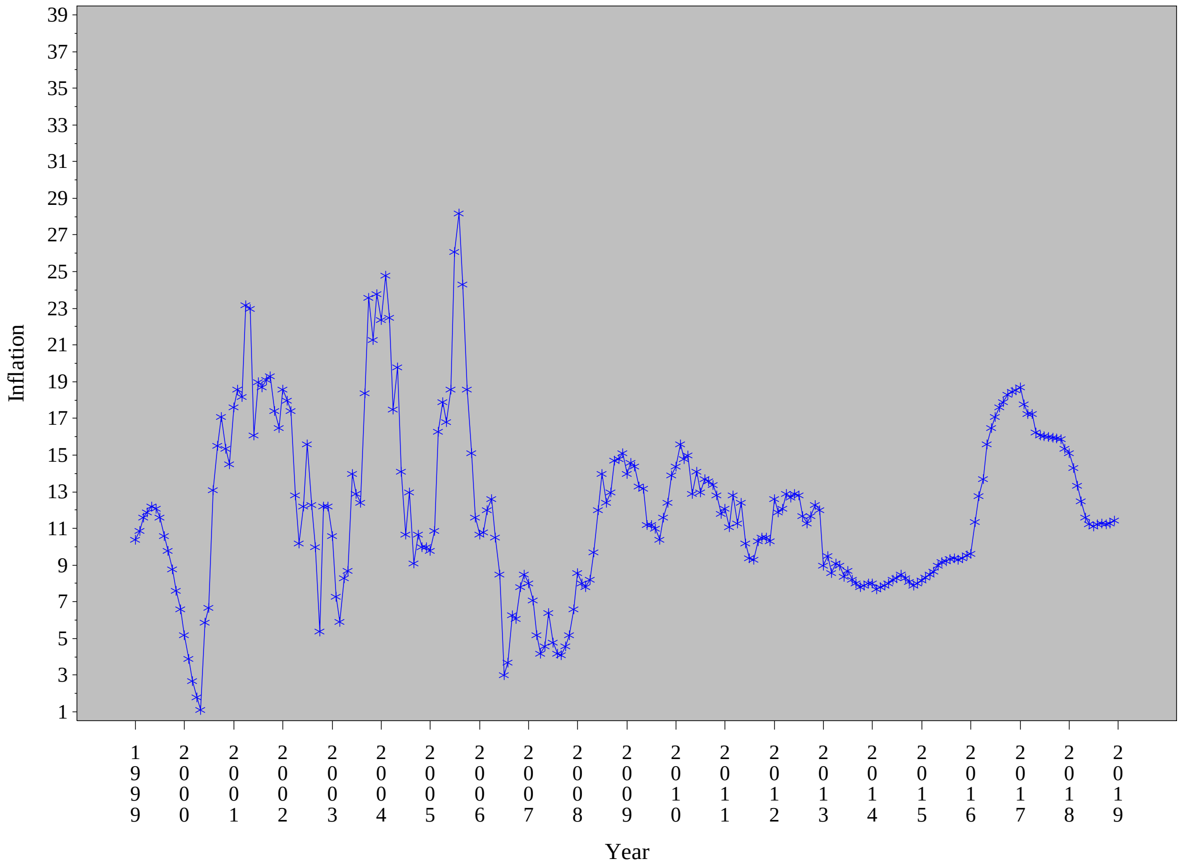

Figure 4.1

Firstly, we have to describe the trend of time series of the Nigerian inflation rate from January, 1999 to December, 2018 is given in Figure 4.1above. It exhibits upward increasing trend and suggests that the given time series is non-stationary. The movement is secular in nature and expect a small shift in the movement in mid-2007.

4.2 Augmented Dickey Fuller Unit Root Test Result

In time series, a unit root test, determines whether a time series variable is non-stationary using an autoregressive model. Augmented Dickey–Fuller test (ADF) and Phillips-Person is a test for a unit root test in our time series data is a negative number. The more negative it is, the stronger the rejection of the hypothesis that there is a unit roots at some level of confidence.

Table 4.1 below the Augmented Dickey Fuller and Phillips-Persons test show that Nigerian Inflation rate is stationary at the first difference, that is $I(1)$ at 1%, 5% and 10% levels of significance with $p-value = 0.001$. Since the order of integration of the difference population series is one (1), then $d = 1$.

Therefore, we denote from the two tables below that the data is stationary at (1).

Table 4.1: Stationarity Test (Augmented Dickey Fuller)

Null Hypothesis: D(INFLATION) has a unit root				
Exogenous: Constant, Linear Trend				
Lag Length: 2 (Automatic - based on AIC, maxlag=13)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-6.299166	0.0000
Test critical values:	1% level		-4.027959	
	5% level		-3.443704	
	10% level		-3.146604	

Table 4.2: Stationarity Test (Phillips-Persons)

Null Hypothesis: D(INFLATION) has a unit root	

Exogenous: Constant, Linear Trend				
Bandwidth: 4 (Newey-West automatic) using Bartlett kernel				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-9.796247	0.0000
Test critical values:	1% level		-4.026942	
	5% level		-3.443201	
	10% level		-3.146309	
*MacKinnon (1996) one-sided p-values.				

4.3 Theory of Model Building for Nigerian Monthly Inflation Series

Trend and prediction of time series can be computed by using ARIMA model.

ARIMA (p,d,q) model is complex a linear model. There are three parts (they do not have to contain always all of these): AR (Autoregressive) – linear combination of the influence of previous values; I (Integrative) – random walk; MA (Moving average) – linear combination of previous errors. These models are very flexible, quite hard for computing and for the understanding of the results.

Fitting a model to time series data involves plotting the data, identifying the dependence orders of the model, parameter estimation, diagnostics tests, and model choice. In this section, a univariate SARIMA methodology is used to model monthly Inflation series in Nigeria.

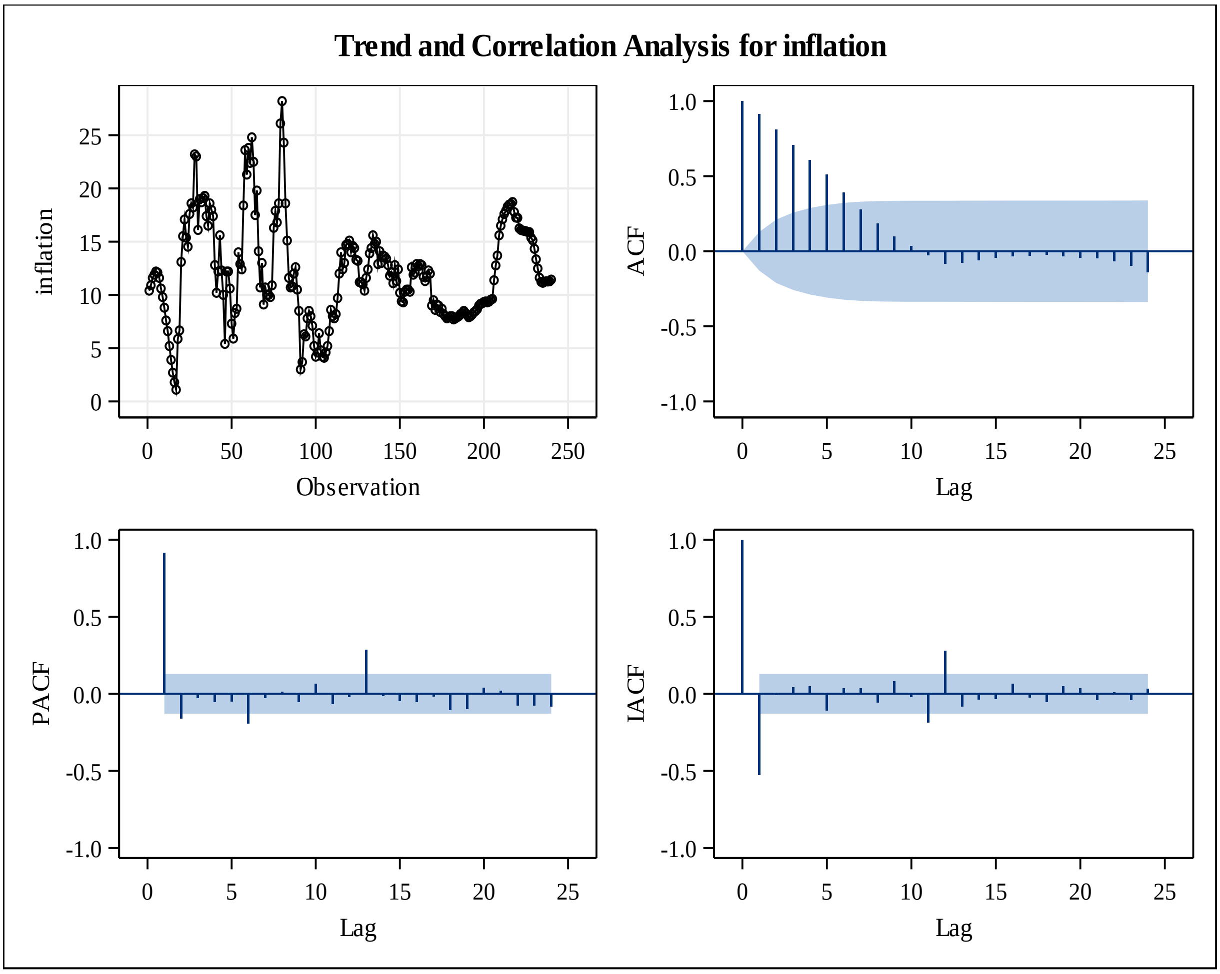

Figure 4.2

If this process contains the seasonal fluctuation, as it is in this model, we can expect also the dependence seasons: y_{t-2s}, y_{t-s}, y_{ts}, y_{t+s}, y_{t+2s},..., where *s* is the length of the period (in this case 12).

This process is called SARIMA (*p*,*d*,*q*) (*P*,*D*,*Q*)s , where

p is order of process AR,

q is the order of process MA,

d is the order of difference,

P is order of seasonal process AR,

Q is the order of MA,

D is order of seasonal difference,

s is the length of seasonal period.

The equation of this model is:

$\Phi_P(B)$ is the autoregressive operator

$\theta_q(B)$ is the operator of moving averages

$\Phi_P(B^s)$ is the seasonal autoregressive operator

$\Theta_Q(B^s)$ is the seasonal operator of moving averages

$\{\alpha_t\}$ is white noise

4.4.3 Model Identification

Once the degree of differencing has been determined, the autoregressive and moving-average orders are selected by examining the sample autocorrelations and sample partial autocorrelations. To use the sample autocorrelation and sample partial autocorrelation functions for tentative model parameter identification. We consider the ACF and PACF shown in diagram above. Using table 4.1 and Table 4.2 above as guides, the preliminary values of *p, q, P and Q* are chosen. Because we are dealing with estimates, it will not always be clear whether the sample ACF or PACF is tailing off or cutting off. Also, two models that are seemingly different can actually be very similar *(*Shumway and Stoffer, *2010)*. With this in mind, we should not worry about being so precise at this stage of the model fitting. At this stage, a few preliminary values of *p, q*, *P* and Q should be at hand and we can start estimating the parameters.

Table 4.3 The Autocorrelation and Partial correlation table

Date: 15/05/19 Time: 21:11
Sample: 1 240
Included observations: 240

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|******|	.|******|	1	0.894	0.894	112.74	0.000
.|***** |	**|. |	2	0.749	-0.253	192.37	0.000
.|**** |	.|. |	3	0.610	-0.012	245.70	0.000
.|**** |	.|. |	4	0.491	-0.004	280.50	0.000
.|*** |	*|. |	5	0.380	-0.070	301.48	0.000
.|** |	.|. |	6	0.292	0.042	313.96	0.000
.|** |	.|. |	7	0.232	0.042	321.91	0.000
.|* |	*|. |	8	0.165	-0.136	325.95	0.000
.|* |	.|. |	9	0.111	0.055	327.79	0.000
.|. |	*|. |	10	0.058	-0.075	328.30	0.000
.|. |	.|. |	11	0.027	0.067	328.41	0.000
.|. |	.|* |	12	0.029	0.121	328.54	0.000
.|. |	.|* |	13	0.073	0.154	329.37	0.000
.|* |	*|. |	14	0.113	-0.075	331.36	0.000
.|* |	*|. |	15	0.124	-0.075	333.79	0.000
.|* |	.|. |	16	0.129	0.028	336.44	0.000
.|* |	.|. |	17	0.140	0.068	339.57	0.000
.|* |	*|. |	18	0.119	-0.164	341.85	0.000
.|. |	**|. |	19	0.043	-0.229	342.15	0.000
.|. |	*|. |	20	-0.048	-0.105	342.53	0.000
*|. |	.|. |	21	-0.123	0.030	345.02	0.000
*|. |	.|. |	22	-0.176	0.042	350.18	0.000
**|. |	.|. |	23	-0.223	-0.040	358.54	0.000
**|. |	.|. |	24	-0.253	0.007	369.37	0.000
**|. |	.|. |	25	-0.262	0.041	381.10	0.000

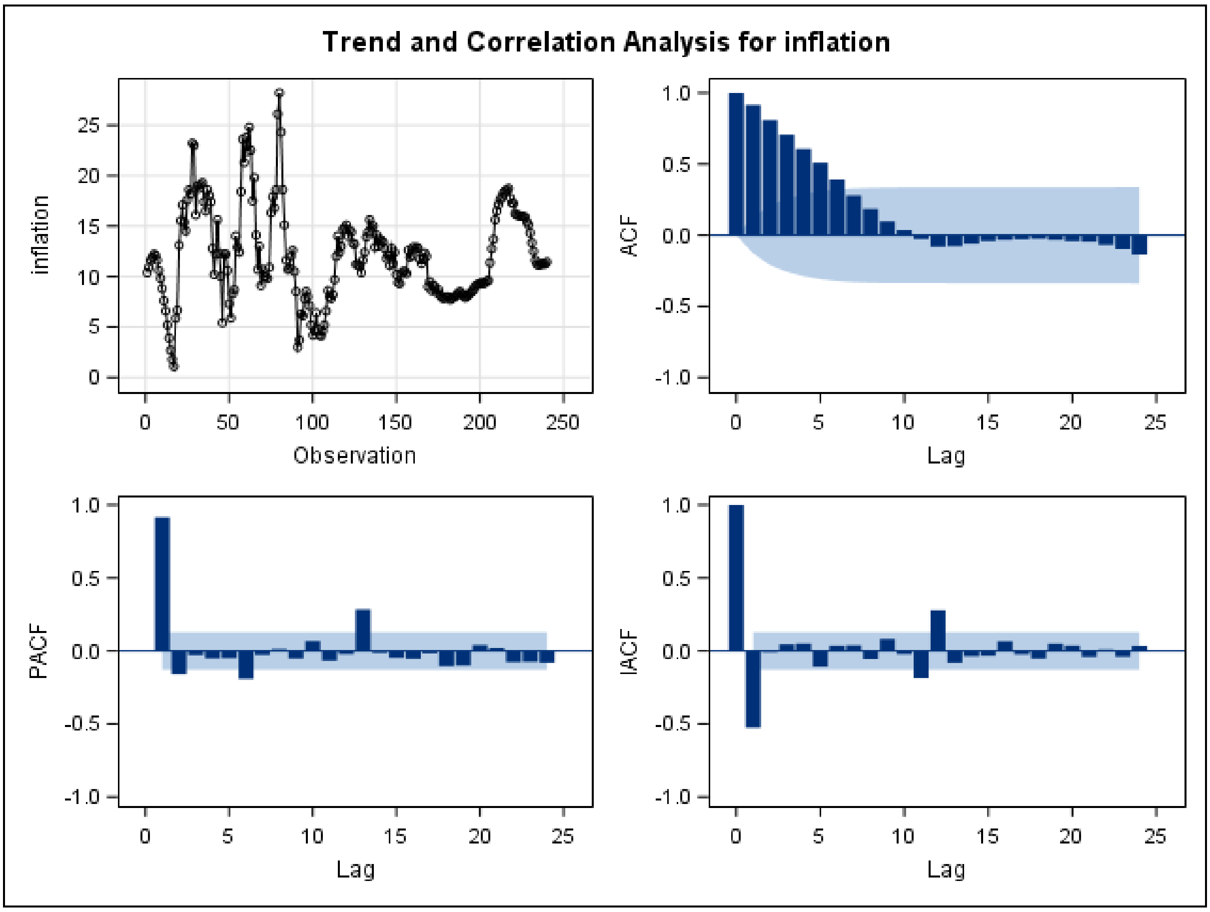

Figure 4.3

First, concentrating on figure 4.3 the seasonal lags, the characteristics of the ACF and PACF in tend to show a strong peak at $h = 12$ in the autocorrelation function, combined with peaks at $h = 12 \; and \; 24$ in the partial autocorrelation function. Hence it appears that either: (i) the ACF is cutting off after lag 12 and the PACF is tailing off in the seasonal lags, (ii) the ACF and PACF are both tailing off in the seasonal lags. Table 4.5 above suggests either (i) a seasonal moving average of order $Q = 1$, or (ii) due to the fact that both the ACF and PACF may be tailing off at the seasonal lags, perhaps both components, $P = 1$ and $Q = 0$, are needed. To identify the between-season model, we focus the lags h = 1, 2,…,24 and identify order based on Table 5. First, we set the ACF to be tailing-off and the PACF to be cutting after lag 1, we identify p = 1 and q = 1. Also it is possible to think of the PACF to be tailing-off and the ACF to cutting after lag 1, leading to identify P = 0 and Q = 1.

Fitting the model suggested by these observations, we obtain:

$SARIMA\ (1,1,0) \times (1,1,0)_{12}$

4.4.4 Parameter Estimation

General equation of the chosen model is:

$$\Phi_1(B^{12})\Phi_1(B)(1-B)^1(1-B^{12})^1 y_t \alpha_t \tag{36}$$

After the modification of the equation and substitution of estimated values, we get the following equation and it describes the dynamics of our time series:

$$u_t = (1+0.41L)(1-0.61L^{12})u_t = (1-0.19L)(1+0.96L^{12})\varepsilon_t$$

In this section, we assume we have n observations $x_1, \dots, x_n$ from a causal and invertible Gaussian $ARMA\ (p,q) \times (P,Q)_{12}$ process in which initial order of parameters, $p, q, Pand\ Q$are known. Our goal is to estimate the value of parameters: $\theta_1, \dots, \theta_p, \emptyset_1, \dots, \emptyset_q, \Phi_1, \dots, \Phi_q, \Theta_1, \dots, \Theta_p$.

For $SARIMA\ (1,1,1) \times (1,0,1)_{12}$ model, the fitted model is given as;

$$X_t = -0.095 + u_t$$

$$u_t = (1+0.41L)(1-0.61L^{12})u_t = (1-0.19L)(1+0.96L^{12})\varepsilon_t \tag{37}$$

where $AIC = 3.35863, SIC = 3.47235\ and\ HQC = 3.40483$

For $SARIMA\ (1,1,1) \times (0,0,1)_{12}$ model, the fitted model is obtained as;

$$X_t = -0.008 + u_t$$

$$(1+0.08L)u_t = (1+0.28L)(1-0.97L^{12})\varepsilon_t \tag{38}$$

where $AIC = 3.27004, SIC = 3.35571\ and\ HQC = 3.30485$.

While for $SARIMA\ (1,1,1) \times (1,0,0)_{12}$ model, the fitted model is given as;

$$X_t = -0.030 + u_t$$

$$(1 + 0.02L)(1 - 0.23L^{12})u_t = (1 + 0.29L)\varepsilon_t \quad (39)$$

where $AIC = 3.63860, SIC = 3.72958\ and\ HQC = 3.67556.$

and for $SARIMA(0,1,1) \times (1,0,1)_{12}$model, the fitted model is obtained as;

$$X_t = -0.074 + u_t$$

$$(1 - 0.62L^{12})u_t = (1 + 0.21L)(1 - 0.96L^{12})\varepsilon_t \quad (40)$$

where $AIC = 3.34805,\ SIC = 3.43856\ and\ HQC = 3.38482.$

In order to select the best model to analyze monthly inflation rate in Nigeria, the AIC and SBC were used to compare selected models fit. The model with the smaller information criteria is said to fit the data better. Since $SARIMA\ (1,1,1) \times (0,0,1)_{12}$ model has the lowest AIC and SBC, then this model is believed to estimate Nigerian monthly inflation rate better than the other models.

4.4.5 Diagnostic Checking

In this section, we shall assess how well the selected model fits Nigerian inflation rate. If the model fits the data well, the residuals of the fitted model are random (Chatfield, 1991). In ARIMA modeling, the selection of the best model to analyze data is directly related to how well the residual analysis performs (Kadri et al., 2005). Therefore, several diagnostic statistics and plots of the residuals can be used to examine the goodness of fit of the selected model to the data.

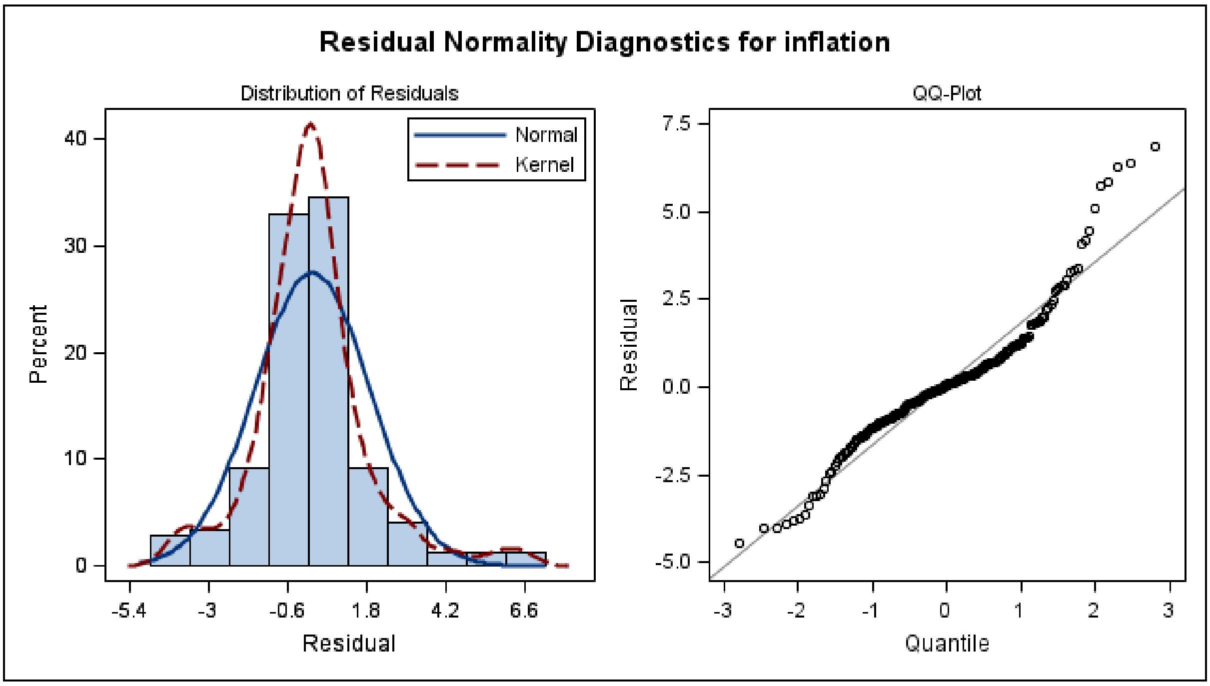

Figure 4.4

Figures 4.4 above shows the histograms and QQ-plot of the residuals. As expected, the curves significantly reflect a normal distribution.

Test statistic values of the Breusch and Pagan (B-P) test for the homoscedasticity of the residuals are also presented in Table 11 (Appendix). All calculated values are found to be smaller than the respective critical values, indicating that the residual variance is constant. Therefore, the hypothesis that the residuals are white noise cannot be rejected, indicating that the fitted model is adequate. That is, $SARIMA\ (1,1,1) \times (0,0,1)_{12}$ model is adequate for modeling Nigeria inflation rate.

To test whether the residual from the fitted model come from normally distributed series, we use histogram and QQ-plot of the residual test. The histogram and QQ-plot of the residual shown in Figures 4.3 and 4.4 above, show that the residuals come from a normal distribution.

4.4.6. Forecasting

Since the model diagnostic tests show that all the parameter estimates are significant and the residual series is white noise, the estimation and diagnostic checking stages of the modeling process is complete. We can now proceed to forecasting the inflation series with fitted $SARIMA\ (1,1,0) \times (1,1,0)_{12}$ model. Forecasting refers to the process of predicting future inflation values from a known time series. In this study, the forecasting is performed as follows:

According to (23), the $SARIMA\ (1,1,1) \times (0,0,1)_{12}$ model can be written as

$$(1 - \Phi_1 B^1)\nabla^1 X_t = (1 + \Theta_1 \mathrm{B}^1)(1 + \Theta_{12} \mathrm{B}^{12})e_t \qquad (41)$$

This equation can also be multiplied out and rewritten in a form that is used in forecasting as shown in (42) below.

$$X_t = \Phi_1(X_{t-1}) + e_t + \Theta_1 e_{t-1} + \Theta_{12} e_{t-12} \qquad (42)$$

where $B^1 X_t = X_{t-1}$.

The above equation can be re-expressed as:

$$X_{t+m} = \Phi_1(X_{t+m-1}) + e_{t+m} + \Theta_1 e_{t+m-1} + \Theta_{12} e_{t+m-12} \qquad (43)$$

In order to forecast one period ahead that is X_{t+1}, (43) is increased by one unit throughout to become:

$$X_{t+1} = (1 + \phi)X_t - \phi X_{t-1} + e_{t+1} - \Theta_{12} e_{t-11} - \Theta_1 e_t + \Theta_1 \Theta_{12} e_{t-12} \qquad (44)$$

The term $e_{t+1}$is not known because the expected value of future random errors has been taken as zero. There are 240 data points from January 1999 to December 2018 used to build the SARIMA model.

From table 8, using $\phi = 0.08, \Theta_1 = 0.28, \Theta_{12} = 0.97\ and\ \Theta_1 \Theta_{12} = 0.2716$

Thus, (44) is given as

$$\hat{X}_{t+1} = 1.08X_{240} - 0.08X_{239} + \hat{e}_{t+1} - 0.97\hat{e}_{t-11} - 0.28\hat{e}_t + 0.2716e_{t-12} \quad (45)$$

In order to forecast inflation for the period 240 (that is, December 2018), (45) is given by

$$\hat{X}_{240} = 1.08X_{240} - 0.08X_{239} + \hat{e}_{239} - 0.97\hat{e}_{227} - 0.28\hat{e}_{239} + 0.2716\hat{e}_{226} \quad \hat{e}_{240} = 0$$

The forecast quantity for a year period was gotten using Statistical analysis software

Once our model has been obtained and its parameters estimated, we can use it to make our prediction. Table 4.5 and figure 4.5 respectively below summarizes 12 months' upfront inflation forecast from January 2019 to December 2019 while detailed statistics is in the appendix.

TABLE 4.5. 12- MONTH FORECASTED INFLATION FOR January 2019 TO December 2019

Month(s)	**Forecast %**
January	11.48
February	11.51
March	11.79
April	11.87
May	12.11
June	12.06
July	12.07
August	11.89
September	11.85
October	11.73
November	11.71
December	11.54

Figure 4.5 Table forecast of the Nigeria inflation

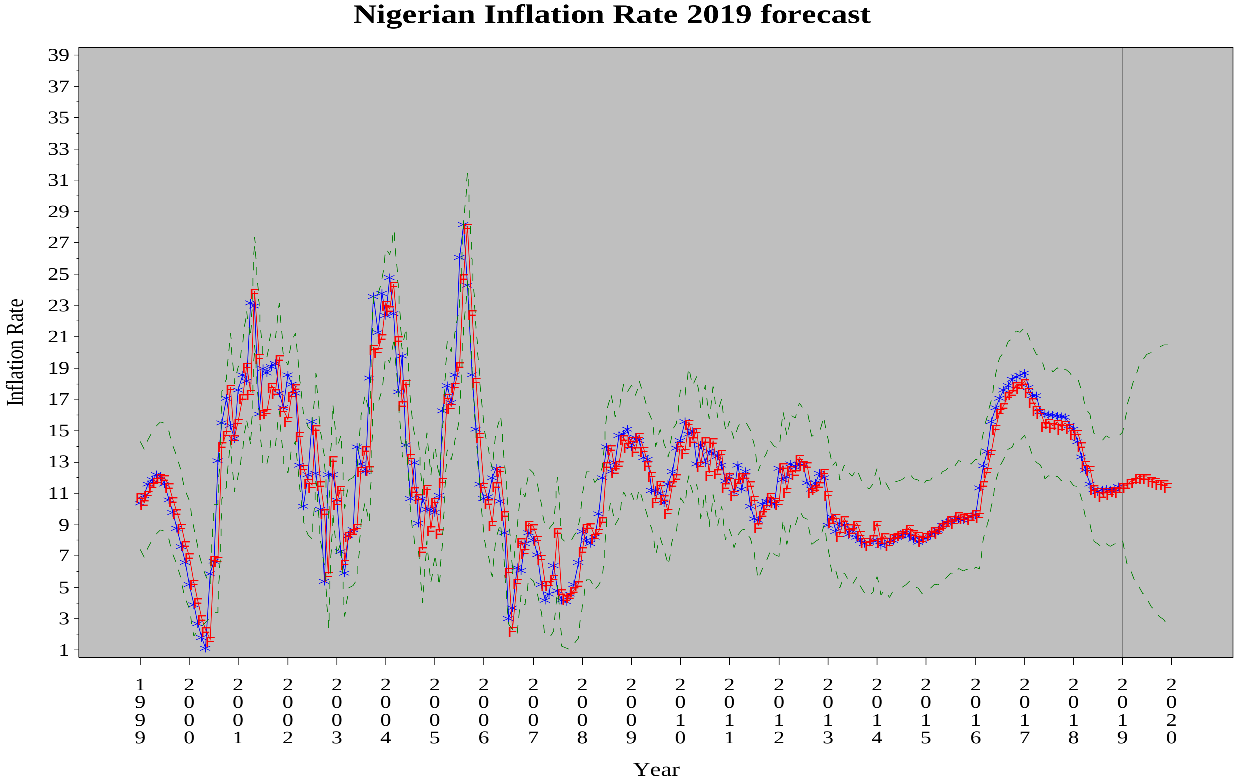

Figure 4.5

Forecasting Accuracy Evaluation

If the fitted $SARIMA\ (1,1,1) \times (1,1,0)_{12}$ model has to perform well in forecasting, the forecast error will be relatively small. The accuracy of forecasts is usually measured using root mean square error (RMSE), mean absolute error (MAE), Mean absolute percentage error (MAPE) and Theil's inequality coefficient (Theil-U). The result shows that the Mean Absolute Percentage Error (MAPE) turns out to be 3.56%, which is relatively less than 4% and Theil's inequality coefficient (U-statistic) turn out to be 0.018, which is relatively close to zero. Besides this result, the bias and variance proportion are also very small, which are 0.047 and 0.001, respectively. Thus, measures indicate that the forecasting inaccuracy is low.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

The purpose of this study is to analyze the structure and pattern of Nigeria monthly inflation rates from January 1999 – December 2018 using seasonal autoregressive integrated moving average (SARIMA). The aim and objectives of this research work are to examine and explore the monthly inflation rates in Nigeria by constructing and analyzing the model. We as well, forecast in-sample values and determine the forecast performance of the models used.

The findings from this study show that the aim and objectives of the research were achieved and the findings are outlined below:

The time plots showed that each year there is upward increase in the trend and suggests that the given time series is non-stationary. The movement is secular in nature and expect a small shift in the movement in mid-2005.

The augmented dickey fuller and Phillips-Persons test show that Nigeria Inflation rate is stationary at the first difference, that is$I(1)$.

The SARIMA model identified and fitted is $SARIMA\ (1,1,1) \times (0,0,1)_{12},$. This model were used to estimate parameters using Akaike Information Criterion (AIC) and Schwartz's Bayesian Criterion (SBC), $SARIMA\ (1,1,1) \times (0,0,1)_{12}$ said to estimate Nigerian monthly inflation rate better than the other models because it has the lowest values.

The forecasted values of future Nigerian inflation between January 2019- December 2019 was obtained using the model i.e. $SARIMA\ (1,1,1) \times (0,0,1)_{12}$. The forecasting results in general revealed a decreasing pattern of inflation rates over forecast period of 12 months.

5.2 Conclusion

In this study, we made use of some Nigerian inflation rates from January, 1999 to December, 2019 to determine the structure and pattern of the Inflation rates using Seasonal Autoregressive Moving Integrated Average (SARIMA) model following the Box Jenkins approach. Four models were considered and in order to select the best model to analyze monthly inflation rate in Nigeria, the AIC and SBC were used to compare selected models fit. The model with the smaller information criteria is then said to be the best model. Therefore $SARIMA\ (1,1,1) \times (0,0,1)_{12}$ model was picked as the champion model to estimate Nigerian monthly inflation rate better than the other models.

This study had shown the forecast performance from January, 2019 to December, 2019. In the light of the forecast results, policy makers should gain insight into more appropriate economic and monetary policy in order to combat such increase in Inflation rate which may occur in the month of July, 2019 causing unnecessary panic and can probably lead to unexpected increase in the inflation rate of Nigeria.

Finally, this study has shown that the inflation rates in Nigeria are non-stationary. Then, if all the recommendations below are considered by the government, policy makers, ministries, financial organizations and the private sector, Nigerian economy will develop and grow rapidly. Monetary policy must be transparent and corruption must be properly checked. Nigerian government needs to give room to statistician to participate more in the planning and execution of government economic policies.

5.3 Recommendations

Employing the SARIMA process, the study estimated four parsimonious models for inflation rates and evaluated these models based on their sample performance. In order to improve the economy, this calls for concerted efforts in several fronts.

The study recommends that the Central Bank of Nigeria uses the SARIMA model in their forecasting toolkit to predict inflation as some central banks are doing. The study further recommends for the Central Bank of Nigeria to use the SARIMA model in addition to the econometric models to predict other macroeconomic variables such as the exchange rate and GDP.

Also, Nigeria government must support statistician and econometrician in various universities and research institute by providing adequate fund in order to carry out economic research that will explain and improve the economy.

Central Bank of Nigeria is mandated to pursue a primary objective of achieving and maintaining price stability. To achieve this objective, the CBN has to adopt inflation targeting as its monetary policy framework. Inflation targeting requires the CBN to be able to predict inflation with much more precision to guide policy discussions at the Monetary Policy Committee (MPC).

It is very obvious that Nigerian government uses monetary and fiscal policies measures as tools for combating inflation and meeting various macro-economic objectives. Evaluation of these policies showed that they do not work due to negligence of the correlation that exists between government expenditure, money supply and inflation. Expenditure management and budget discipline should be taken seriously by Nigerian government. This can be achieved by ensuring that all expenditures made match with revenue. Nigerian government should also act productively in balancing its fiscal and monetary policies, as well as institutional intervention with expectation on inflation.

This will prevent unexpected and unplanned reaction of prices which may have a counter-productive impact on the economy.

Provision of extension services and new technology by the government to the agricultural sector could enhance increased productivity in the sector. In addition, spending a significant proportion of the oil revenues on infrastructural facilities would greatly enhance workers' productivity which would result in both short- and long-term growth. It is important that the government of Nigeria prioritize agricultural sector again with more sense of responsibility and strong effort in reducing the impact of corruption on the implementation of policies.

Finally, there is the need to increase central bank independence in order to reduce the effect of fiscal pressure on monetary policy. Provisions in the CBN Amendment Decree of 1998 that made provision for operational autonomy of CBN is in the right direction. So also, the buying and selling of securities in the open market operation (OMO) should be reconsidered. The use of OMO to stabilize price at the expense of output should be discouraged. The mopping of excess liquidity with the aim of stabilizing the economy has a negative impact on growth by raising lending rate and reducing investment.

5.4 Limitation of Study

This scope of this research has been limited to lack of adequate and relevant information from the central bank of Nigeria (CBN), financial constraint, and photocopy of some relevant material. There is also the inadequate of materials in the libraries that deals or discuss more on SARIMA model.

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