Synthetic Reflection Seismogram, an Application to Earth Model

By
Gambo Titus Mdurvwa

OCTOBER, 1992

SYNTHETIC REFLECTION SEISMOGRAM,

AN APPLICATION TO EARTH MODEL

A THESIS

SUBMITTED TO

SCHOOL OF SCIENCE AND SCIENCE EDUCATION

ABUBAKAR TAFAWA BALEWA UNIVERSITY OF TECHNOLOGY,

BAUCHI

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF B.TECH (HONS) PHYSICS

IN

PHYSICS PROGRAMME

OCTOBER, 1992

APPROVED

Supervisor: Signature------------------------

Name:--Othniel K. Likkason

Date:--5/2/95-------------

Program Coordinator Signature: Md Fazlul Haque

Name--DR. M. F. HAQUE----

Date:-_-29/5/95-----------

Dean of School Signature:----------------

Name:--Prof. J. P. Fabiyi

Date:---29/6/95-----------

DEDICATION

To my Parents Mr. and Mrs. T.D. Mdurvwa;

My Late Uncle, Dr. Wabada Mamza;

My Brothers and Sisters.

ACKNOWLEDGEMENT

My profound gratitude goes to God almighty for his steadfast love and mercy in keeping me and making all things possible.

My gratitude also goes to my hard working supervisor, Mr. O.K. Likkason, for putting in all his time, energy and also his patience in making sure I produce what is expected. I'll like also to thank my head of department, Professor E.D. Mshelia and other lecturers: Dr. M.F. Haque, Mr. Y.H. Ngadda, Mr. O.W. Olasoji and Mr. A. Sanni, for they have contributed so much during the course of my study.

I wish to say a big thank you to my colleagues: Charles Osayande, Yakubu Pam, Femi Ogundele, Adedayo Adeleye, Sylvester Fierkwap and Joseph Ibrahim for the good working relationship we had over the years. I cannot forget Mr. Joseph Oboh for the help he rendered.

The gracious guidance and hospitality I enjoyed from Alhaji Usman Bagudu while in Jos during my research work is unforgettable.

I am particularly grateful to Mr. F. Okoro for the enornous help he rendered in the processing of this thesis.

ABSTRACT

The basic field activity in seismic surveying is the collection of seismic data in form of seismograms. This is analogous to digital time series that record the amplitude of ground motion as a function of time during the passage of seismic wavetrain. The acquisition of seismograms involves the conversion of seismic ground motion into electrical signals, amplifications and filtering of the signals and registration on a chart recorder and/or tape recording.

By seismic modelling "synthetic seismograms" are constructed for earth models in order to derive insight into the physical significant of reflection events. With these identification of precise origin of the "reflector" in terms of subsurface lithology can be inferred.

TABLE OF CONTENTS

TABLE OF CONTENTS Cont'd

LIST OF FIGURES

TABLE OF CONTENTS Cont'd

CHAPTER I

1.1 INTRODUCTION AND LITERATURE REVIEW

Seismogram is a record of the resultant ground velocities (generally of its vertical component) which may be due to the sum of large number of impulses of waves of different types travelling by different path. Seismogram is a single field record which may contain more information about the sub-surface conditions. Before it can be interpreted, the event which appeared on the record must be carefully identified; only then can the measurement of time, frequency and amplitude of this event be used in interpreting the subsurface structure.

By assuming that shot produced a certain waveform, the reflection wave form can be recorded after modification because of its passage through a sequence of layers with given velocities and densities. The wave impinges on the first interface where its energy is partitioned into transmitted and reflected wave. The resulting seismic record is simply the super-position of these waves which are ultimately reflected back to the geophone station and Snell's law determines the ray paths. Usually horizontal bedding is assumed and only the ray path normal to the reflection section is tracked. Density variations are frequently ignored so that the reflected and transmition coefficients are based on velocities only. Multiples especially short-paths are ignored. The result is called a synthetic seismogram.

The transient seismic signal like that due to explosion or earthquake produces a contin. . ; back-ground noise in the earth which is measurable with time-order seismograph over a wide range of period. This is characterised by a sudden release of elastic waves. These are longitudinal, transverse and Rayl. gh or Lone (surface) waves. The analysis is principally based on the theory of elastic body waves of dilatational and shear types and on the wave theory for travelling surface wave. Through the analysis of their travelling time from a point of origin to receiving seismograph station, they permit deduction of layering and elastic constants of the interior earth.

In order to analyse a seismogram, it is useful to know the form of the velocity impulse which is generated from explosion source into the ground, since this is the principal unit from which the seismogram is constructed. A seismic pulse propagates outward from a seismic source at velocity determined by physical properties of the surrounding rock. If the pulse travels through a homogeneous rock it will travel with the same velocity in all directions away from the source. Seismic rays are defined as a period swing is compared with the first.

Benioff in 1935 observed that a seismograph other than the pendulum type possessed the advantage of recording very long period disturbances. This instrument measures a component of ground strain instead of ground displacement. During the passage of seismic wave, this instrument records variation in distance between two points on the ground some 20 metres apart. The variation is measured against a standard length tube, originally of steel, but later by fused quartz extenometer. The recording is electromagnetic. Benioff's strain seismograph was first to record the earthquake period up to an order of one hour. Benioff also investigated properties of seismograph designed to measure ground dilatation directly.

Various instruments have been brought to bear with the view of realizing optimum levels for particular purposes, for example instrument may be designed to produce seismogram in which the dynamical magnification is nearly constant over a wide range of earth period and to reveal earth motions of unusually short or usually long periods, or to enable specific part of the seismic spectrum to be closely studied.

instrumentation is usually arranged so that a reasonable time picture of the vertical velocity is obtained of the motion whose frequency component lies within a chosen band. The motion whose frequency component lies outside this band will not be produced accurately. This must be kept in mind when the actual seismograms are compared with the theoretical calculation. If the offset of the instrumentation is allowed on the seismogram, the observed motion may be compared with the prediction from the wave theory. The earth is a very complicated model to handle theoretically. However, the initial disturbance produced by dynamite explosions is not generally a simple pulse.

Regardless of the type of transducer employed in the seismometer, it must respond to the ground displacement or its time derivative. No point of the seismometer remains truely fixed during the arrival of seismic wave, but a mass of large inertia, loosely coupled to the frame of instrument remains nearly so. It would be convenient for the seismograph to be designed so that its seismogram gives a fairly close picture of the relevant component of the actual earth movements. Seismograph must be damped if it is to give a satisfactory result, otherwise the early movement in the explosion or earthquake would set up a free vibration which would make later event indistinguishable. It is best for the damping to be such that after a sharp displacement of the ground the second element that tends to remain at rest. The other magnetic field moves in response to the seismic waves. The coil has only one degree of freedom and it is used so that it will be sensitive to vertical motion only. There is mutually dependent elements in a three-component detectors and sometimes usual to determine the direction for which the waves come or distinguish the type of waves (p, s or Raleigh waves).

The geophones are usually arranged in groups (arrays) spread over a distance and connected electrically so that in effect the entire group acts as a single large detector. Such an arrangement discriminates against seismic waves travelling in certain direction. Those travelling horizontally reaches different detector in the group at different times so that the wave peaks and troughs cancel, whereas the waves travelling vertically affect each detector at the same time so that the offsets add.

The signal from the detector is transmitted to the recording equipment over a cable or streamer where it is amplified and recorded. The output level of geophones or detectors varies tremendously during the recording. The seismic recording system is linear over ranges of 100 dB or more. Seismic amplifier employs various scheme to compress the range of seismic signal without loss of amplitude information, It also incorporates adjustable filters and permit discrimination on the bases of frequency. The to the subject. Seismograph is employed to record the translational component of the local earth movement. A seismograph is an instrument which provides a useful record of some characteristic ground motion during the arrival of seismic wave. There are two methods in use for seismic analysis; the most commonly used is based on the principle of inertia and the pendulum seismometer is used as the inertial sensor. The other is based on the deformation of small parth of the earth and strainmeter or strain-seismometer is the sensor.

There are two ideal seismographs for measurement of the local earth movement; one for measuring the horizontal component and the other for the vertical component. The constructional details differ but same type of differential equation representing the motion of the seismograph relative to the ground occur in both.

The usual modern seismological usage is called the entire instrumentation that record the motion of the ground as a continuous function of time, the seismograph, but the component that responds to the ground motion, the seismometer (or geophone) is the heart of the instrument.

Geophone is predominantly an electromagnetic device. A coil moving in a uniform magnetic field generates voltage that is proportional to the velocity of the motion. Oftenly the coil is an inertial element weathering effect must be eliminated for accuracy to be realized.

Seismic reflection profile, or sounding provide trail time data that must be converted to velocity function before depth interpretation can be made. The resolutions of seismic reflection survey depends on the wavelength of the energy-source-receiving system used with resolution increases as wave-length decreases. Penetration or depth survey also depends upon the wavelength and the energy of the source, with penetration increases as wavelength decreases. As a result of these relationship, high resolution seismic reflections survey is usually limited to shallow depth penetration. Interpretation of seismic reflection survey requires an understanding of both the equipment used and the geology of the site. Some high powered seismic source produces energy in form of wavetrain rather than a single energy pulse. The wavetrain produces series of reflectives from each reflection. The characteristics of reflection surfaces (velocity discontinuity) and the lithologies affect the reflecting signal enhancement.

1.2 INSTRUMENTATION

The most general types of local movement in an elastic body is represented by terms which correspond to translational rotation and strain. In the case of seismic waves the translational motion is much relevant of seismic energy travelling along a ray path in an isotropic medium and are everywhere perpendicular, to the wavefront. A ray has no relevance except i it gives a useful concept in discussing travel path of seismic energy through the ground. It should be noted that the propagation of seismic wave is accounted by the velocity with which the seismic energy travels through a medium. It is not the same as the velocity of the particle of the medium perturbed by passage of waves. The associated oscillatory ground motion involves the particle velocities that depend on the amplitude of the waves.

In seismic reflection survey, the travel-times are measured from the reflected waveform from interfaces of media of different acoustic impedances. In such a situation, the velocity varies much more as a function of depth due to differing physical properties of individual layers than horizontal from lateral facies changes within the individual layers. The depth of investigation is usually larger compared with the distance of shot from the receiver. This method gives records of information from large number of horizons down to the depth of several thousand of metres. This technique possesses appreciable accuracy, particularly, when the changes in depth of reflectors rather than absolute depths are required. However, the variability of the weathering layer produces a scattered observed reflections times. The

CHAPTER II

2.0 SEISMIC WAVES

2.1 INTRODUCTION

Seismic waves are parcels of elastic strain energy that propagate outward from seismic source such as explosion. These waves are of low frequency and are quickly damped. Source suitable for seismic survey should generate a short time wave train known as pulse that typically contains wide range of frequencies. Except at the immediate vicinity of the source, the train associated with the passage of seismic pulse is minute and is assumed to be elastic. On this assumption, the propagation of seismic waves is determined by elastic moduli and density of material through which they passed.

The seismic waves are motions that can be observed on a seismogram, with the exception of the direct disturbances of the instrument. The seismic waves which arise through a sudden explosion propagate through the whole of the interior or along the surfaceof the earth.

2.2 TYPE OF SEISMIC WAVES

There are two groups of seismic waves; the body and the surface waves. When an elastically homogeneous ground is suddenly stressed, three elastic pulses travel outward at different speeds. Two are body waves and they propagate as spherical waves which are affected on a minor extent by the free surface ground. They differ in ground motion within the pulse in the direction of propagation (i.e. radial). The faster one called the P-waves or dilatational, longitudinal, irrotational or compressional waves. Its speed depends on the density and compressibility of the earth. Normal to the P-wave is the S-wave, known also as shear, transverse or rotational wave. It is slower than the P-wave and its speed depends on density and rigidity of the earth.

The surface waves are generally complex and travel through the crust near the surface. Their energy is not quickly dissipated with distance. For a homogeneous ground, Surface disturbances are caused by waves known as Rayleigh waves. They comprise the combination of longitudinal and transverse motion with a definite phase relationship between them. The amplitude of these waves decrease exponentially with depth. The particle motion is confined to a vertical plane which includes the direction of propagation of the waves. The path of an element of the medium during the passage of the Rayliegh wave cycles follows an ellipse lying in a plane of propagation.

The figure 2.1 below shows an elastically homogeneous ground stressed at point P near its surface.

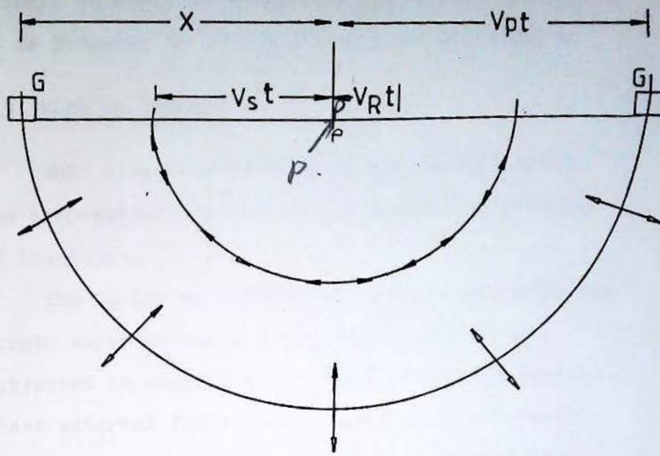


Fig. 2.1 Pulse of P- , S- and Rayleigh wave types at some time after their initiation at point P.

G is the geophone, P the source and X is the distance between the source and the geophone. V_s, V_p and V_R are the velocities of the S-wave, P-wave and Rayleigh wave respectively and $V_s t$, $V_p t$ and $V_R t$ their respective distances in relative term.

In an infinite homogeneous isotropic medium, only P- and S-waves exist. However, the medium does not extend to infinity in all directions, but are bounded by the surface. Hence we restrict ourselves to only body wave because surface waves do not give information of deep interior of the earth. When seismic velocity is given without any specification it is referred to as the velocity of the p-wave.

2.3 ELASTIC THEORY

When a wave is propagated through the earth, the propagation depends on the elastic properties of themedium.

The theory of elasticity is concerned with the strain experienced by a deformable matter when subjected to stress (i.e. applied external force). These external forces are opposed by the internal forces which resist the change in shape and size. This makes the body to return to its original conditions when the external force is removed. Similarly fluids resist change in size (volume) but not shape. These properties of resisting changes in shape and size and of returning to its undeformed conditions when the external force is removed is called elasticity. Perfectly elastic body recovers completely after being deformed.

The theory of elasticity relates forces which are applied to the external surface of the body to the resulting changes in shape and size. The relationship between the applied force and the deformation is expressed in terms of the concept of stress and strain. It will be assumed that the medium is made up of particles which are sufficiently closely packed and whose distribution is continuous.

2.3.1 The Stress

When an external force is applied to a body, internal forces are set up in it. Stress is the measure of the intensity of those balanced internal forces. The stress is the ratio of the normal force to the area on which the force is applied.

$$T = \text{Stress} = \frac{\text{force}}{\text{Area}} = \frac{F}{A}$$

If the force varies from point to point, the stress also varies and its value at any point is found by taking the infinitesinal element of area centred at a point and dividing the total force acting on this area by the magnitude of the area. If the force is perpendicular to the area, the stress is said to be normal and when tangential to the element of the area, it is called shearing stress.

Taking an infinitesimal areal element and assuming that the upper and the lower part of the continuous body can be separated with respect to the plane as illustrated in fugure 2.2 below. The surface force exerted on this area can be expressed as

$$\left.\begin{aligned} F_x &= T_{zx}(r)\ dxdy \\ F_y &= T_{zy}(r)\ dxdy \\ F_z &= T_{zz}(r)\ dxdy \end{aligned}\right\} \qquad \text{-----} \qquad 2.1$$

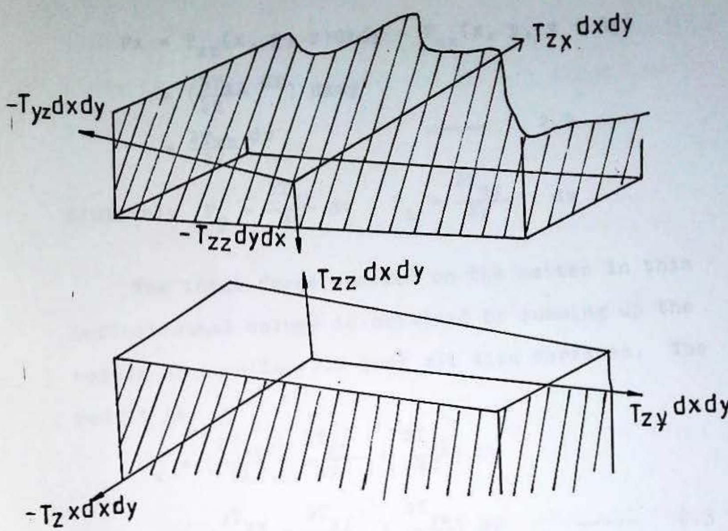


Fig. 2.2 The Surface forces. Two tangential volume which are next to each other one drawn separately.

The proportionality constants $T\ \alpha, \beta\ (r)$ $(\alpha, \beta, = x,y,z)$ are a function of the surface d x dy whose subscripts are generalized co-ordinate since the body is continuous, there should be a reaction of the surface on the matter on the side of the plane dxdy, with the same magnitude but opposite signs and are corresponding component of equation 2.1.

Considering an infinitesimal volume dv, the force on the upper dxdy plane is given by equation 2.1. The net force on volume element dv through the two dxdy surfaces is

$$Fx = T_{xz}(x, y, z)dxdy - T_{zx}(x, y, z + dz)$$

$$= (\frac{\partial T_{xz}}{\partial z} dz) \, dxdy$$

$$= \frac{\partial T_{xz}}{\partial z} dv \qquad ----- \qquad 2.2$$

Similarly $F_y = \frac{\partial T_{zy}}{\partial z} dv$, $F_z = \frac{\partial T_{zz}}{\partial z}$. dv .

The total force exerted on the matter in this infinitesimal volume is obtained by summing up the result of equation 2.2 over all size surfaces. The result is

$$F_x = (\frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z}) \, dv$$

$$F_y = (\frac{\partial T_{yx}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial T_{yz}}{\partial z}) \, dv \qquad ----- \qquad 2.3$$

$$F_z = (\frac{\partial T_{zx}}{\partial x} + \frac{\partial T_{zy}}{\partial y} + \frac{\partial T_{zz}}{\partial z}) \, dv$$

The nine quantities $T_{\alpha\beta}$ of the second rank tensor is called stress tensor. The diagonal tensor T_{xx}, T_{yy}, T_{zz} give the normal stress and the other six terms give the shearing or tangential stress.

In stress tensor any two tangential force lying in the same plane and are directed opposite to each other must be equal (i.e. body in static equilibrium). The stress tensor is said to be symmetrical i.e. $T_{\alpha\beta} = T_{\beta\alpha}$.

It is this quantity of cross shear that reduced the nine component of a stress to six. In addition, a pair of shearing stresses such as $T_{\alpha\beta}$, constitute a couple tending to rotate the element about the z-axis.

2.3.2 The Strain

When a body is subjected to stress, change in shape and size occurs. These changes are called strain. Just as the state of stress at a point, strain can be represented by nine components, $S_{\alpha\beta}(\alpha, \beta = x, y, z)$. S_{xx}, S_{yy}, S_{zz} are simply contraction or expansion, S_{xy} S_{zy} .... are shear strain. Strain like stress are symmetrical i.e. $S_{\alpha\beta} = S_{\beta\alpha}$, so that the nine components are reduced to only six independent components.

Consider two neighbouring particle in a solid at P(x,y,z) and Q(x+dx,y+dy,z+dz). Now suppose that the body deforms in some manner under an applied force so that the particle P is displaced by an amount U to a new position P'(x+u. y+v. z+ω), then if the displacement suffered by the particle at Q is by amount U + dU then the component of dU(du,dv,dω) as

$$dU = \frac{\partial u}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial \omega}{\partial z}.dz \qquad ----- \qquad 2.4$$

using the notations $x^1 = x$, $x^2 = y$, $x^3 = z$ equation 2.3 can be contracted into a single formula (with α, β = 1, 2, 3)

$$F_{\alpha\beta} = (\frac{\partial T_{\beta\alpha}}{\partial x^{\beta}})dv \qquad ----- \qquad 2.5$$

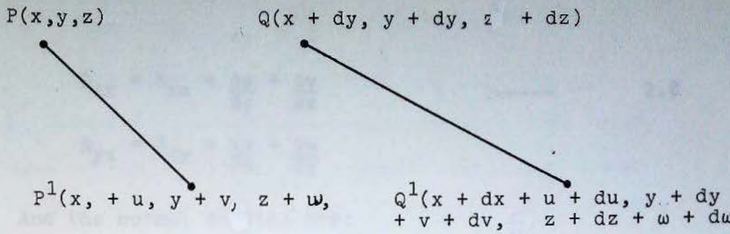


Fig. 2.3 Displacement of neighbouring point with an elastic continuum

The displacement of matter in all at r of the continuous body is given by directional vector $\mathbb{U}(r)$. The displacement itself, however, does not produce any stress, but the difference in displacement between neighbouring point in the investigation of the dynamic of continuous body are the derivatives of u, v, and ω with respect to x, y, z,. These nine derivatives are a second rank tensor, since $\mathbb{U}$ and r are vectors with the antisymmetric part given by

$$\theta_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$$

$$\theta_y = \frac{\partial y}{\partial z} - \frac{\partial w}{\partial x} \qquad ----- \qquad 2.6$$

$$\theta_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

The real part of the strain is given by the symmetric part and is the shear strain given by

$$S_{xy} = S_{yx} = \tfrac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad ----- \qquad 2.7$$

$$S_{zx} = S_{xz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \quad \text{-----} \quad 2.8$$

$$S_{yz} = S_{zy} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

And the normal strains are:

$$S_{xx} = \frac{\partial u}{\partial x}$$

$$S_{yy} = \frac{\partial v}{\partial y} \quad \text{-----} \quad 2.9$$

$$S_{zz} = \frac{\partial w}{\partial z}$$

2.2.3 Cubical Dilatation

Cubical dilatation or simply dilatation is the term often used to specify the functional volume increase at point P in a limiting case when the direction of small volume at P shrinks to zero. To a first order this quantity ϕ is equal to the sum of the normal strains as:

$$\phi = S_{xx} + S_{yy} + S_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad \text{-----} \quad 2.10$$

$$= \vec{\nabla}.\vec{u}$$

The dilatation is just the divergence of the displacement vector. Being scalar, it is invariant under rotation of the axis. A negative dilatation is sometimes called compression.

2.3.4 Elastic Properties of the Earth

Elastic properties express the relationship between stresses and recoverable strains. For every type of applied stress in a body, there is a linear relationship between it and its elastic strain. To a certain value of stress, known as yield strength of material. This is given by Hook's law. The linear relationship between stress and strain in the elastic field is specified for any material by its known elastic moduli, expressing a particular type of stress to the resultant strain. In general, Hook's law leads to complicated relations but when the medium is isotropic, that is, when the properties are invariant in all directions, it can be expressed in a relatively simple form as

$$\xi = \tfrac{1}{2} \Lambda (S_{xx}+S_{yy}+S_{zz})^2 = M(S_{xx}^2 + S_{yy}^2 + S_{zz}^2) + 2S_{xy} + 2S_{yz} + 2S_{xz} \qquad \text{-----} \qquad 2.11$$

This is the most generally acceptable equation for the energy density ξ, a scalar function. Two elastic constants Λ and M, are called Lame's constants. From equation 2.11 we obtain

$$T_{xx} = \quad = (\Lambda + 2M)\, S_{xx} + \quad (S_{xx} + S_{zz})$$

$$T_{yy} = (\Lambda + 2M) S_{yy} + \Lambda(S_{zz} + S_{xx})$$

$$T_{zz} = (\Lambda + 2M) S_{zz} + \Lambda(S_{xx} + S_{yy}) \qquad \text{---}\ 2.12$$

$$T_{xy} = 2MS_{xy}$$

$$T_{yz} = 2MS_{yz}$$

$$T_{zx} = 2MS_{zx}$$

Taking a simple case of elastic body under a hydrostatic pressure P; then

$$\left.\begin{array}{l} T_{xx} = T_{yy} = T_{zz} = -P \\ T_{xy} = T_{zy} = T_{yz} = 0 \end{array}\right\} \qquad \text{-----} \qquad 2.13$$

In equations 2.12, we see similar symmetry existing for $S_{\alpha\beta}$ and thus

$$S_{xx} = S_{yy} = S_{zz} = \frac{1}{3}\ \frac{\partial u_x}{\partial x} = \frac{1}{3}\ \nabla.u$$

$$= \frac{1}{3}\ \frac{V - V_o}{V_o} \qquad \text{-----} \qquad 2.14$$

Where V_o is the natural volume of the elastic body, while V is the volume under pressure

$$\frac{V_o - V}{V_o} = \frac{1}{K} P \qquad \text{-----} \qquad 2.15$$

where $K = \Lambda + \frac{2}{3} M$ ----- 2.16

the quantity K is called the bulk modulus and is the ratio : $\frac{\text{Volume Stress}}{\text{Volume Strain}} = \frac{P}{\Delta v/v}$ expressed in Nm^{-2}

Another simple case is that of an isotropic elastic body (like the case of a wire) stretched along the x-direction, in which

$$T_{xx} = T \qquad ----- \qquad 2.17$$

is the only non-zero component of the stress tensor and

$$S_{yy} = S_{zz} = -\sigma\, S_{xx} \qquad ----- \qquad 2.18$$

$$\text{where } \sigma = \frac{\Lambda}{2(\Lambda + M)} \qquad ----- \qquad 2.19$$

with σ as the poisson ratio and gives the ratio of sidewise contraction to the length-wise stretch in the deformation. Also

$$T_{xx} = T = ES_{xx} \qquad ----- \qquad 2.20$$

$$\text{where } E = \frac{M(3\Lambda + 2M)}{\Lambda + M} \qquad ----- \qquad 2.21$$

E is called the Young's modulus defined by

$$E = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{\Delta L/L} \text{ with usual notations}$$

expressed in Nm^{-2}

2.4 EQUATION OF MOTION

We consider how time-dependent stress may be transmitted through an unbound elastic solid. Assuming the material to be macroscopically homogeneous and in equilibrium with respect to the surface in the undisturbed configurations. If the disturbances pass through the material, the displacement of the point $\vec{P}(x,y,z)$ at any instance t during the passage shall be specified by a vector $\bar{u}(x,y,z)$. According to Newton's second law of motion,

$$m \frac{d^2u}{\partial t^2} = \Sigma \, dF \qquad ----- \qquad 2.22$$

where u is the amplitude of variations i.e. instantaneous value of the displacement for the position of equilibrium at time t, m = mass of the material and df is the instantaneous force on the material.

Enumerating all surface forces acting on small parallelopiped Δv in z direction fig. 2.4., there is only one component in this direction across each of the pairs of opposite surfaces. For example the faces perpendicular to O_y has the expression

$$(T_{yz} + \tfrac{1}{2}\, \partial\, T_{yz}/_{\partial y}\, \Delta y)\Delta z \Delta x -$$

$$(T_{yz} - \tfrac{1}{2} \frac{\partial T_{yz}}{\partial y} \Delta y)\Delta z \Delta x = \frac{\partial T_{yz}}{\partial y} \cdot \Delta v \qquad ----- \qquad 2.23$$

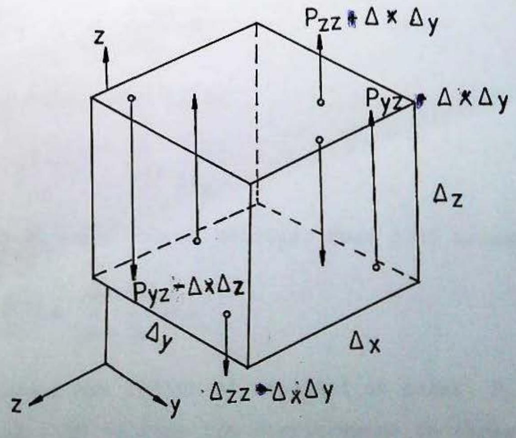


Fig. 2.4 Cubic faces in the z-direction that act on volume element at P

similar expressions can be obtained for the other pairs of faces as

$$\frac{\partial T_{x,y}}{\partial x}\Delta v \quad \text{and} \quad \frac{\partial T_{zz}}{\partial z}\Delta v$$

The sum of the three terms gives the net unbalanced surface force on v in the direction of $_z$ or using usual notation, $x = x_1$, $y = x_2$, $z = x_3$, we obtain

$$F_3 = \sum_{k=1}^{3} \frac{\partial T_{K3}}{\partial x_K}\Delta v \qquad ----- \qquad 2.24$$

In general,

$$F_i = \sum_{k=1}^{3} \frac{\partial T_{Ki}}{\partial x_K}\Delta v$$

Neglecting, gravitational force and substituting in F_i

$$\frac{m\partial^2 u}{\partial t^2} = \sum_{k=1}^{3} \frac{\partial T_{Ki}}{\partial x_K}\Delta v \qquad ----- \qquad 2.25$$

dividing both sides by Δv

$$= \frac{m}{\Delta v}\frac{d^2 u}{dt^2} = \sum_{k=1}^{3} \frac{\partial T_{Ki}}{\partial x_K}$$

but $\frac{m}{\Delta v} = \rho$, the volume density: thus 2.25 becomes

$$\rho \frac{d^2 u}{\partial t^2} = \sum_{k=1}^{3} \frac{\partial T_{Ki}}{\partial x_K}$$

which guides the motion of material at point P. Equations 2.25 relates the displacement to stress

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \quad ----- \quad 2.26$$

from equation 2.13

$$T_{xx} = 2MS_{xx} + \Lambda S_{xx} + \Lambda S_{yy} + \Lambda S_{zz}$$

$$= 2MS_{xx} + \Lambda (S_{xx} + S_{yy} + S_{zz})$$

$$T_{xx} = 2MS_{xx} + \Lambda \phi$$

$$\frac{\partial T_{xx}}{\partial x} = \Lambda \frac{\partial \phi}{\partial x} + \frac{2M\partial S_{xx}}{\partial x}$$

$$T_{xy} = MS_{xy}$$

$$T_{zx} = MS_{xz}$$

$$T_{zy} = MS_{yz}$$

$$\frac{\partial T_{xy}}{\partial y} = \frac{M\partial S_{xx}}{\partial y} \quad ----- \quad 2.28$$

$$\frac{\partial T_{xy}}{\partial x} = \frac{M\partial S_{xy}}{\partial x} \quad ----- \quad 2.29$$

Substituting equations 2.27, 2.28 and 2.29 into 2.26

$$\rho \frac{d^2 u}{dt} = \Lambda \frac{\partial \phi}{\partial x} + 2M \frac{\partial S_{xx}}{\partial x} + M \frac{\partial S_{xy}}{\partial y} + \frac{M \partial S_{xz}}{\partial z} \quad ----- \quad 2.30$$

but $S_{xx} = \frac{\partial u}{\partial x}$, $S_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$, $S_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$

Equation 2.30 becomes

$$\rho \frac{\delta^2 u}{\partial t^2} = \Lambda \frac{\partial \phi}{\partial x} + 2M \frac{\partial^2 u}{\partial x^2} + M\left[\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

$$= \Lambda \frac{\partial \phi}{\partial x} + M\left[2\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial^2 v}{\partial x \partial x} + \frac{\partial^2 u}{\partial y^2}\right) + \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial t}\right)\right]$$

$$= \frac{\partial\phi}{\partial x} + M\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} + (\frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 u}{\partial y^2}) + (\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x\partial z})\}$$

$$= \frac{\partial\phi}{\partial x} + M\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}) + (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z})\}$$

$$= \frac{\partial\phi}{\partial x} + M\{(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) + M\frac{\partial}{\partial x}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})\}$$

but

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \phi$$

$$\rho\frac{\delta^2 u}{ct^2} = \Lambda\frac{\partial\phi}{\partial x} + M\nabla^2 u + M\frac{\partial\phi}{\partial x}$$

$$\frac{\partial\phi}{\partial x} + M\frac{\partial\phi}{\partial x} + M\nabla^2 u$$

$$= (\Lambda + M)\frac{\partial\phi}{\partial x} + M\nabla^2 u \qquad ----- \qquad 2.31(a)$$

By analogy we can write the equation for v and w

$$\rho\frac{\partial^2 v}{\partial t^2} = (\Lambda + M)\frac{\partial\phi}{\partial y} + M\nabla^2 v \qquad ----- \qquad 2.31(b)$$

$$\rho\frac{\partial^2 w}{\partial t^2} = (\Lambda + M)\frac{\partial\phi}{\partial y} + M\nabla^2 w \qquad ----- \qquad 2.31(c)$$

To obtain the equation of wave, we differentiate these equations with respect to x, y and z and the result together gives

$$\frac{\rho\partial^2}{\partial t^2}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) = (\Lambda + M)(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2})$$

$$+ M\nabla^2(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})$$

Thus

$$\rho\frac{\partial^2\phi}{\partial t^2} = (\Lambda + 2M)\nabla^2\phi$$

$$\text{or} \quad \frac{1}{\alpha^2}\frac{\partial^2\phi}{\partial t^2} = \nabla^2\phi$$

$$\text{where} \quad \alpha^2 = \frac{\Lambda + 2M}{\rho} \qquad 2.32$$

By substituting the derivatives of equation 2.31c with respect to z and the derivatives of equation 2.31b with respect to y we obtain

$$\rho \frac{\partial^2}{\partial t^2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial \cdot}\right) = M\nabla^2 \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

and thus

$$\frac{1}{\beta^2}\frac{\delta^2\theta x}{\partial t^2} = \nabla^2\theta x$$

$$\text{with } \beta^2 = \frac{M}{\rho} \qquad \text{------} \qquad 2.33$$

By substituting appropriate derivatives, we obtain similar result for θ_y, θ_z. We can write in general

$$\frac{1}{v^2}\frac{\delta^2\psi}{\partial t^2} = \nabla^2\psi \qquad \text{-----} \qquad 2.34$$

where v is a constant velocity. The quantity ψ has not been defined, it can only be identified as some disturbances which propagate from one point to another with the speed v. In an homogeneous isotropic medium, equations 2.32 and 2.33 must be satisfied. We can identify the function ϕ and θ_x with ψ and conclude that two types of waves can be propagated in a homogeneous isotropic medium: One corresponding to changes in one or more components of the rotation given in equation 2.25. The P wave has velocity α and the S-waves has the velocity β

$$\left.\begin{aligned} \alpha &= \left(\frac{\Lambda + 2M}{\rho}\right)^{\frac{1}{2}} \\ \beta & \quad (M/\rho)^{\frac{1}{2}} \end{aligned}\right\} \quad \text{-----} \quad 2.35$$

Since the elastic constant is positive, α is always greater than β. Writing for the ratio β/α

$$\gamma^2 = \frac{\beta^2}{\alpha^2} = \frac{M}{\Lambda + 2M} = \frac{\frac{1}{2} - \sigma}{1 - \sigma}$$

$$\gamma = \left(\frac{\frac{1}{2} - \sigma}{1 - \sigma}\right)^{\frac{1}{2}} \quad \text{-----} \quad 2.36$$

where γ is also a lame's constant

The quantity γ decreases from 0.5 to zero, σ increase from zero to its maximum value, $\frac{1}{\sqrt{2}}$, thus, the velocity ot S-wave ranges from zero up to 70% of the velocity of the p-waves. For fluid M is zero and hence β and γ are also zero therefore S-waves do not propagata through fluids.

2.5 WAVE PROPAGATION IN LAYERED MEDIA

The problem of plane wave propagation in layered media can best be treated by means of filter theory developed by many workers. They include Baranov and Kuntz (1960) Wuensikel(1960) Goupilland, (1969) Kuntz and D'Erceville (1962) Trorey, (1962) Freitel and Rolison (1966). The method was originally used by Abel (1946) and Crook (1948) in optics. This method has tremendous success in takling the problem of generating a synthetic seismogram and removing objectionable reverberation from field seismogram.

2.5.1 Reflection and Transmission at Interfaces

Consider a system of two homogeneous media (semi infinite elastic lasers) in contact seperated by a plane boundary. Let a plane compressional wave be propagated downward in the positive y direction at normal incidence to the plane separating the two media. For simplicity consider only the reflected and transmitted compressional waves resulting at the interface.

Let ψ_i be the displacement due to incident wave in the first medium and it is given by

$$\psi_i = A_i \, e^{iw(y/\alpha_1 - t)} \qquad \text{------} \qquad 2.37$$

where $\alpha_1 = \dfrac{\Lambda + 2M}{\rho_1}$, is the compressional wave velocity and Λ and M are Lame's constant, ρ = volume density, t = time and w = angular frequency of incident sinusoidal wave.

The reflected and transmitted waves ψ_r, and ψ_t respectively are:

$$\left.\begin{aligned} \Psi_r &= A_r e^{-iw(y/\alpha_1 + t)} \\ \Psi_t &= A_t e^{-iw(y/\alpha_1 + t)} \end{aligned}\right\} \qquad \text{------} \qquad 2.38$$

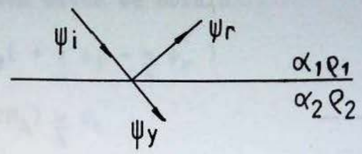


Fig. 2.5 A ray diagram showing the direction and symbols for a downward direction of the wave.

To determine the amplitudes of the reflected wave A_r and that of the transmitted ray waves A_t, two conditions have to be satisfied; first, at any boundary the displacement is continuous i.e.

$$\Psi_i = \Psi_r + \Psi_t \qquad \text{-----} \qquad 2.39$$

where Ψ_r is displacement due to reflected wave, Ψ_t the displacement due to transmitted wave or The condition $y = 0$ gives

$$A_i = A_r + A_t \qquad \text{-----} \qquad 2.40$$

The second condition is that the normal stress is also continuous. If u, v and w are displacements in the x, y and z directions then the normal stress in the y direction is

$$T_{yy} = \Lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2M\,\frac{\partial v}{\partial y} \qquad \text{-----} \qquad 2.41$$

or wince all derivatives with respect to x and z are zero for plane waves propagating in the y-direction

$$T_{yy} = (\Lambda + 2M)\,\frac{\partial v}{\partial y} \qquad \text{-----} \qquad 2.42$$

Substituting the appropriate term for the incident and transmitted waves we obtain

$$\Lambda_1 + 2M_1 (+ \frac{w}{\alpha} \psi_i - \frac{w}{\alpha} \psi_r)$$
$$\simeq + (\Lambda_2 + 2M_2) \frac{w}{\alpha} \psi_t \qquad ----- \qquad 2.43$$

and at y = 0

$$\frac{\Lambda_1 + 2 M_1}{\alpha_1} (A_i - A_r) = \frac{(\Lambda_2 + 2M_2)}{\alpha_2} A_t \qquad ----- \qquad 2.44$$

The accoustic impedence z, is defined as the product of density and velocity

$$Z_1 = \rho_1 \alpha_1 = (\frac{\Lambda_1 + 2M_1}{\alpha_1^2}) \alpha_1 = \frac{\Lambda_1 + 2M_1}{\alpha_1} \qquad ---- \qquad 2.45$$

Substituting equation 2.44 into equation 2.45 implies

$$A_i - A_r = \frac{Z_2}{Z_1} A_t \qquad ----- \qquad 2.46$$

The amplitude of the reflected and the transmitted curve may be solved from equation 2.40 and 2.46, to give

$$A_t = \frac{2}{1 + Z_{2/Z_1}} A_i \qquad ----- \qquad 2.47$$

The reflected amplitude is

$$A_r = -A_i + (\frac{2}{1 + Z_2/Z_1}) A_i \qquad ----- \qquad 2.48$$

or $$A_r = -A_i [1 - \frac{2Z_1}{Z_1 + Z_2}]$$

$$= -A_i [\frac{Z_1 + Z_2 - 2Z_1}{Z_1 + Z_2}]$$

$$= A_i [\frac{Z_1 - Z_2}{Z_1 + Z_2}] \qquad ----- \qquad 2.49$$

The reflection and transmission coefficients r and t are defined respectively as

$$r = \frac{Z_1 - Z_2}{Z_1 + Z_2} = \frac{\rho_1\alpha_1 - \rho_2\alpha_2}{\rho_1\alpha_1 + \rho_2\alpha_2} \quad \text{-----} \quad 2.50$$

$$t = \frac{2Z_1}{Z_1 + Z_2} = \frac{2\rho_1\alpha_1}{\rho_1\alpha_1 + \rho_2\alpha_2} \quad \text{-----} \quad 2.51$$

Similarly the coefficients for the waves in an upward direction are respectively

$$r^1 = \frac{Z_2 - Z_1}{Z_1 + Z_2} \quad \text{-----} \quad 2.52$$

and

$$t^1 = \frac{2Z_2}{Z_1 + Z_2} \quad \text{-----} \quad 2.53$$

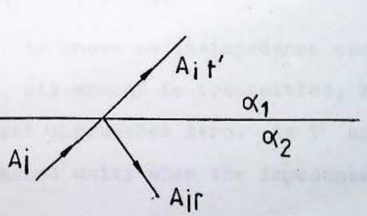


Fig. 2.6 Ray diagram showing the direction of waves in an upward direction.

The function of incident energy which is reflected is also given by

$$R_E = \left(\frac{Z_2 - Z_1}{Z_1 + Z_2} \right)^2 \quad \text{-----} \quad 2.52$$

and

$$T = \left(\frac{2Z_1}{Z_2 + Z_1} \right)^2 = \frac{4\, Z_1^2}{(Z_2 + Z_1)^2} = \frac{4Z_1^{\,2}}{Z_2^2 + Z_1^2 + 2Z_1 Z_2}$$

$$4\,\frac{Z_1^2}{Z_2^2} \quad \frac{\quad}{1 + \frac{Z_1^2}{Z_2^2} + \frac{2Z_1}{Z_2}}$$

let $\frac{Z_1}{Z_2} = \delta$; then

$$T = \frac{4\delta^2}{1 + \delta^2 + 2\delta} = \frac{4\,\delta^2}{(\delta + 1)^2} \quad \text{------} \quad 2.53$$

The quantity δ is known as theimpedance constant. If $\delta = 1$, $R = 0$, all energy is transmitted, as the impedance contrast approaches zero. As T approaches zero, R approaches unity when the impedance contrast increases.

2.6 DIFFRACTION

Diffraction occurs whenever a wave encounters a feature whose radius of curvature is smaller than the wavelength. The seismic energy travels along other paths beside those given by Snell's law. Diffraction is a very important process since the seismic wavelength is large (often 100 m or more) compared to the geological dimensions. The law of diffraction are complex but at distances greater than several wave-length from the diffracting source, the diffracting wave is essentially that given by Huygens (Trory, 1970).

From the diagram fig. 2.7, AB is the plane wave-front which is incident normally on a faulted bed CO at time $t = t_o$. The position of the wavefront reaches the surface of the bed COD. AT $t = to + \Delta t$ the point at the right of 0 will advance to the position GH while the point at the left of 0 will be reflected and reaches the position E F . 0 marks the transition point between the centre which gives rise to the down ward travelling wavefront GH. The diffracted wave fronts extend into the symmetry geometrical shadow, GN and into the region FM.

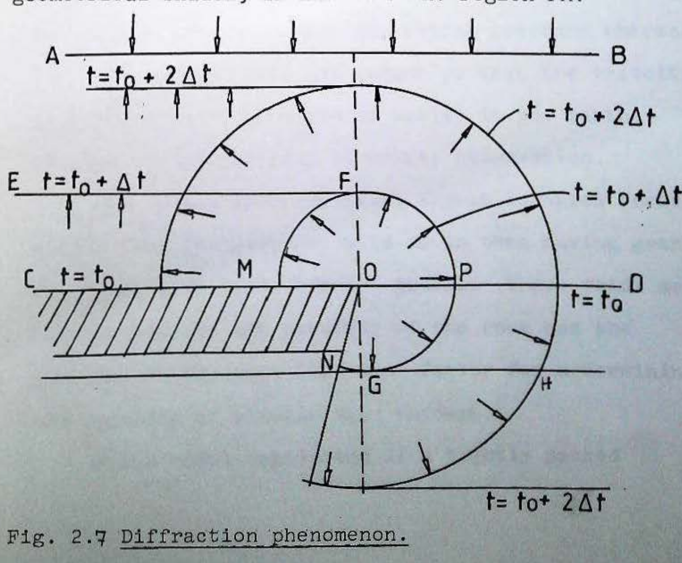


Fig. 2.7 Diffraction phenomenon.

CHAPTER III

3.0 SEISMIC TECHNIQUES

3.1 SEISMIC VELOCITY

Seismic velocity can be defined as the speed with which seismic waves travel. There are factors which affect the velocity of seismic wave in a medium. The velocity of p-waves in a homogeneous solid is a function only of elastic constants and density as seen earlier.

The elastic constants which are properties of intermolecular forces are independent of pressure where as density increases with pressure because rocks are moderately compressible. The numerator in the expression of velocity (equation 2.36) would not change very much with increasing pressure whereas the denominator would get larger so that the velocity would decrease with depth of burial in the earth. This is infact contrary to actual observation.

One of the most important aspect in which rocks differ from homogeneous solid is in them having granular structure with voids between grains. These voids are responsible for the porosity of the rock and the porosity is the most important factor for determining the velocity of elastic wave through.

For a model consisting of a tightly packed spherical particles under a pressure, the air spaces between the spheres are relatively smaller. The elastic constants of such a pack vary with pressure. The p-wave velocity varies as the 1/6th power of the pressure. Thus in terms of depth of burial Z and the formation of resistivity R, the velocity v is $v = 2 \times 10^3 (ZR)^{1/6}$. The duration of each individual measurement is so large, indicating the pressence of the other factors which have not been taken into account.

Faust (1953) earlier included the age of rock as a factor in determining the velocity. An older rock might be expected to have a higher velocity, having been subjected for a longer time to pressure, concentration etc. which increase the velocity.

The pore spaces for rocks are filled with a fluid whose elastic constants and density also affect the seismic velocity. The fluid is under pressure which is usually different from that resulting from the weight of overlying rocks. The effective pressure of the granular matrix is the difference between overburden and th efluid pressure. The formation fluids are under abnormal pressures approaching the over burden pressure. The seismic velocity in this case is exceptionally low, a fact which is sometimes used to predict abnormal fluid pressure from velocity measurement.

The variation of velocity with depth, usually referred to as velocity function is frequently a reasonable tool for the study of velocity increase with depth. Areas of moderately uniform geology exhibit little variation in lateral velocity as per radial ve velocity.

3.2 THE WEATHERED OR LOW VELOCITY LAYER

Seismic velocity which are lower than the velocity of water implies that gas fills at some of the space (Watkin et al., 1972). Such low velocity is usually seen only near the surface in the zone called the weathered layer or low velocity layer (LVL). This layer is usually 4 - 50 m thick, characterised by Seismic velocity which are not only low (250 - 1000 km/sec), but variably high at times. Frequently the base of the LVL coincides with the water table indicating that the LVL corresponds to the aerated zone above the water saturated zone, but this is not always the case. The significance of LVL are:

(i) High rate of absorbtion of seismic energy

(ii) Rapid changes in velocity have disproportionately large effect on travel times of seismic waves.

(iii) The marked velocity charge at the base of the LVL sharply affect seismic ray so that their travel through the LVL is nearly vertical regardless their directions of travel beneath the LVL.

(iv) The very high impedance contrast at the base of LVL makes it evidently, a reflector very significant in multiples.

3.3 VELOCITY MEASUREMENT

3.3.1 Conventional Well Survey

This is one of the most accurate method for determining layer velocity using borehole. These methods of shooting a well and sonic (or continuous velocity survey) are two

In shooting method the geophone or hydrophone is suspended by means of cable and time required for energy to travel from a short fired near the well down to the geophone is recorded. The geophone is made to withstand high temperature and pressure in oil wells. The cable supporting the geophone serves to measure the depth of the geophone output to the surface where it is recorded.

The vertical travel time t, to the depth of z, is obtained by multiplying the observed time by the factor of $z/(z^2 + x^2)$ to correct for actual start distance x being the offset of the geophone (Fig. 3.1) The average velocity between the surface and depth z is given by ratio z/t. By subtracting the depth and time for two shots, interval velocity v_t can be found, the average velocity in the depth interval $(Z_m - Z)$ is determined by means of the formular

$$V_t = \frac{Z_m - Z_n}{t_m - t_n}$$

Shooting a well gives the average velocity with accuracy of measurement. For marine well surveys, air gun energy source is used as explosion.

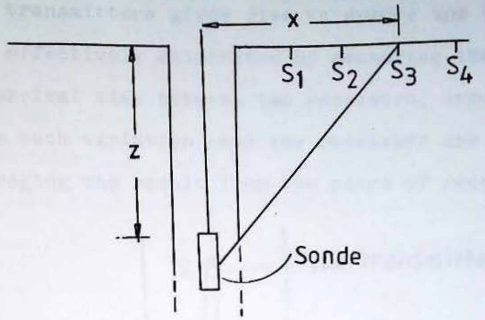


Fig. 3.1 Shooting a well for velocity

3.3.2 Velocity Logging

Velocity logs show the interval velocities of the formation through a well as a function of depth. The continuous velocity survey makes use of one or two pulse generators and two or four detectors: all integrated in a single unit called a Sonde. A semi-logging sonde consists of two sources of seismic pulses S_1 and S_2, and detectors, R_1 to R_4, the span distances from R_1 to R_3 and from R_2 to R_4 being 2 ft. (Fig. 3.2). The velocity is found by measuring the travel time difference for the pulse travelling from S_1 to R_2 and R_3, and for a pulse going from S_2 to R_3 and R_1. The Sonde is run in boreholes filled with drilling mud which has a seismic velocity of roughly 1500 m/sec, however the first energy arrivals are the P-waves which have travelled in the rock surrounding the borehole. The variation in borehole size or mud cake thickness near the transmitters gives rise to errors and th errors are effectively eliminated by measuring the difference in arrival time between two receivers, errors resulting from such variation near the receivers are reduced by averaging the result from two pairs of receivers.

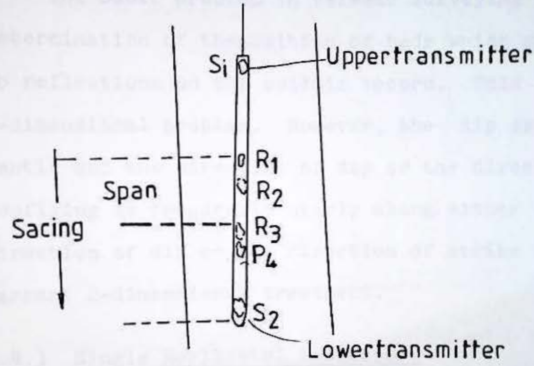


Fig. 3.2 Sonic logging. Borehole-enslated logging sonic

Other methods of velocity measurements like the x^2-T^2 method and t - Δtn method will be discussed in the next sections.

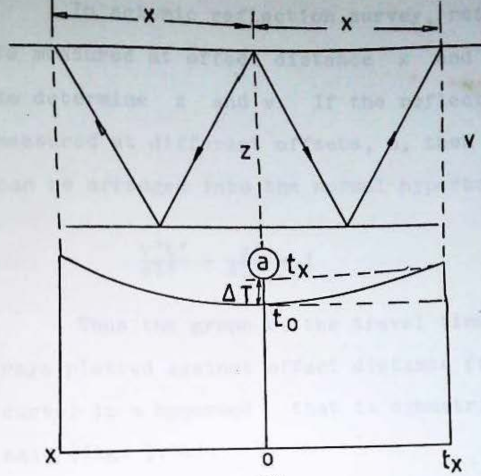
3.4 GEOMETRY OF SEISMIC PATH

The exact interpretation of reflection data requires a knowledge of the velocity at a point along the reflection path. With such detailed knowledge of velocities, analyses are made much easier. Assuming a simple distribution of velocity which is closed enough to give usable result, that the velocityis also assumed constant between the surface and the reflecting bed. Although such assumption is rarely even approximately true, it leads to simple formula which have great bearings in our problem.

The basic problem in seismic surveying is the determination of theposition of beds which give rise to reflections on the seismic record. This is a 3-dimensional problem. However, the dip is often very gentle and the direction of dip or the direction of profiling is frequently nearly along either the direction of dip or the direction of strike to warrant 2-dimensional treatment.

3.4.1 Single Horizontal Reflector

For a single horizontal reflector, the geometry is shown in figure 3.3. It is a case of a single horizontal reflector lying at the depth Z beneath a homogeneous top layer of velocity v.



(b)

Fig. 3.3(a) Reflected Seismic waves from a horizontal layer

(b) time-distant curve from the horizontal reflection.

The actual length of the reflected path is

$$h = 2 \left(\frac{x}{2}\right)^2 + Z \qquad \text{------} \qquad 3.1$$

The equation of the travel time t of the reflected ray from source to a detector or at horizontal offset, or short detector separation x is given by the ratio of travel path length to the Velocity i.e.

$$t = 2\left\{\frac{\left(\frac{x}{2}\right)^2 + h^2}{v^2}\right\}^{\frac{1}{2}} \qquad \text{-----} \qquad 3.2$$

or

$$t = \frac{(x^2 + 4h^2)^{\frac{1}{2}}}{v} \qquad \text{-----} \qquad 3.3$$

In seismic reflection survey, reflection time t is measured at offset distance x and it is required to determine z and v. If the reflection times are measured at different offsets, n, then equation 3.3 can be arranged into the normal hyperbolic form to give

$$\frac{V^2t^2}{4Z^2} - \frac{x^2}{4Z^2} = 1 \qquad \text{-----} \qquad 3.4$$

Thus the graph of the travel time of reflected rays plotted against offset distance (time - distance curve) is a hyperbol that is symmetrical to the time axis (fig. 3.1b).

Substituting x = 0 in equation 3.3 the travel time t_o of a vertically reflected ray is obtained as

$$t_o = \frac{2Z}{V} \qquad \text{----} \qquad 3.5$$

This represents the intercept on the curve (fig. 3.3b), equation 3.1 can be rewritten as

$$t^2 = \frac{4Z^2}{V^2} + \frac{x^2}{V^2} \qquad \text{----} \qquad 3.6$$

or

$$t^2 = t_o{}^2 + \frac{x^2}{V^2} \qquad \text{----} \qquad 3.7$$

which in a close form gives

$$t = t_o \; \{1 + (\frac{x^2}{vt})^2\}^{\frac{1}{2}} \qquad \text{----} \qquad 3.8$$

The binomial expansion of equation 3.8 in

$$t = t_o \; \{1 + \frac{1}{2}(\frac{x}{vt})^2 - \frac{1}{8}(\frac{x}{vt})^4 + \text{---}\} \qquad \text{----} \qquad 3.9$$

Thus for small offsets, x/vt << 1 which is the normal case in reflection surveying. This equation may be truncated after the first term to obtain

$$t = t_o[1 + \frac{1}{2}(\frac{x}{vt_o})^2] \quad ----- \quad 3.10$$

This is the more convenient form of time-distance equation for reflected rays and it is used in various ways in processing and interpretating reflection data.

The difference in travel times t_1 and t_2 for a given reflection for two geophones location x_1 and x_2 is known as move out and it is represented by ΔT. From equation 3.10

$$t_2 - t_1 = \frac{x_2^2 - x_1^2}{2v^2 t_o} \quad ----- \quad 3.11$$

Normal moveout (NMO), ΔTn, at a distance x is the difference in travel-time t between repeated arrival at x and zero offset (which gives up hole travel time t_o) as shown in fig. 3.3b.

Thus $$\Delta Tn = t - t_o = \frac{x^2}{2v^2 t_o} \quad ------ \quad 3.12$$

The NMO is a function of the offset, velocity and the reflection depth Z (with $Z = vt_o/2$). The concept of moveout is fundamental to the recognition, correlation and enhance-ment of the reflection events, and to the calculation of velocity using reflection data. It is implicitly or explicitly used at several stages in the interpretation of reflection seismic.

3.3.2 Multiple Layer Horizontal Reflectors

In a multilayered ground rays reflected from the nth interface undergo reflection at all higher interface to produce a complex travel time as shown in fig. 3.4

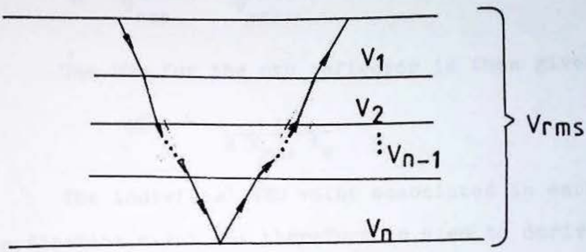


Fig. 3.4 Complex trail path of reflectal ray through a multilayered ground

The offset of trail though several layers is to replace the velocity in equation 3.9 and 3.10 by the average velocity $\bar{v}$ or to a closer approximation, The rms velocity V_{rms} of the layers overlying the reflection.

The V_{rms} for the nth interfaces is

$$V_{rms} = \left[\frac{\sum_{i=1}^{n} V_i^2 t_i}{\sum_{i=1}^{n} t_i} \right]^{\frac{1}{2}} \qquad \text{-----} \qquad 3.13$$

V_i is the interval velocity of the ith layer and t_i is the one-way time of reflected ray through the ith layer.

The total time t_n of the ray reflected from the nth surface, at a depth Z is given from equation 3.10 as:

$$t_n = \frac{x^2}{V_{rms}} + \frac{4Z^2}{V_{rms}} \quad -----$$

The NMO for the nth reflector is then given by

$$\Delta Tn = \frac{x^2}{2 V_{rms} t_o}$$

The individual NMO value associated in each reflection event may therefore be used to derive a V_{rms} value for the layers above a reflector. The value of V_{rms} down to different reflector can be used to compute interval velocity using formulation by Dix (1955). By this the interval velocity Vn for nth interval is

$$Vn = \{\frac{V^2_{rms} t_n - V^2_{rms\ n-1} t_{n-1}}{t_n - t_1}\}^{\frac{1}{2}} \quad ----- \quad 3.13$$

where $V_{rms_{n-1}}$, t_{n-1} and V_{rms_n}, t_n are root mean square velocity and reflectal ray time up to the $(n-1)^{th}$ and n^{th} reflector respectively.

3.3.3 Dipping reflector

As shown in figure 3.5a, the value of the dip is denoted by

$$v^2t^2 = x^2 + 4Z^n - 4xZ \cos\left(\frac{\pi}{2} + \theta\right)$$

or $$\sigma t = \frac{(x^2 + 4Z^2 - 4xZ \sin\theta)^{\frac{1}{2}}}{V} \quad \text{-----} \quad 3.14$$

which on transformation gives

$$\frac{V^2t^2}{4Z^2 \cos^2\theta} - \frac{(x - 2Z \sin\theta)^2}{4Z^2 \cos^2\theta} = 1 \quad \text{-----} \quad 3.15$$

The axis of symmetry of the parabola is no longer the time axis as in fig. 3.5b.

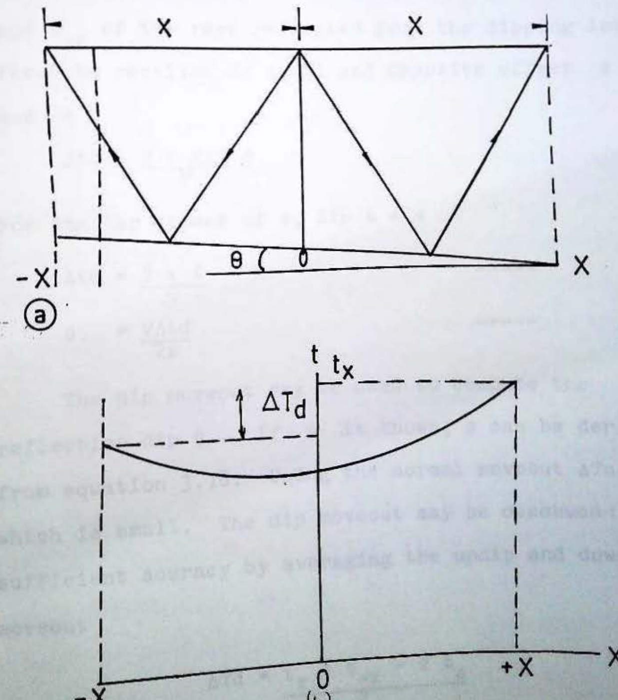


Fig. 3.5a Geometry of reflected ray path

b. time-distant curve for reflection from dipping bed

As in the case of horizontal reflection the following truncated binomial expansion can be obtained

$$t = t_o \left[1 - \frac{x^2 + 4xZ \text{ Cos}}{2V^2t^2} \right] \quad \text{------ } 3.16$$

Considering two receivers of equal offsets x, updip and downdip times can be calculated for a central shot point (fig. 3.5a). The reflected ray paths are different in lengths and the two rays will therefore have different travel times. Dip moveout td is defined as the difference in travel times t_x and t_{-x} of the rays reflected from the dipping interfaces to receiver of equal and opposite offset x and -x

$$\Delta td = \frac{2 \text{ x Sin } \theta}{V}$$

For smaller values of θ, Sin θ = θ

$$\Delta td = \frac{2 \text{ x } \theta}{V} \quad \text{-----} \quad 3.17$$

$$\theta = \frac{V \Delta td}{2x} \quad \text{-----} \quad 3.18$$

The dip moveout may be used to compute the reflection dip θ . If v is known, θ can be derived from equation 3.18. Using the normal moveout ΔTn, which is small. The dip moveout may be observed with sufficient acuracy by averaging the updip and downdip moveout

$$\Delta Td = \frac{t_x + t_{-x} - 2\ t_o}{2}$$

3.5.1 Measurement of Velocity Using $X^2 - T^2$ Method

The arrival time of reflected energy depends not only on the reflection depth and velocity above the reflector but also on the offset distance.

The X^2-T^2 method is based upon equation 3.7 but v is represented by V_{rms} for multiple horixontal layer that is

$$t^2 = \frac{x^2}{V^2_{rms}} + t_o^2 \text{ , which}$$

by plotting a graph of t^2 as a function of x^2, a straight line graph is obtained whose slope is $1/V_{rms}$ and whose intercept is to^2 from which the depth to reflector can be determined.

If the regular seismic profile does not have a sufficient large range of X value to make it easier to find V with the accuracy required for interpretation purpose, spread long-offset profiles are shot generally using arrangement described by Dix (1955).

3.5.2 Measurement of Velocity Using t - Δtn method

This is based upon equation 3.12. With symmetrical spread we can calculate Δtn from the arrival times of a reflection event at the shot point to and at outside geophone groups, t_1 and t_2. The dip moveout is eliminated by averaging the moveout on the opposite side of the point

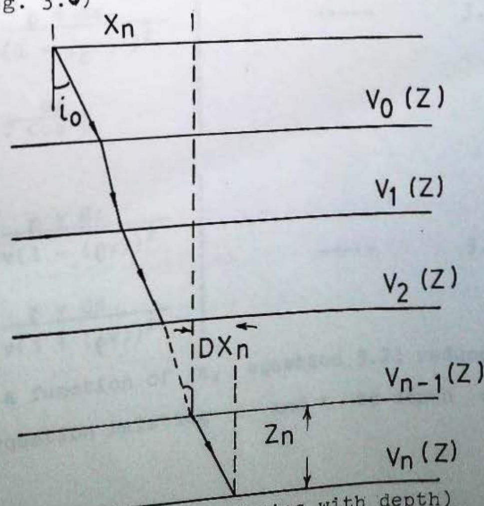
$$\Delta tn = \tfrac{1}{2}[(t_1-t_o)+(t_1-t_o)] = \tfrac{1}{2}(t_1+t_2)-t_o \quad \text{--- } 3.19$$

Tn is subjected to large error mainly because of the uncertainty of the near surface cornection.

3.6 VELOCITY FUNCTION

Assuming that velocity varies in a systematic continuous manner, then it can be represented by a velocity function. The actual velocity varies extremely rapidly over a short interval. Intergrating these changes over distances of a wavelength, a function which is generally smooth except for discontinuity at marked lithological boundaries can be obtained.

For a medium of large number of beds in which the velocity is constant, two interval equation can be derived. If the number of beds are large, the thickness of each bed can be taken to be infinitesimal and the velocity distribution becomes a certain function of depth (Fig. 3.6)



Varies with depth)

From fig. 3.4 using Snell's law

$$\frac{\text{Sin in}}{V_n} = \frac{\text{Sin } i_o}{V_o} = \rho$$

$$V_n = V_n (Z)$$

$$x_n = Z_n \tan in$$

$$t_n = \frac{Z_n}{V_n \text{ Cos in}}$$

The parameter ρ is a constant and depends on the direction, in which the ray leaves the short point

In the limit as x becomes infinite

$$\frac{\text{Sin i}}{V_3} = \frac{\text{Sin } i_o}{V_o} = \rho \text{ ; } V = V(Z)$$

$$\frac{dx}{dz} = \tan i \text{ , } \frac{dt}{dz} = \frac{1}{V \text{ Cos i}}$$

$$x = \int_o^z \tan i \, dz \text{ , } t = \int_o^z dz/_V \cos i \text{ ,}$$

i.e.

$$\left. \begin{aligned} x &= \int_o^z \frac{\rho \, v \, dz}{(1 - (\rho v)^2)^{\frac{1}{2}}} \\ t &= \int_o^z \frac{dz}{V \text{ Cos i}} \end{aligned} \right\} \quad \text{-----} \quad 3.20$$

or

$$\left. \begin{aligned} x &= \int_o^z \frac{\rho \, v \, dz}{v(1 - (\rho v))^{\frac{1}{2}}} \\ t &= \int_o^z \frac{\rho \, v \, dz}{v(1 + (\rho v))^{\frac{1}{2}}} \end{aligned} \right\} \quad \text{-----} \quad 3.21$$

Since v is a function of z, equation 3.21 reduces to integral equation relating x and t to depth z.

Now to express the velocity v as a continuous function of z and integrate equation 3.21, the linear increase of velocity with depth is given as

$$V = V_o + KZ \qquad \text{-----} \qquad 3.22$$

V_o is velocity of the horizontal datum plane and V_1 the velocity at depth Z below the datum plane with K as a constant.

Let $u = \rho v = \sin i$

$du = \rho dv = \rho K\, dz$

Therefore equation 3.21 implies

$$\frac{1}{\rho K} \int_{u_o}^{u} \frac{u\, du}{\sqrt{(1+u^2)}} = \frac{1}{\rho K} (1 - u^2)^{\frac{1}{2}} \Big|_{u_o}^{u}$$

$$[\frac{1}{\rho K} \text{Cos } i]_i^{i_o} = \frac{1}{\rho K}(\text{Cos } i_o - \text{Cos } i) \quad \text{-----} \qquad 3.23$$

$$t = \frac{1}{K} \int_{u_o}^{u} \frac{du}{u\sqrt{(1-u^2)}} = \frac{1}{K} \log \{\frac{u}{1 + \sqrt{(1-u^2)}}\}_{u_o}^{u}$$

$$= \frac{1}{K} \log (\frac{\sin i}{\sin i_o} \cdot \frac{1 + \cos i_o}{1 + \cos i}) = \frac{1}{K} \log (\frac{\tan i/2}{\tan i_o/2}) \text{ -- } 3.23$$

$$\text{Hence } i = 2 \tan^{-1}(e^{kt} \tan i_o/2) \quad \text{------} \qquad 3.24$$

$$z = \frac{1}{K}(v-v_o) = \frac{1}{\rho K} (\sin i - \sin i_o) \quad \text{----} \qquad 3.25$$

Equations (3.23) and 3.25 are parametric equations. The ray path given by equations 3.23 and 3.25 is a circle shown by calculating the radius of curvature r which turns out to be constant

$$r = \frac{(1 + (x')^2)^{3/2}}{x''}$$

$x' = \frac{dx}{dz} = \tan i$ using equations 2.23 and 2.25

$$x'' = \frac{d^2x}{dz^2} = \frac{d(\tan i)}{di}.\frac{di}{dt} = \sec^2 i \frac{di}{dz}$$

$$= \rho K = \sec^3 i \quad \text{using equation 2.25}$$

hence

$$r = \frac{(1 + \tan^2 i)}{\rho K \sec^3 i} = \frac{1}{\rho K} = (\frac{V_o}{K}) \frac{1}{\text{Sin } i} = \text{constant}$$

3.7 MULTIPLE REFLECTION

Apart from the rays that return to the surface after reflection at a single interface known as the primary reflections, there are many paths in layered subsurface after reflection at more than one interface. Thus events which have undergone more than one reflection are called multiples. Variety of possible ray plates involving multiple reflections is shown in Fig. 3.6. Energy of multiples is the product of the energy reflection coefficients for each of the reflectors involved. If R is very small, only the strongest impedance contrast will generate multiples strong enough to be recognised as events.

Multiple reflections tend to have lower amplitude than the primary reflection because of loss of energy at each reflector. However, there are two types of multiples. The first type are reflected at high frequency coefficient and therefore have amplitude comparable to primary reflection (i.e. travel path is long compared with primary reflection from the interface) and hence long path multiples appear as separate event on seismic record. This multiples are called long path multiples. The second type called short-path multiple whose rays from a burial impulse on land are reflected back to the surface or the base of the weathered layer to produce a reflection events that arrive a short time after the primary is known as ghost reflection.

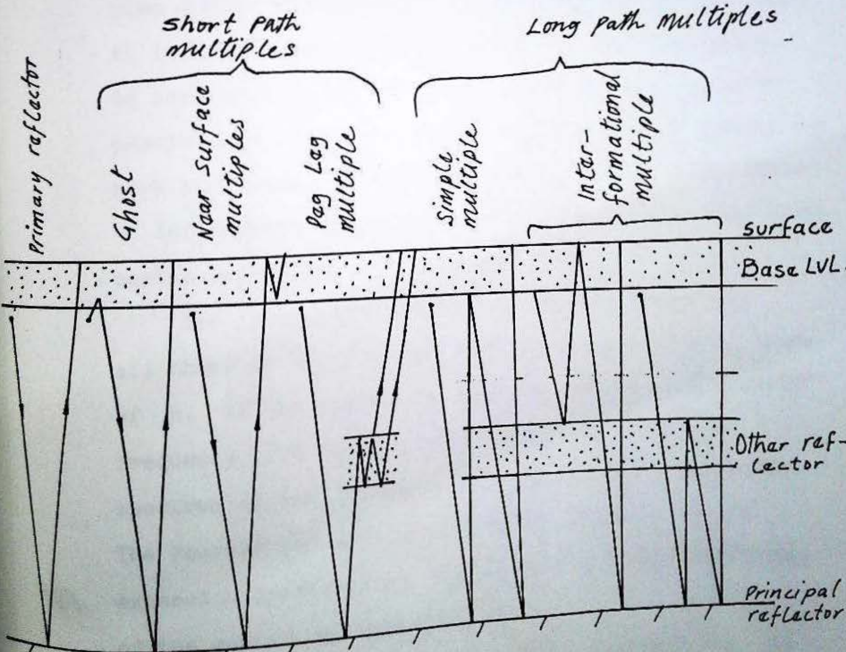


Fig. 3.7 Multiple Refelctions

CHAPTER IV

4.1 REVIEW OF FOURIER TRANSFORM AND CONSTRUCTION OF SYNTHETIC SEISMOGRAMS

Fourier transformation involves the transformation of a function from the time domain to the frequency domain. This process is called Fourier analysis. The inverse process is called Fourier synthesis and involves the transformation from frequency domain to the time domain. There is supposed to be no lost of information in the transformation. Starting with the wave-form in time domain transformed into frequency domain and then it is transformed into characteristic waveform which is identical to the original waveform. This makes it possible to do part of the processing in time domain and part in frequency domain. There is loss of small amount of information in the actual transform due to trunclated series expansions and round-off errors.

The frequencies present in Fourier series are all those of the form $\omega = 2\Pi n/T$ for an integral value of n. If the period T tend to infinity the"quantum" frequency $2\Pi/T$ becomes vanishingly small and the spectrum of the frequency allowed becomes a continuum. The Fourier sum goes over into an integral and the expansion coefficients a_n, b_n and c_n become functions of the continuous variable ω.

For a periodic function $g(t)$ of period T_1 the function can be represented by a complex Fourier Series

$$g(t) = \sum_{n=-\alpha}^{\infty} f(x)\, e^{i\, 2\Pi fxt} \qquad \text{-----} \qquad 4.1$$

where $f_x = n/T$ and

$$f(x) = \frac{1}{T} \int_{-T/2}^{T/2} g(t)\, e^{-i\, 2\Pi fxt} \qquad \text{-----} \qquad 4.2$$

Rewriting equation 4.1 in a real form

$$g(t) = \frac{a_o}{2} + \sum_{n=1}^{\alpha} (a_n \operatorname{Cos} 2\Pi\, fxt + b_n \sin 2\Pi fxt) \text{ --- } 4.3$$

Equation 4.3 is trigonometric with a_n and b_n as real numbers. Assuming that the series converges in the interval $[-\Pi, \Pi]$, it defines a periodic function $f(t)$ with period $T = 2\Pi/\omega$. Equation 4.3 can also be written as

$$g(t) = \sum_{n=0}^{\alpha} C_n \operatorname{Cos}(2\Pi fnt - \phi_n) \qquad \text{-----} \qquad 4.4$$

where ϕ_n is the phase difference, series C_n is known as Fourier series for function of $g(t)$ in the interval $[-\Pi, \Pi]$. The coefficients can be calculated from the following

$$a_o = \frac{2}{T} \int_{-T/2}^{T/2} g(t)dt$$

$$\left.\begin{aligned} a_n &= \frac{2}{T} \int_{-T/2}^{T/2} g(t) \operatorname{Cos} 2\Pi f_n t\, dt \\ b_n &= \frac{2}{T} \int_{-T/2}^{T/2} g(t) \operatorname{Sin} 2\Pi f_n t\, dt \end{aligned}\right\} 4.5$$

$$C_n = a_n \cos \Phi_n + bn \sin \Phi_n = \frac{2}{T} \int_{-T/2}^{T/2} g(t) \cos (f_n t - \Phi_n) dt$$

$$\left. \begin{array}{l} C_n = [a_n^2 + b_n^2]^{\frac{1}{2}} \\ \Phi_n = \tan^{-1} \frac{bn}{a_n} \end{array} \right\} \quad \text{-----} \quad 4.6$$

Equation 4.4 shows that g(t) can be regarded as sum of infinite number of cosine wave function having amplitude C_n and Phase ϕ_n. It thus represents the function, g(t) as a series of cosine waves.

As T becomes larger, it takes longer for g(t) to repeat itself. In the limit when T becomes infinite, g(t) no longer repeat itself. In this case equations 4.1 and 4.2 become

$$\left. \begin{array}{l} g(t) = \frac{1}{\sqrt{2\Pi}} \int_{-\alpha}^{\alpha} f(x)\, e^{ixt}\, dx \\ f(x) = \frac{1}{\sqrt{2\Pi}} \int_{-\alpha}^{\alpha} g(t)\, e^{ixt}\, dt \end{array} \right\} \quad \text{-----} \quad 4.7$$

The function f(x) is the transform of g(t) and g(t) the inverse transform of f(x). Thus

g(t) <-> f(x)

They are referred to as transform pairs.

Fourier theorem expresses f(x) in terms of g(t) in quite a symmetrical fashion. It states that f(x) is the Fourier transform of g(t). The relationship between f(x) and g(t) is reciprocal except for the sign of the exponent.

The Fourier transform g(t) is a real valued function, f(x) is generally complex. The complex conjugate for a real function is

$$g(t) = \frac{1}{2\Pi} \int_{-\alpha}^{\alpha} f(x)\, e^{-ixt}\, dx = g(-t) \text{ -----} \qquad 4.7a$$

When f(x) is an even function of x, however, the Fourier transform g(t) is even too and is real for real f(x). Combining the contribution of t and -t one obtains

$$g(t) = \frac{2}{\sqrt{2\Pi}} \int_{-\alpha}^{\alpha} f(x) \text{ Cos } (xt)\, dt \text{ ------} \qquad 4.8$$

which implies that g(t) = g(-t).

Equation 4.7b. can be written in the form

$$f(x) = \frac{2}{\sqrt{2\Pi}} \int_{o}^{\alpha} g(t) \text{ Cos } (xt)\, dt$$

$$= \frac{2}{\Pi} \int_{o}^{\alpha} \text{Cos } (xt)\, dt \int_{o}^{\alpha} f(t) \text{ Cos } (xt)\, dt \text{ --} \qquad 4.9$$

Similar expressions occur for odd function of x

The equation

$$g(t) = \frac{-2i}{\sqrt{2\Pi}} \int_{o}^{\alpha} f(x) \text{ Sin } (xt)\, dt \text{ -----} \qquad 4.10$$

is an odd function with values that are pure imaginary for real f(x). The reciprocal formular becomes

$$f(x) = \frac{2i}{\sqrt{2\Pi}} \int_{o}^{\alpha} g(t) \text{ Sin } (xt)\, dt \text{ -----} \qquad 4.11$$

4.2 CONVOLUTION

Convolution is a mathematical operation defining the change of waveform resulting from its passage through a filter. The principal use of filtering in data processing is the smoothening and modification of the wave function being analysed. A filter is a device or a physical process that operates on time history and (usually) changes the time history in some manner. It is entirely convenient to consider the earth as a filter placed between the source and the observing station. Filtering is an inherent characteristics of any transmitting station.

Linear filters are single and the most important category of filters. A linear filter is normally characterised by its time domain response function, g(t), to a unit "spike" impulse $\delta(t)$, where

$$\delta(t) = \infty, \quad t = 0$$

$$= 0, \quad t \neq 0$$

and $\int_{-\alpha}^{\alpha} \delta(t)\, dt = 1$.

The output of the filter to any other imput f(t) is then given by

$$h(t) = \int_{0}^{\alpha} f(\tau)\, g(t - \tau)\, d\tau \qquad \text{-----} \qquad 4.12$$

The integral operation 4.12 is known as convolution which is often represented formally as

$$h(t) = f(t) * g(t) \qquad \text{-----} \qquad 4.13.$$

In geophysics, the output function h(t) is given by the seismogram. The source function f(t) may be known or assumed. The properties of the earth are contained in the impulse response g(t) and could include geometrical spreading of energy, non-elastic attenuation and the introduction of reflection or critically refracted phases at given time. If g(t) can be extracted from the seismogram, it can provide a clue to the earth structure. Reflection horizon, for example, simply introduces spikes of length proportional to the reflection coefficients at discrete time. A synthetic seismogram can be obtained by computing g(t) for a given model and performing a convolution for an assumed shape.

It is the convolution theorem that gives the relationship between Fourier transformation and convolution. This establishes an equivalence of filter by multiplication of signal spectrum by the spectrum of filters.

4.3 CONSTRUCTION OF SEISMOGRAM

Let the assumed data for construction of the seismogram be these discrete input signals shown in table below. The data are taken at time interval t = 0.02.

1.0	1.89	5.40	8.10	9.81	9.9	7.47
1.26	-4.05	-9.50	-13.68	-14.13	-11.79	-7.83
-2.86	1.0	3.51	4.23	4.14	3.28	2.25
1.26	0.5	0.36	0.18	0.01	0.0018	0.00

The velocity models are shown in fig. 4.1

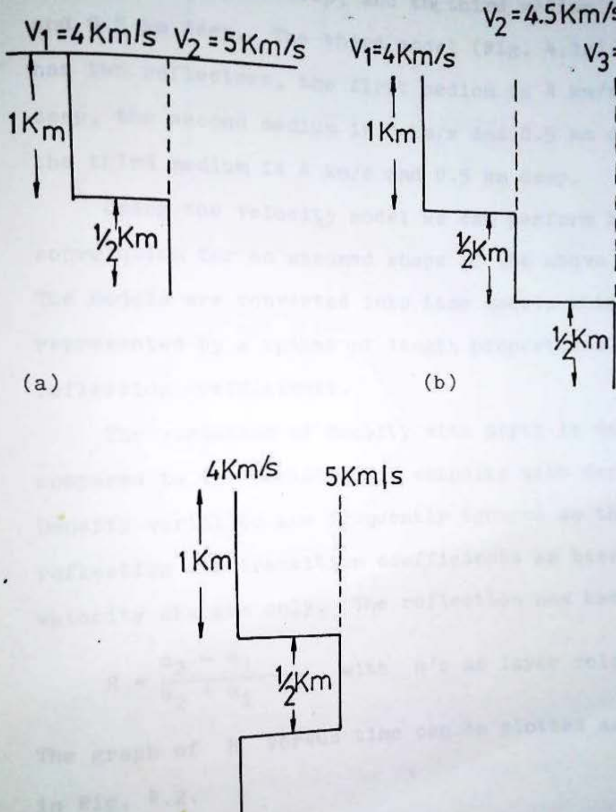


Fig. 4.1 shows velocity models

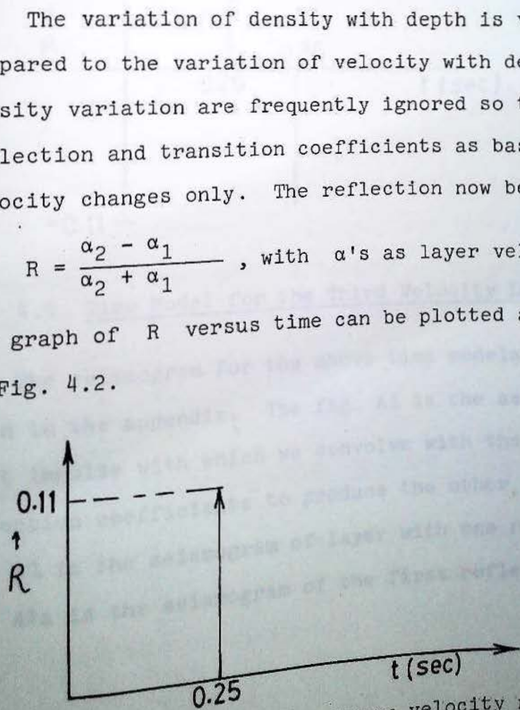
The velocity model has a single reflector. The velocity of the first medium is 4 km/s and is 1 km deep and the second medium has the velocity of 5 km/s and is $\frac{1}{2}$ km deep. The second model (Fig. 4.1b) has two reflectors, the first medium has the velocity of 4 km/s, and 1 km deep, the second medium has the velocity of 4.5 km and ½ km deep, and the third medium is 5 km/s and 0.5 km deep. The third model (Fig. 4.1c) also has two reflectors, the first medium is 4 km/s and 1 km deep, the second medium is 5 km/s and 0.5 km deep and the third medium is 4 km/s and 0.5 km deep.

Using the velocity model we can perform the convolution for an assumed shape of the above data. The models are converted into time models which we represented by a spikes of length proportional to the reflection coefficients.

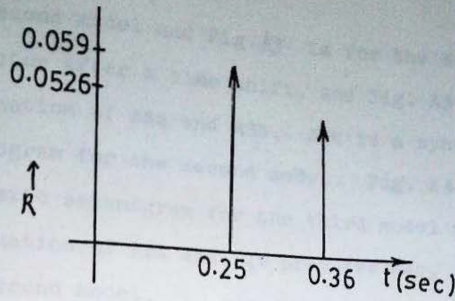
The variation of density with depth is very small compared to the variation of velocity with depth. Density variation are frequently ignored so that the reflection and transition coefficients as based on velocity changes only. The reflection now becomes

$$R = \frac{\alpha_2 - \alpha_1}{\alpha_2 + \alpha_1}$$, with α's as layer velocities.

The graph of R versus time can be plotted as shown in Fig. 4.2.



for one-layer velocity model



(b)

Fig. 4.2 Time Model for the Second Velocity

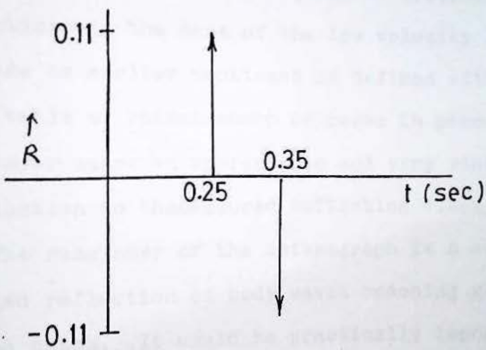


Fig. 4.4 Time Model for the Third Velocity Layer

The seismogram for the above time models were as shown in the appendix. The fig. A1 is the assumed input impulse with which we convolve with the reflection coefficients to produce the other seismograms. Fig. A2 is the seismogram of layer with one reflector. Fig. A3a is the seismogram of the first reflector of the second model and Fig. A3 is for the second reflector after a time shift, and Fig. A3c is the combination of A3a and A3b. A3c is a synthetic seismogram for the second model. Fig. A4 is the synthetic seismogram for the third model and the combination of A4a and A4b produces A4c, just like the second model.

In reflection seismogram that could be obtained in a routine work, the earliest arrival wave are not normally due to direct waves, but to critical refractions at the base of the low velocity layer. Its base as earlier mentioned is defined either by water table or rather sharp decrease in porosity. This layer makes an appreciable and very viable contribution to themeasured reflection times.

The remainder of the seismograph is a complex of unwanted reflection of body waves reaching geophores by some paths. It would be practically impossible to resolve this complex pattern without multi-channel record, on which events can be correlated from trace to trace and then time distance relationships compared with the theoretical ones.

4.4 FIRST ARRIVAL AND DATUM CORRECTION

The variability of the weathered layer leads to scattering of observed reflection times which must be eliminated as much as possible if accuracy is to be realised. The weathering correction determines and accounts for excess time introduced by the thickness of the weathered layer. The usual practice is to correct for both weathered layer thickness and ground topography in one step known as datum correction. Usually these corrections are made to a reference datum such that the shot and geophones are effectively placed at the same flat surface called datum level. In other words, the observed times are reduced to the value they should have if both the shot and geophone wave placed as in figure 4.5.

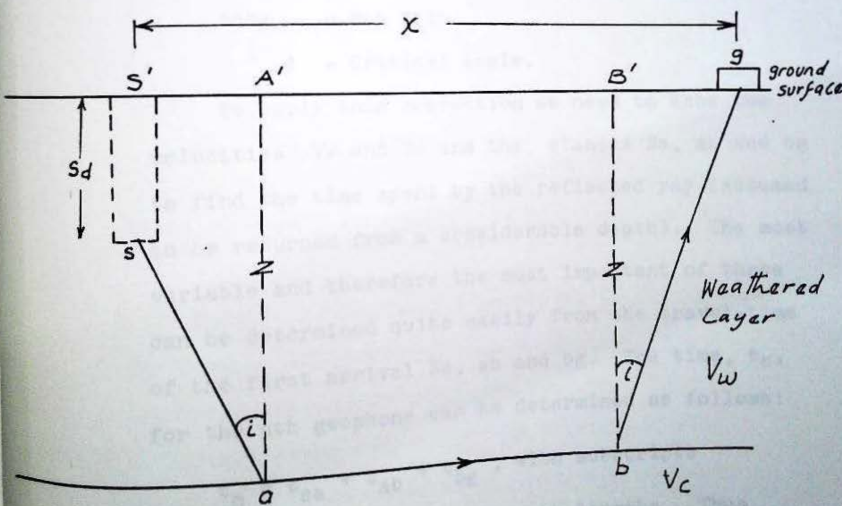


Fig. 4.5 Simplified two-layer medium with weathering Correction

With reference to fig. 4.5, the general assumptions made are:

(1) the ground surface is horizontal,

(2) layers are isotropic with each layer having constant interval velocity and

(3) ray paths are straight.

With the symbols defining the following:

S_d - Shot hole depth.

x - horizontal distance between shot point S and the geophone station g.

Z - the weathering thickness.

Vw - velocity in weathered layer.

Vc - Velocity in consolidated layer

Sabg - ray path.

i - Critical angle.

To apply this correction we need to know the velocities Vw and Vc and the stances Sa, ab and bg to find the time spent by the reflected ray (assumed to be returned from a considerable depth). The most variable and therefore the most important of these can be determined quite easily from the travel-time of the first arrival Sa, ab and bg. The time, t_n, for the nth geophone can be determined as follows:

$t_n = t_{sa} + t_{ab} + t_{bg}$, with subscripts representing effects of the path lengths. Thus

$$t_n = \frac{Sa}{Vw} + \frac{ab}{Vc} + \frac{bg}{Vw} \qquad \text{-----} \quad 4.14$$

But from fig. 4.5

$$Sa = \frac{Z - Sa}{\text{Cos } i} \quad \text{-----} \quad 4.15a$$

$$bg = \frac{Z}{\text{Cos } i} \quad \text{-----} \quad 4.15b$$

and $ab = x - (2Z - Sd) \tan i$ ----- 4.15c.

Also $\text{Sin } i = \frac{Vw}{Vc}$ (Snell's law) ----- 4.16a.

$$\text{Cos } i = \frac{(Vc^2 - Vw^2)^{\frac{1}{2}}}{Vc} \quad \text{-----} \quad 4.16b$$

$$\tan i = \frac{Vw}{(Vc^2 - Vw^2)^{\frac{1}{2}}} \quad \text{-----} \quad 4.16c$$

hence substituting equations 4.15 and 4.16 and simplifying we have

$$t = \frac{x}{Vc} + (2Z - Sd)\frac{(Vc^2 - Vw^2)^{\frac{1}{2}}}{Vc.Vw} \quad \text{-----} \quad 4.17$$

Thus

$$tn = t - \frac{x}{Vc} = (2Z - Sd)\frac{(Vc^2 - Vw^2)^{\frac{1}{2}}}{Vc.Vw} \quad \text{---} \quad 4.18$$

Hence $z = \frac{tn}{2} \cdot \frac{Vc \cdot Vw}{(Vc^2 - Vw^2)^{\frac{1}{2}}} + \frac{Sd}{2}$ ------ 4.19

Equations 4.18 and 4.19 are generally equations for the determination of the time in weathering and weathering thickness respectively.

Other cases which might be considered are

<u>Case 1:</u> When the shot is at the ground surface. In this case Sd = 0 and equations 4.18 and 4.19 become respectively ----- 4.20

$$tn = 2Z \cdot \frac{(V^2 - Vw^2)^{\frac{1}{2}}}{Vc.Vw}$$

and

$$Z = \frac{tn}{2} \cdot \frac{Vc.Vw}{(Vc^2 - Vw^2)^{\frac{1}{2}}} \quad \text{------} \quad 4.21$$

Case 2: Deep hole with

(a) Shot point at the base of weathered layer, that is Sd = Z

The equations 4.18 and 4.19 reduce respectively to

$$tn = Z.\frac{(Vc^2 - Vw^2)^{\frac{1}{2}}}{Vc.Vw} \quad \text{------} \quad 4.22$$

and $$Z = tn - \frac{Vc.Vw}{(Vc^2 - Vw^2)^{\frac{1}{2}}} \quad \text{------} \quad 4.23$$

(b) Short Point in theconsolidated layer

i.e. Sd $\approx$ Z (See Fig. 4.6). There are three possibilities hence;

(i) Sd = 2Z

Then equation 4.18

$$tn = 0$$

and $$t = \frac{x}{Vc}$$

(ii) α is small such that Cos $\alpha \approx 1$ and Sd $\approx$ Z which is same as Case (a) above

(iii) α is large such that Cos $\alpha \neq 1$. The trend time t is then given by

$$t = \frac{x - Z \tan i}{Vc \text{ Cos}} + \frac{Z}{Vw \text{ Cos } i} \quad \text{-----} \quad 4.24$$

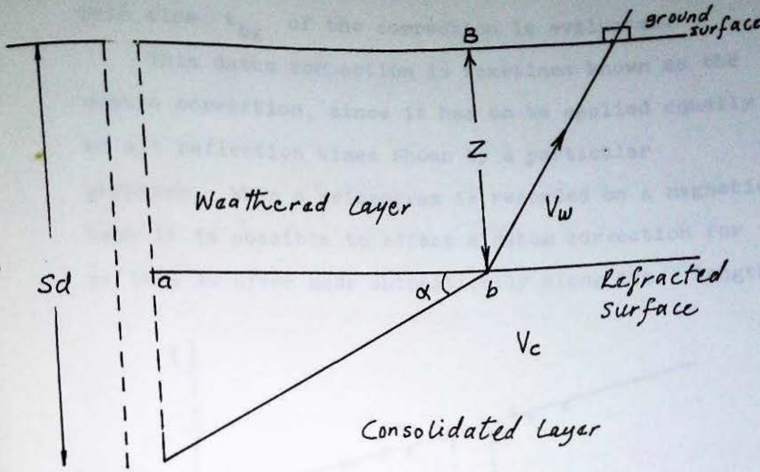


Fig. 4.6 Shot in Consolidated layer

The graph of first arrival time against the geophone ranges x_n will have the form shown in Fig. 4.7 provided Z does not show a systematic increase or decrease over the spread, the best time through the points (unbroken lines) of the figure will have a slope of $1/v_c$ and the departures of the point from a line parallel to this through the origin will be the required time tw_n. With V_c known from the first arrival times and with the elevation of the shot and the datum surface, it becomes easier to determine the time spent on the path Sa. Similarly Vw is measured in borehole or on the surface, and hence the elevation of g can be found and the remaining path time t_{bg} of the correction is evaluated.

This datum correction is sometimes known as the static correction, since it has to be applied equally to all reflection times shown by a particular geophone. When a seismogram is recorded on a magnetic tape it is possible to effect a datum correction for as this is often made automatically along its length.

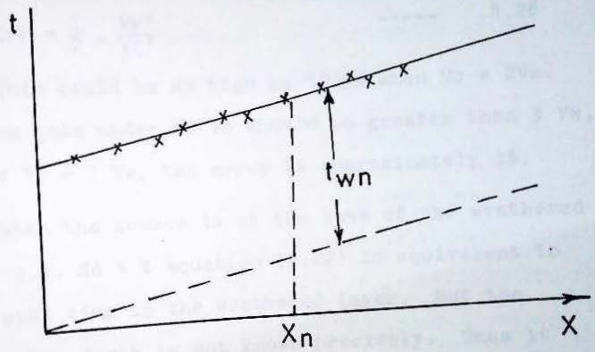


Fig. 4.7 First arrival time versus geophone ranges X_n

4.4.1 Applications

1. When the source is at the ground surface, Sd = 0, equations 4.20 represents the total time in the layer and it requires modification from the practical method in the field.

2. When the source is within the weathered layer i.e. Sd < Z, then from equation 4.17, the weathered depth is

given by

$$2Z = tn + \frac{Sd(Vc^2 - Vw^2)^{\frac{1}{2}}}{Vc.Vw} \qquad \text{-----} \qquad 4.25$$

For easier calculation, the second term in the RHS of equation 4.25 is generally approximated to Sd/Vw. Expanding the term under the radical using Maclaurin series and neglecting fourth and higher terms, the errors in the above approximation Err is given as

$$Err = \frac{1}{2} \cdot \frac{Vw^2}{Vc^2} \qquad \text{-----} \qquad 4.26$$

This could be as high as 12.5% when Vc = 2Vw. To keep this under 5% Vc should be greater than 3 Vw, and if Vc = 7 Vw, the error is approximately 1%.

(3) When the source is at the base of the weathered layer i.e. Sd = Z equation (4.22) is equivalent to the total time in the weathered layer. But the weathering depth is not known precisely. Thus it cannot be easily ascertained when Sd equals Z .

(4) When the source is in the consolidated layer, i.e. with Sd Z, the following specifics apply:

(i) Sd = 2Z

From equation 4.17, the intercept time Tn and the total time in the weathered layer are respectively equal to zero. Hence there will be no enough information to determine the weathering depth from the records. Theoretically all first arrival times are direct waves. This condition is synonymous with Vc = Vw, which shows no distinction between the materials of the weathered and the consolidated layers.

(ii) Sd ≠ 2Z .

(a) When α is as small such that Cos ≈ 1, then it is equivalent to Case (3) above.

(b) When α is large such that Cos $\alpha \neq 1$, α is not usually determined in the field. Thus the travel time and hence the intercept time tn cannot be considered (see equation (4.24)). In an extreme case, where $\alpha = 90°$, equation (4.23) itself also becomes indeterminable.

4.5 IDENTIFYING AND "PICKING" REFLECTOR

In reflection survey, the range of shot is usually small compared to the depth of the reflector. This makes the reflectal pulse to arrive almost simultaneously at all geophones and so appears as series of peaks and troughs alligned almost simultaneously on the seismogram. An allignment of this kind which can be followed across nearly all traces of several records is likely to be a reflection. This is usually "picked" for measurement of travel time. Oftenly spurious reflections are eliminated. Fine measurements are made on the remaining and are used to calculate the depth and dip of reflector.

Geological interfaces are generally complex and since the reflection pulse is the same with the Ricket wavelength, the number of Ricket pulses, erect and inverted, and with various time delays will be added to the actual ground motion. This form is further distorted by the recording apparatus which adds extra "loops" to the pulse and introduces complicated distortions. This makes the first peak and trough to which the measurements are made have no absolute significance. It also makes the "seismic horizon" plot to be about 100 ft or more from the "lithographical horrizon".

Besides the surface wave a reflection seismogram also contains a large number of other events which could be classified as "noise" and can be reduced by suitable combination of output of several geophones on recording channel. It has now been known that many of these events are multiple reflections travelling by some ray paths. The multiples known as ghost always follow a strong single reflection at times twice the uphole times.

Ghost reflections as seismic record can be removed by reverberation filtering. Weathered layers occupy some regions of solid earth consisting of surface rocks which are aerated, leached, altered chemically and are not saturated with water. The velocity of the weathered layer is not regular from place to place and the values lie between 1500 ms^{-1} and 5000 ms^{-1}. The delay in arrival time as a result can be cornelated if the velocity is known, but the spectra alteration can only be carried out by an inverse filter.

Any velocity discontinuity within weathered layer produces a ghost which is similar to direct or primary reflection.

Diffractions are indistinguishable from reflection on the basis of character. There is variation of amplitude of diffraction, usually maximum at some point along the time of profiling and decrease rapidly as one goes away from the point. If discontinuities such as faults are present in the reflector, they may show themselves not much by the discontinuous change in travel time of the reflected pulse (which is often small in amplitude near the fault), but by generating a spherical diffracted waveforms centred discontinuously as shown in fig. 2.7

4.6 PLOTTING THE POSITION OF REFLECTORS

After spurious reflection has been eliminated, time measurements are made on the remaining and are used to calculate the depth and dip of reflector. Let v_i be constant velocity down to the reflector with the dip wholly in the line of seismic profile.

Plotting seismic cross-section on a sheet of graph paper helps to prepare a composite picture of the output signal. The shot point location is marked at the top of the shot on the horizontal line indicating the datum plane. The reflection events are each plotted below the corresponding shot point according to their arrival times. Plots are often made also of the surface elevation the depth of the base of the low level layer and the first break arrival times.

Cross-section are called time section of the vertical scale in linear section with time or depth section. A cross-section is migrated when the reflection are plotted vertically below the shot point. A migrated section is one for which it is assumed that the seismic line is normal to strike so that the dip moveout indicates true dip and plotting their actual location produces the recorded events. The scale on migrated section is usually linear with depth.

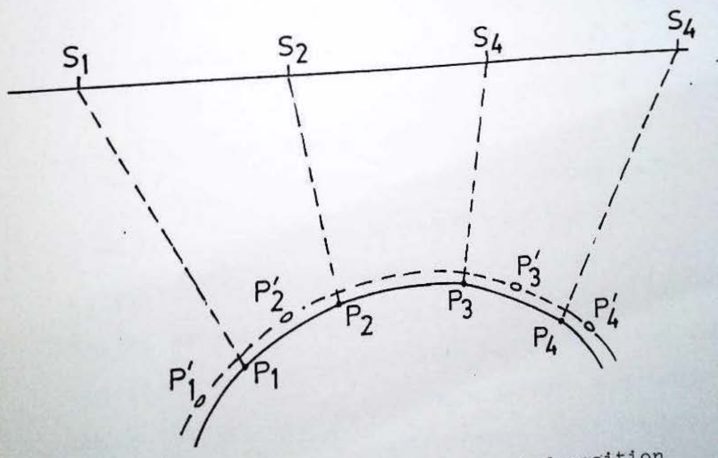


Fig. 4.30 True and Vertically plotted position of reflected points

The profile prepared directly from a seismogram is a time section and is of little practical value for most purpose until it is converted into depth section.

This convertion may change its appearance considerably, particularly if the velocity varies laterally as is commonly the case. Moreover spurious events present in the individual record will be as prominent in the profile as the wanted reflection which make interpretations difficult. These should not be read a denial of the value of continuous profile which sometimes draws attention to important events whose corollation would not be apparent from the individual record event when side by side.

CHAPTER V

SUMMARY AND CONCLUSION

Seismologists have demonstrated that, in important cases, seismogram of seismic wave can be computed rather more realistic. A number of alternative procedure has been used. Seismological investigation of earth's interior now rely not only the travel time, but also on the amplitude of seismic wave. Computation of seismogram aids significantly in the quantitative interpretation of the observed seismogram. Comparism of the actual and synthetic seismogram helps to determine which event represent primary and which multiple. In many areas the synthetic seismogram is a reasonable approximation to actual seismic record and is therefore useful in correlating reflection events with particular horizon.

A draw back of many synthetic seismogram has been their limitation to larger wave lengths. Because of this limitation in theoretical approximation many construction has been restricted to seismic wave with period above two seconds, and model which fit the data well at such periods have been found not to give acceptable fits to high frequency acceleration recorded near a fault. The problem is an important one since for engineering purpose due concern is mainly with acceleration record rather than with displacement.

In attempting to analyse a seismogram, one needs to know the velocity impulse which is generated by an explosive source in homogeneous ground. The classical treatment of this problem is due to Ricker (1953), and showed that the "Spike" impulse velocity which must exist very near to the source is transferred in the early stages of its propagation. This elementary impulse, which is known is the "Ricker wavelength" and is broader and smaller in amplitude at greater distance but it is essentially unaltered in form. To analyse a seismogram, it is necessary to remember that the Ricker wavelength of ground velocity is distorted by the recording apparatus (Angkly, 1958), the most common form of distortion being the addition of entra loop.

The broadening of the pulse by further selective attenuation of high frequency component as it is generally different in amplitude takes place at the rate which is depended on the mechanical properties of the ground. The hand component of the rock transmit the pulse with little attenuation while loose soils attenuate it heavily with accompanying rapid broadening. Attenuation and broadening are much less readily measured than velocity and so velocity, has far been used as an indicator of the mechanical properties of the medium.

An important use of synthetic seismograms is in study of the effect of changes in the reflecting layers on the seismic record. This might then provide a clue in looking for ancient stream channels or other features of interest. Synthetic seismograms are important in indicating the features which may help to identify stratigraphic traps.

The seismic reflection method has been applied most extensively in search of oil, although it cannot of course detect the oil directly, but only the geological structures most favourable for its accumulation. In prospecting for other mineral, the method is little used, mainly because they generally occur in a geological contact which is too complicated for accurate interpretation of measurements. This same factor and theexpense of the conventional seismic survey has hitherto hinted the extent to which civil engineers has been able to use seismic method in site exploration, but in recent years much experience has been gained in solving cheaply the class of problem in which low velocity "soils" overlay a high velocity "bed rock" at no depth.

This research was done without actual field practice, though it could serve as reference for anyone who intends to dig further on how to obtain a synthetic seismogram.

LIST OF REFERENCE

(1) Bullen, K.E., Bruce, A.B., Introduction to Theory of Seismology Fourth Ed. Cambridge University Press 1985.

(2) Courant, R., Introduction to Calculus and Analysis Volume 2. Second Ed., A Wiley Interscience Publication, 1974.

(3) Duffieux, P.M., The Fourier Transform and its Application to Optics Second Ed. John Wiley and Sons, Inc. Canada (1983).

(4) Galand, G.D., Introduction to Geophysics Mantle, Core, and Crust Second Ed. W.B. Saunders Company, USA. 1971.

(5) Frant, F.S., Interpretation Theory in Applied Geophysics, Second Ed. W.B. Saunders Company USA 1979.

(6) Markus, B. Introduction to Seismology, 2nd Ed., Birkhauser Verlag, Boston 1973.

(7) Jacobs J.A., Russel, R.B., Wilson J.T., Physics and Geology 2nd Ed. McGraw-Hill International Series, 1979.

(8) Jeckins, A.F., White, H.E., Fundamental of Optics. Fourth Ed. McGraw-Hill Books Company.

(9) Kanasewich, E.R; Time Sequence Analysis in Geophysics Second Ed. Alberta Press 1975.

(10) Rzhevsky, V; Novie, G. Physics of the Rock Second Ed. MIR Publishers Moscow, USSR. 1953.

(11) Stacey, D.F., Physics of the Earth Second Ed. John Wiley & Sons 1969.

(12) Steward E.G. Fourier Optics an Introduction. Second Ed. Ellis Harwood Limited Chechester 1983.

(13) Telford, W.M. Geldard, L.P., Sheriff, R.E. Keys, D.A., Applied Geophysics, Cambridge University Press, 1980.

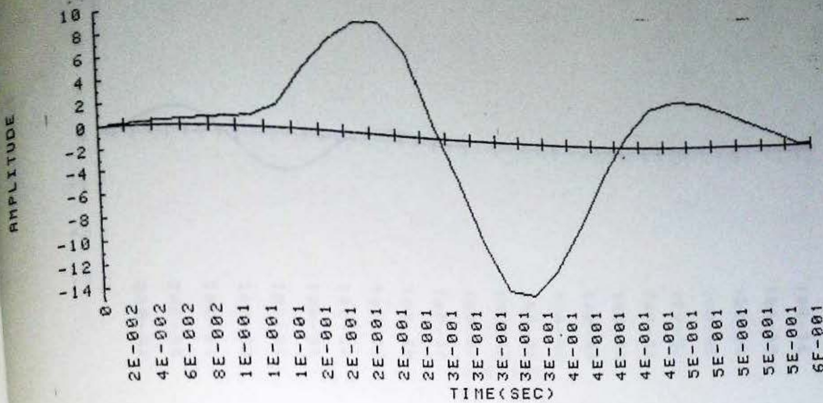


Fig. A1: The assumed input signal.

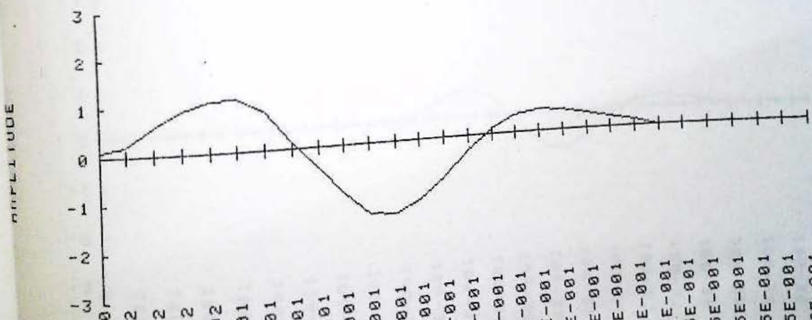


Fig. A2: Synthetic seismogram for a single layer with reflection coefficient of R = 0.11

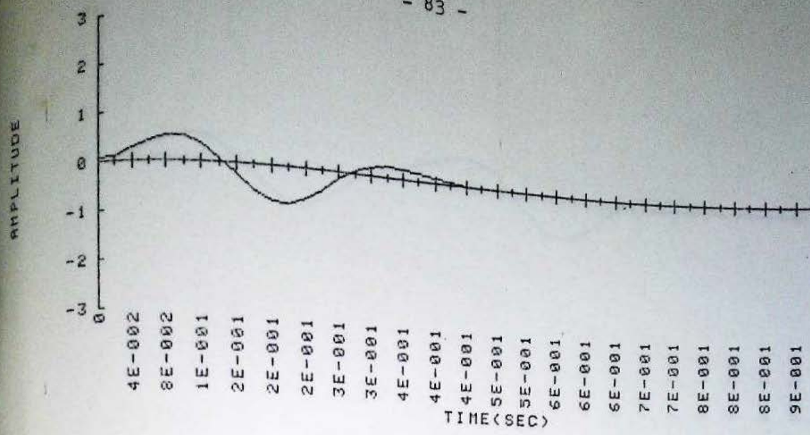


Fig. A3a - Seismogram for the first reflector of the second model R = 0.053

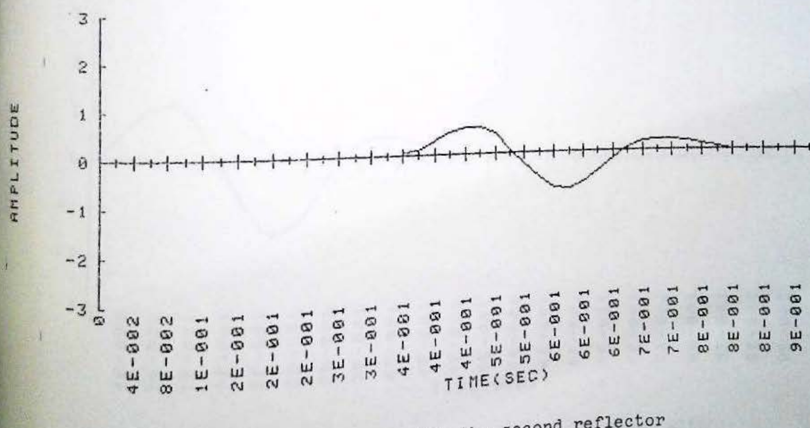


Fig. A3b - Seismogram for the second reflector of the second model. R = 0.059.

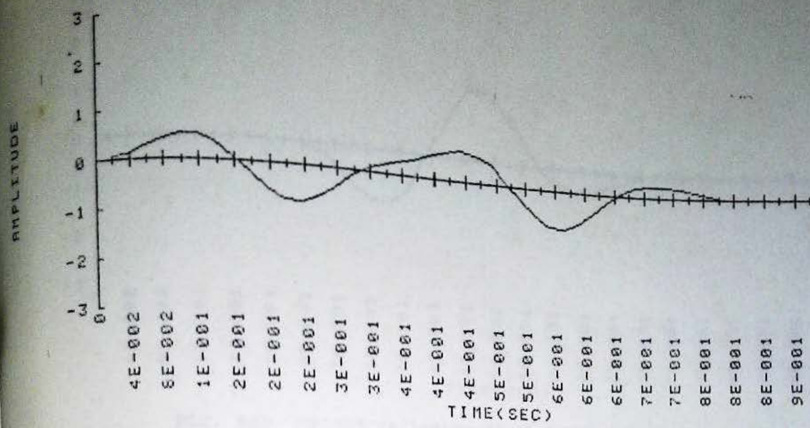


Fig. A3c Synthetic Seismogram of two layered medium for the second model.

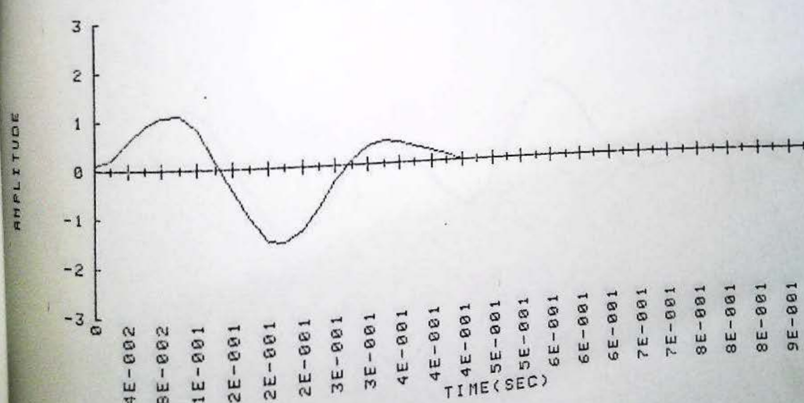


Fig. A4a First reflector for the third model with $R = 0.11$

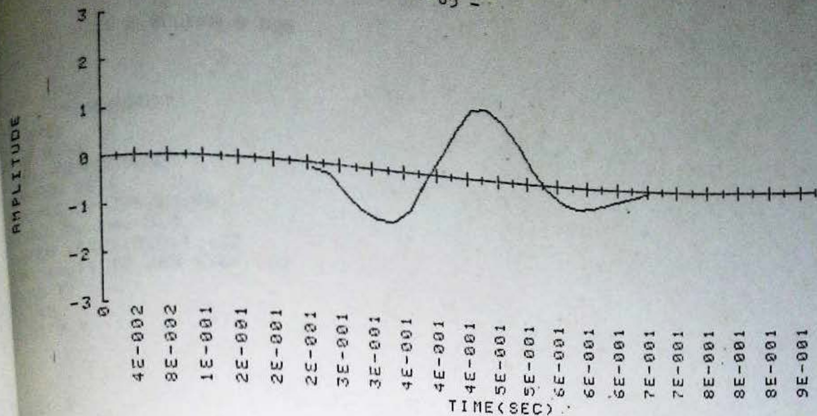
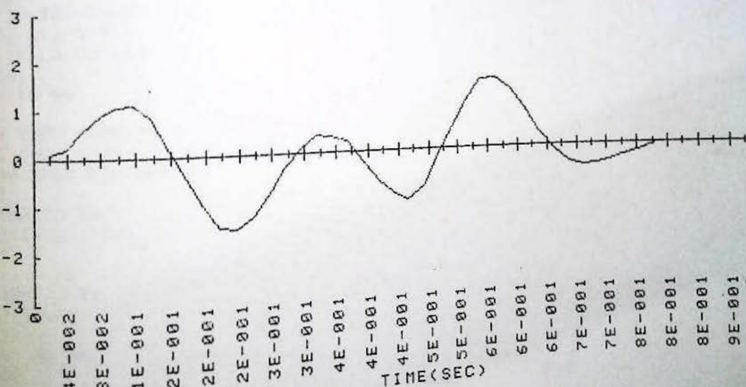


Fig. A4b Second reflector for the third model, R = -0.11



A4c Synthetic seismogram for the third model with double layers.

```
10 PLOTTER IS 1 @ GCLEAR @ DEG
20 GRAPHALL
30 CSIZE 5,.7
40 MOVE 120,0
50 LABEL "TIME(SEC)"
60 LDIR 90
70 MOVE 5,46
80 LABEL "AMPLITUDE"
90 PLOTTER IS 1
100 LOCATE 30,226,25,98
110 SCALE 0,.56,-3,3
120 LAXES .02,1,0,0,1,1,3
140 FOR X=.11 TO .65 STEP .02
150 READ Y
155 Y=Y*G
160 PLOT X,Y
170 NEXT X
180 END
190 DATA 1,1.89,5.4,8.1,9.81,9.9,7.47,1.26,-4.05,-9.5,-13.68,-14.13,-11.79,-7.83
,-2.88,1,3.51,4.23,4.14,3.24,2.25,1.26,.05,.036,.018,.01,.0018,0
```

Fig. B1 Program used in plotting the seismograms A1, A2, A3a, A3b, A4a and A4b

```
0 GRAPHALL
) LOCATE 30,225,25,98
 SCALE 0,.91,-3,3
 LAXES .14,.5,0,0,1,1,2
 DIM Y(56)
 FOR I=1 TO 56
 READ A
 IF I<29 THEN A=A*.11 @ Y(I)=A
 IF I>28 THEN A=A*-.11 @ Y(I)=A
) IF I=28 THEN RESTORE
) NEXT I
) FOR I=18 TO 28
) Y(I)=Y(I)+Y(I+10)
 NEXT I
 X=0
 FOR I=29 TO 41
 Y(I)=Y(I+10)
 NEXT I
 FOR I=1 TO 41
 X=X+.02
 PLOT X,Y(I)
 NEXT I
 END
 DATA 1,1.89,5.4,8.1,9.81,9.9,7.47,1.26,-4.05,-9.5,-13.68,-14.13,-11.79,-7.83
 88,1,3.51,4.23,4.14,3.24,2.25,1.26,.05,.036,.018,.01,.0018,0
```

Fig. B Program which combines two seismograms to a single synthetic seismogram (i.e. combined A3a and A3b to give A3c and also combines A4a and A4b to give A4c).