COBB-DOUGLAS PRODUCTION FUNCTION CONSIDERING LABOUR, CAPITAL AND RAW MATERIALS IN CONSTRUCTION TIME COST ANALYSIS

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A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, SCHOOL OF PHYSICAL SCIENCES, IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF SCIENCE IN MATHEMATICS WITH ECONOMICS OF THE MODIBBO ADAMA UNIVERSITY OF TECHNOLOGY, YOLA

FEBRUARY, 2020

DECLARATION

I hereby declare that this thesis was written by me and record of my own research work. It has not been presented before in any previous application for a higher degree. All references cited have been acknowledged.

DOGO, Iliya Dekilo — Date

DEDICATION

This research work is dedicated to my beloved parents Mr. and Mrs. Dekilo Dogo and to my brothers and sisters.

APPROVAL PAGE

This thesis entitled “Cobb-Douglas Production Function Considering Labour, Capital and Raw Materials in Construction Time Cost Analysis” meet the regulation governing the award of Masters of the Modibbo Adama University of Technology, Yola and is approved for its contribution to knowledge and literary presentation.

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ABSTRACT

The purpose of the study was to develop a new framework using Cobb-Douglas Production Function considering labour, capital and Raw Materials Construction time cost analysis. Hybrid Genetic Algorithm was used to find the optimal cost of labour, capital and raw materials by considering the different options. Data were collected from IDD Blocks and Construction Book 2017 in order to test the new framework. The activity choice contain three different options, were the best option was selected using Hybrid Genetic Algorithm. The selected option were tested using Cobb-Douglas Production Function. The findings revealed that the decision maker (Manager) considered both cost and time equally and there will be increase in level of production that may occur in the long run indicated by the scale of production. The research recommended that when using Cobb-Douglas function, level of technology impacts should not be presumed to be one because in reality. There is high significant improvement in technology day in day out as many types of machines are launched to do a task in a shorter time. Therefore, technology impact should not be presumed as one in any time nearby.

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CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

The Cobb-Douglas Functional form of production function is widely used to represent the relationship of output to inputs. This can be applied to research domain construction of time cost analysis. In this thesis, we shall explore the function of Cobb-Douglas to analysis of time and cost. Time and cost are the main goal of construction management. Planners are searching for the optimal schedules which will give both early completion time and small total cost. The schedule problem name Time-Cost Trade-off Problem (TCTP) is attended to by previous researchers. The optimal schedules should provide early completion with minimum cost. Some were formulated using linear programming or integer programming (Perera, 1980; Liu *et al*, 1985; Moussorakis & Haksever, 2004). The problem models were solved by either the exact method (Perera, 1980) or the approximate method like genetic algorithm (Goldberg, 1980).

There has been lack of theoretical base to model cost function associated with time of activity. The TCTP was presented first in 1959 in the work of Kelly and walker (1961). They gave a solution based on linear programming of Activity on Edge (AOE) network. Fulkerson (1961) and Kelly (1979), gave another solution based on maximal flow algorithm. The problem of time and cost can be solved using minimum flow algorithm. Then Hajdu and Klafszkzy (1992) showed the acceleration of the method. The solutions also is based on maximal flow algorithm.

The cost and duration of any project highly depends on cost variances, which is referred to as any deviation in the budget or cost. The benchmark for estimating the cost variance is derived from the comprehensive planning stage where design, specification, scope, time and cost are developed for the award of contract, which is the final negotiated and agreed price at the construction commencement (Ng *et al*, 2008).

Considering many construction crews are composed of labour, equipment and raw materials, Cobb-Douglas production function CDPF (Cobb and Douglas 1928; Varian 1992). Uses two inputs (Labour L and Equipment K) to determine the output. But in this research we are using three parameters; Labour inputs L, equipment/capital inputs K and raw materials R to determine formation which can be used to explain many types of production activities.

$$Q = AL^{\alpha} K^{\beta} R^{\Gamma} \qquad (1.1)$$

where Q = production rate; L = labour input, K = equipment/capital; R = raw materials input; A = technology. α, β and γ are output elasticity of labour, equipment/capital and raw materials respectively. We shall use the following situations to explain the origin of the crashing cost.

- Constant return to scale ($\alpha+\beta+\gamma = 1$)
- Decrease return to scale ($\alpha+\beta+\gamma < 1$)
- Increase return tom scale ($\alpha+\beta+\gamma > 1$)

Using one of these situations will produce a suitable tool to model construction schedule crashing activities using CDPF which can theoretically be explained by increasing one of the parameters (L, K and R). To end this new framework for TCTP in construction using CDPF and GA is proposed to find the optimal solution in the case of cost and time.

1.1.1 Return to scale

The long run refers to a time period when the production function is defined on the basic of variable factors only. No fix factors exit in the long run and factors become variable.

Thus, the scale of production can be changed as input change proportionality. Thus return to scale defined as the change in output as factors input change in same proportion. It is a long run concept.

1.1.2 Increase return to scale

Increase in output that is proportionally greater than a simultaneous and equal percentage change in the use of all inputs, resulting in a decline in cost. External economics of scale might be one of the reasons behind such increase in output in increasing returns to scale. Thus when input double, output more than doubles in the case.

1.1.3 Decrease return to scale

This occurs when an increase in all inputs (labour, capital and raw materials) lead to a less than proportional increase in output, we call it decrease returns to scale or diminishing returns to scale. In this case, internal or external economics are normally overpowered by internal or external dis-economics. Thus, if we double inputs, the output will increase but by less than double.

1.1.4 Constant return to scale

When the output increases exactly in proportion to an increase in all the inputs or factors of production, it is called constant returns to scale. For example, if twice the inputs are used in proportion, the output also doubles. Thus, constant return to scale is reaching when internal and external economics and dis-economics balance each other out.

A regular example of constant returns to scale is the commonly used Cobb-Douglas Production Function (CDPF). The figure given below captures how the production looks like in case of increasing/decreasing and constant returns to scale.

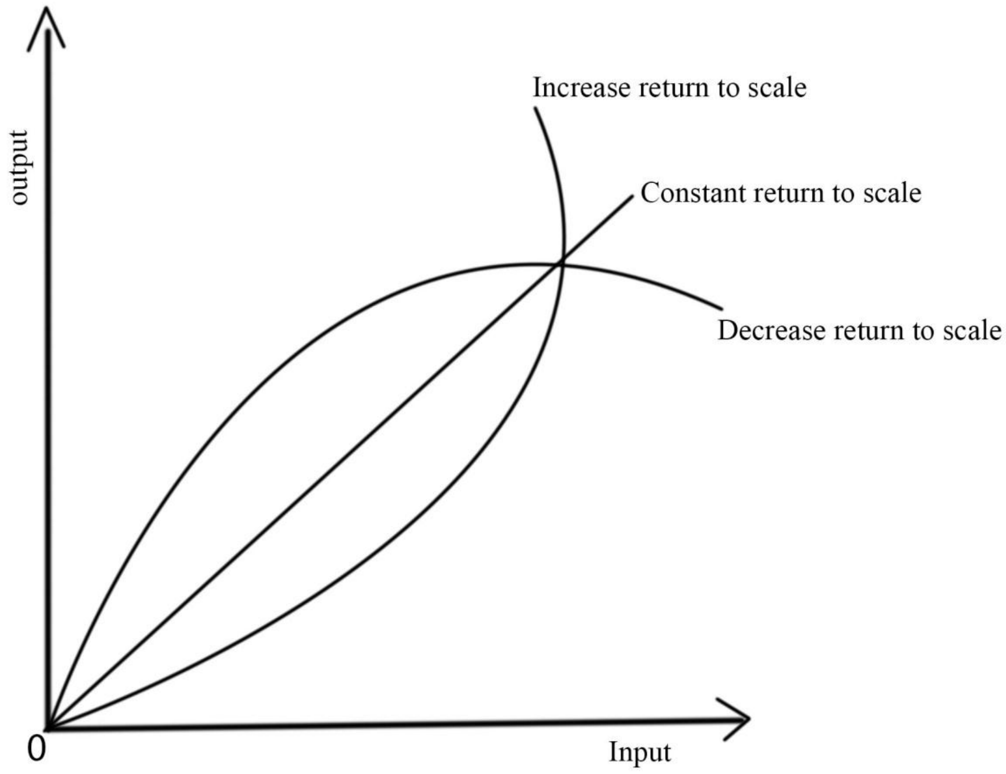


Figure 1.1 Return to scale graph

1.2 Statement of the Problem

In existing literatures, crashing cost function was either assumed without justification or come from ad-hoc regression analysis of historical data of some actual project. Hassani, (2012). Based project duration on labour and equipment only as CDPF.

This project will fill the gap in time-cost tradeoff research by considering three parameters namely labour, capital and raw materials which are very vital in construction.

1.3 Aim and Objectives of the Study

The aim of the research is to develop a new framework for time and cost scheduling in construction using Cobb-Douglas production function based on labour, equipment/capital and raw materials.

The objectives are to:

1. find optimum solution by analyzing different options in crashing duration of a project.
2. make sensitivity analysis on the parameters .
3. write a computer program using MATLAB 8.5 for the implementation.

1.4 Justification of the Study

Despite many researches on time and cost analysis or in crashing scheduling of time and cost, very little study can be found exploring or explaining the source of cost increase during activity crashing. The proposed research will have a good and wide insight of sources of cost increase using CDPF.

Evensmo and Karlsen (2008) were among few researchers who try to explain the origin of cost during activity crashing. A significant limitation in their approach is lack of considering change in labour, equipment and raw materials inputs during crashing.

1.5 Scope of the Study

The research will be limited to the following; using CDPF which can be used to explain many types of production activities, some important features of constant return to scale, increase return to scale and decrease return to scale and also genetic algorithm inputs during crashing.

1.6 Definition of Terms

1.6.1 Cobb-Douglas production function (CDPF)**:** This is the relationship between two or more inputs of factors of production.

1.6.2 Time-cost tradeoff problem (TCTP)**:** This is the process to identify suitable construction activities for speeding up and for deciding how much as to attain the best possible saving of both time and cost.

1.6.3 Production**:** Is the process of combining inputs to make outputs.

1.6.4 Production function**:** The relationship between input and output.

1.6.5 Capital**:** We mean fixed capital and working capital. This in particular include equipment.

CHAPTER TWO

LITERATURE REVIEW

2.1 Crashing Time and Cost Function

Various time and cost tradeoff problem scheduling models have been proposed in many literature, some were formulated by using linear programming or integer programming (Perera, 1980; Liu *et al.*, 1995; Moussarakis and Haksever, 2004). The relationship between cost and time has been well studied. Cost function such as linear (Bazaraa and Shetty 1979, Fulkersen 1961), non-linear (Moussarakis and Haksever 2010), discrete (Kelly, 1961) and convex (Lamberson & Hocking, 1970).

2.2 Objective Function

Paul Douglas explained that his first formulation of the Cobb-Douglas production function was developed in 1927. When seeking for a functional form to relate estimate he had calculated for workers and capital, he spoke with a mathematician and colleague Charles Cobb, who suggested a function in the form; $Y = AL^{\alpha} K^{1-\alpha}$

Where Y = output, A = Level of technology, L = Labour, K = Capital, α = elasticity of Labour and 1–α = elasticity of Capital.

Estimating the function using least squares, he obtained a result for exponents of labour and capital which was subsequently confirmed by the National Bureau of Economic Research (Cobb and Douglas; 1928).

Abeselom (2008) stated that, construction project cost management is the process which complements the broad function of estimating and tendering, scheduling cost control and financial control. Accordingly, contractors need to have a cost management system which spans from the tendering up to the completion stage which integrates estimating, tendering, budgeting and controlling. Construction cost management is the entire process, which ensures that the contract amount is within the cost limit of client’s approved budget (Eldash 2012).

According to PMBOK guidelines (2013), project cost is predominantly concerned with the cost of the resources require to complete schedule project activities during the speculated time, and this includes cost of using tendering, construction, maintaining and supporting results of the project

Existing research on construction TCTP can be classified on using different objective function based on heuristic method and mathematical programming model they proposed. Some researchers consider multi-objectives functions and they try to find the optimum solution and some minimize project cost.

Feng *et al.* (1997) proposed a genetic algorithm to draw a Pareto set for a discrete time-cost tradeoff problem. They consider a multi-objective criteria problem to find the optimum solution, which ended up in a Pareto set.

Aghaie and Mokhtari (2009) proposed a nonlinear mix integer programming to increase the probability of completion of the project in a given due date. They also assumed that each activity duration follow an exponential distribution. Ke *et al.*, (2009) proposed integrating stochastic simulation and genetic algorithm to increase the probability of completion of a project by a specific due date. While Cohen et al. (2007) wanted to minimize the expected cost related by the project in order to minimize cost.

Benjaoran and Sae-Tae (2011) considered minimization of the summation of the weighted deviation from the managing goals and each deviation is assigned to measure a particular aspect subject to project time, project cost, activity time and labor resources (Multi-objectives).

The assumption of constant labor share is necessary for the use of the Cobb-Douglas production function method, the abstraction of labor share movement over time across countries as in Denis et al. (2010) for instance may be debatable. In particular, the assumption of the constant labor share is not fully appropriate for a converging economy which has not yet reached its steady-state and in which GDP per capital is not growing at a rate of technological progress.

Construction project management incorporates set of project objectives which may be accomplished by implementing series of operations subject to resource restraints. It is a challenging task in practice and there may be potential conflicts between the specified activity with regard to time, cost, scope and quality, and the constraints imposed on all of the physical resources demanded. A project manager should require knowledge and attention that focuses on different areas from project cost, the manager is the one to identify required resources and keep budget control. Chris Hendrickson (2008).

Leu *et al.* (2001) determined project crashing regarding project total cost, which include direct and indirect cost.

2.3 Risk and Uncertainty

Uncertainty is a fundamental factor of human life. So all economic transactions involve an element of risk wherever uncertainty is, there is risk. It is essential to know the difference between risk and uncertainty. Risk is a situation where the possibility of happening (outcomes) or non-happening of an event can be qualified and measured. On the other hand, uncertainty is a situation where this possibility cannot be measured. Thus in a risky situation, there are more than one possible happening (outcomes). The risk-taker is aware of all possible happenings and know the probability of each outcome occurring (Chris, 2008).

Construction projects are mostly long term projects, so there are different sources of uncertainty which affect the decision of managers or decision makers. Indeed, how much a project takes longer to be done; it is more susceptible to be affected by market sources of uncertainty.

Monte-Carlo simulation was used in order to better understand the sensitivity of objective value, total budget needed to do the project, and the optimum duration considering stochastic labour and equipment cost and elasticity. Simulation works based on a model which tries to maximize the objective value. Indeed, there are two main approaches in the literature. Most authors analyze the risk associated with construction project management based on the probability of occurrence and the cost of occurrence (Zavadskas et al. 2008). Using historical data or in some cases they use expert recognition of a system. So, the approach is called data driven analysis. In some other cases, based on a model, authors try to analyze the systems which are called model driven analysis.

As Abeselom (2008) noted that, contractors on receipt of work tender, prepare cost estimate based on the estimates, they quote the estimated price of the works and then agrees to executing the work followed by drawing up their plan of work based on the quantities and cost reflected in the Bill of Quantities (BOQ) which forecast the contractors' commitment to resources, input cost and the profit which they expect. Once construction commences, contractors attempt to accomplish the work in a way that keep the cost in carrying the work, with the money that will be reimbursed to them as a result of valuation of completed works. These processes comprise the task which most contractors are involved and which need systematic approach.

Hassani (2012) considered total crashing cost for all activities in a network. The mathematical model, find a combination of labour and equipment, minimizing the total cost for all activities. So many other authors choose different approach to cope with the tine and cost in project crashing. Some also proposed an evolutionary algorithm to fit the problem of time and cost.

2.4 Models

The Cobb-Douglas function form can be estimated as linear relationship using the expresson.

$$In\ (Y) = \ a_i + \sum a\ \ In\ (I_i)$$

Where Y = output, I_i = inputs, a_i = model Coefficient

The model can also be written as

$$Y = (I_i)^{ai} \quad (I_2)^{a2} \ldots$$

As noted, the common Cobb-Douglas function used in macroeconomic modelling is

$$Y = \ L^{\alpha}\ \ K^{\beta}$$

Where k is capital and L is labour and the model exponents are sum to one.

Ghazanfari (2009) assumed fuzzy variables. Via Possibility Goal Programming, the cost was minimized while considering the minimum duration. The main contribution of this fuzzy approach is the use of vagueness in the cost function during the project execution. Zheng & Ng (2004) also presented fuzzy set theory regarding the uncertainty included in TCTP problem. They also used GA as a meta-heuristic algorithm to develop a Pareto set of time and cost.

2.5 Labour, Capital and Raw Material

Labour as one of the factors of production, it is perishable in nature. Labour actually means any type of physical or mental exertion. In economics term, labour is the effort exerted to produce any goods and services. It includes all types of human effort- physical exertion, mental, use of intellect, etc. done in exchange for an economic reward.

Labour is a unique factor of production in comparison with others. It is directly related to human effort unlike others. So there are certain special factors we must take into consideration when it comes to labour. Fair treatment of workers, rest time, suitable working environment, and idle time etc. are just some such factors (Jhingan, 2007).

Nevertheless, labour as a factor of production has a weak bargaining power with the buyers of the services. It cannot be stored, isn't very mobile and has no standard or reserve price. So generally labourers are force to work for whatever wages the employer offers. In comparison to the employer the labourer has very little bargaining power.

Capital has been defined as that part of person's wealth, other than land which yields an income or which aids in production of further wealth. Obviously, if wealth is unused or hoarded, it cannot be considered capital. In the ordinary language, capital is used in the sense of money, but when we talk of capital as factor of production, it is quite wrong to confuse capital with money. There is no doubt that money is a form of wealth and it yields income, when it is lent out. But it cannot be called capital.

Capital is man-made, it is therefore possible to increases its supply when the situation required, it involves the element time. That is why payment for capital is calculated per annum. The use of capital makes roundabout method of production possible. Its application increases efficiency and the production power of all the factors with which it is combined and used (Jhingan, 2007).

Moreover, capital may be divided into fixed capital and working capital. Fixed capitals are the durable-use producer goods which are used in production again and again till they wear out. Machinery, tools, railways, tractor, factories etc. are fixed capital or equipment.

According to Jhingan (2007), raw material, also known as a feedstock, unprocessed material or primary commodity is a basic material that is used to produce goods, finished products, feed stock for future finished products. As feedstock the term contains more materials that are bottleneck asserts and are highly important with regards to producing other products. An example of this is crude oil which is a raw material and a feedstock used in the production of industrial chemicals, fuels, plastics and pharmaceutical goods. Often a time managers and producers neglect the cost of these products, which by main looking is the most essential element in production.

CHAPTER THREE

METHODOLOGY

3.1 Problem Description

The deterministic approach is based on critical path method and has been the most widely used method in planning and controlling of the construction projects.

An objective function containing both cost and time (which are normalized to be comparable) will be used to do time-cost tradeoff analysis. Cobb-Douglas production function will be applied to each activity regarding labour, capital (equipment) and raw materials. Time and cost are used as the objective function which shows user priorities and preference for either duration or cost. The main idea of the study is from Hassani, A., (2012).

3.2 Model

In this research, we intend to assume discrete time-cost relationship. We shall obtain total cost of labour, capital and raw materials based on the amount of work for each activity. Considering the production function of an activity as

$$Q = (L, K, R) \tag{3.1}$$

where Q = production output rate, L = labour, K = Capital and R = Raw materials

From equation (1.1), we assume that the technology impact A is constant and equal to 1 for simplicity without affecting the result and conclusion. We are concerned with the allocation of labour, capital and raw materials during project crashing.

In Felipe and Adams (2005), $\alpha = 0.3$ and $\beta = 0.7$ are often reported based on the environment and the sum allocated. Since the values of A, α, β and γ are treated as parameter and will not affect the overall framework, we assume A = 1, $\alpha = 0.2$, $\beta = 0.4$ and $r = 0.4$ based on the environment and the total sum allocated on each parameter.

To get

$$Q = A\,L^{0.2}\,\mathrm{K}^{0.4}\,R^{0.4} \tag{3.2}$$

So cost function of an activity

$$TC = C_1 L + C_2 K + C_3\mathrm{R} \tag{3.3}$$

Where TC = total cost of activity, C_1L is the cost rate for labor input,C_2K is the cost rate for capital and C_3R is the cost rate for raw materials.

In this work, to minimize the cost of labour$C_1 L$, cost of capital $C_2 K$ and cost of raw materials $C_3\mathrm{R}$

Considering the production function of work needed

$$Q_0 = \frac{w}{t_0} \tag{3.4}$$

Where

Q_0 is normal production output rate

$w = production\ during\ t_0$

During crashing, we consider

$$Q_1\ \frac{w}{t_1} \tag{3.5}$$

Q_1 is crash production output rate from t_0 to t_1

During crashing again

$$Q_2 = \frac{w}{t_2} \tag{3.6}$$

Q_2 is crash production output rate from t_1 to t_2

3.3 Source of Data

Discrete time-cost relationship was assumed. The cost of labour, cost equipment/capital and cost of raw materials based on the amount of each activity shall be obtained. In this research work, data from IDD Blocks and Construction Data book 2017 was used.

The company was incorporated in August 2011 as IDD Blocks and Construction Nig. LTD.

It services range from:

1. Building construction and renovation.
2. Electrical services including solar power installation.
3. Landscaping and erosion control.
4. Water supply development and drainage.
5. Plumbing services including installation of sanitary fitting.
6. Supply of essential drugs and furniture.

   Backed by an experienced management team, they are recognize for their commitment to safety, sustainable development of the commutative and environment in which we operate.

They have a mission statement as to provide and deliver superior quality work, dependable services and cost effective prospect to their client with mutually beneficial result.

They weave their expertise around the area of construction management to the desire of their client.

We assigned value for work needed (w) which is defined base on decision maker preference.

Let assume w = 10000 for a specific activity and normal time t_0 = 20 days.

Considering the production function of the work

$$Q_0 = \frac{w}{t_0} = \frac{1000}{20} = 500$$

If crashing duration t_1 and t_2 equals to 16 and 10 days respectively, and supply of labour and raw materials increase, then

$$Q_1 = \frac{10000}{16} = 650$$

$$Q_2 = \frac{10000}{10} = 1000$$

The cost function is defined as

$TC = C_1L + C_2K + C_3R$

Where C_1L is the cost rate for labour input

C_2K is the cost rate for equipment/capital

C_3R is the cost rate for raw materials

To reduce activity from t_0 to t_1 and from $t_1\ to\ t_2$, output rate increases by $Q_1 - Q_0$ per time unit and also $Q_2 - Q_1$ per time unit to the extra input of labour or capital or raw materials.

3.4 Derivation of the new scheme

Given the Cobb-Douglas production function

$$Q = DL^{\alpha}\, K^{\beta}\, R^{r} \tag{3.7}$$

$$Subject\ to\ AL + BK + CR \tag{3.8}$$

$$where\ \ D = level\ of\ technology,$$

$$A = input\ of\ labour,$$

$$B = input\ of\ capital$$

$$C = input\ of\ raw\ material$$

$$\alpha, \beta\ and\ \text{г} = elasticity\ of\ labour, capital\ and\ raw\ material\ respectively$$

$$Forming\ the\ langrangian\ function\ (L^{*}), we\ have$$

$$L^{*} = AL + BK + CR + \lambda \left(Q - L^{\alpha} K^{\beta} R^{\text{г}}\right)$$

$$\lambda = Lagrangian\ Multiplier$$

$$\frac{\partial L^{*}}{\partial L} = A - \lambda \alpha L^{\alpha - 1} K^{\beta} R^{\text{г}} \qquad = 0 \tag{3.9}$$

$$\frac{\partial L^{*}}{\partial K} = B - \lambda \beta L^{\alpha} K^{\beta - 1} R^{\text{г}} = 0 \tag{3.10}$$

$$\frac{\partial L^{*}}{\partial R} = C - \lambda \text{г} L^{\alpha} K^{\beta} R^{\text{г} - 1} = 0 \tag{3.11}$$

$$\frac{\partial L^{*}}{\partial \text{л}} = Q - L^{\alpha} K^{\beta} R^{\text{г}} = 0 \tag{3.12}$$

From eqn 3.9 $\quad A = \lambda\, \alpha L^{\alpha - 1} K^{\beta} R^{\text{г}}$

$$\lambda = \frac{A}{\alpha L^{\alpha - 1} K^{\beta} R^{\text{г}}} \tag{3.13}$$

From eqn 3.10 $B = \lambda\beta L^{\alpha} K^{\beta-1} R^{\Gamma}$

$$\lambda = \frac{B}{\beta L^{\alpha} K^{\beta-1} R^{\Gamma}} \tag{3.14}$$

From eqn3.11 $C = \lambda\Gamma L^{\alpha} K^{\beta} R^{\gamma-1}$

$$\lambda = \frac{C}{\Gamma L^{\alpha} K^{\beta} R^{\Gamma-1}} \tag{3.15}$$

Equating eqn (3.13) and (3.14)

$$\frac{A}{\alpha L^{\alpha-1} K^{\beta} R^{\Gamma}} = \frac{B}{\beta L^{\alpha} K^{\beta-1} R^{\Gamma}}$$

$$\frac{A}{B} = \frac{\alpha L^{\alpha-1} K^{\beta} R^{\Gamma}}{\beta L^{\alpha} K^{\beta-1} R^{\Gamma}}$$

$$\frac{A}{B} = \frac{\alpha L^{\alpha-1-\alpha} K^{\beta-(\beta-1)} R^{\Gamma-\Gamma}}{\beta} \text{B}\alpha$$

$$\frac{A}{B} = \frac{\alpha L^{-1} K^{\beta-\beta+1} R^{0}}{\beta} \qquad but\ R^{0} = 1$$

$$\frac{A}{B} = \frac{\alpha L^{-1}}{\beta} K$$

$$\frac{A}{B} = \frac{\alpha K}{\beta L}$$

Making L the subject, we have

$$A\beta L = B\alpha K$$

$$L = \frac{B\alpha K}{\beta A} \tag{3.16}$$

Euating eqn (3.12)and (3.13)

$$\frac{B}{\beta L^{\alpha} K^{\beta-1} R^{г}} = \frac{C}{г L^{\alpha} K^{\beta} R^{г-1}}$$

cross multiply

$$\frac{B}{C} = \frac{\beta L^{\alpha} K^{\beta-1} R^{г}}{{}^{\gamma}L^{\alpha} K^{\beta} R^{г-1}}$$

$$\frac{B}{C} = \frac{\beta L^{\alpha-\alpha} K^{\beta-1-\beta} R^{г(г-1)}}{г}$$

$$\frac{B}{C} = \frac{\beta L^{0} K^{-1} R}{\text{г}} \qquad But\ L^{0} = 1$$

$$\frac{B}{C} = \frac{\beta R}{\text{г} K}$$

Making k the subject, we have

$$B^{\gamma} K = C\beta R$$

$$K = \frac{C\beta R}{B\text{г}} \tag{3.17}$$

Equating eqn (3.11)and (3.13)

$$\frac{A}{\alpha L^{\alpha-1} K^{\beta} R^{\text{г}}} = \frac{C}{\beta L^{\alpha} K^{\beta} R^{\text{г}-1}}$$

$$\frac{A}{C} = \frac{\alpha L^{\alpha-1} K^{\beta} R^{\text{г}}}{\beta L^{\alpha} K^{\beta} R^{\text{г}-1}}$$

$$\frac{A}{C} = \frac{\alpha L^{\alpha-1-\alpha} K^{\beta-\beta} R^{\text{г}-(\text{г}-1)}}{\text{г}}$$

$$\frac{A}{C} = \frac{\alpha L^{-1} K^{0} R}{\Gamma} \qquad K^{0} = 1$$

$$\frac{A}{C} = \frac{\alpha R}{{}^{\gamma}L}$$

Making R the subject, we have

$$R = \frac{A\Gamma L}{\alpha C} \tag{3.18}$$

We can now

$min.\ C_1 L + C_2 K + C_3 \mathrm{R}$

Such that $Q_i = \mathrm{A}L^{\alpha} K^{\beta} R^{\Gamma}$ (3.19)

We defined the objective function as multi-objective which contain cost and duration

$$Z = W_I C_I + W_I T_I \tag{3.20}$$

Where W is defined based on decision maker preference,

$Ci = \frac{max.Cost - Cost\ Solution}{Max.Cost - Min.Cost}$ and $Ti = \frac{Max.time - Time\ Solution}{Max.Time - Min.Time}$

$Ci\ and\ Ti$ are normalized scores of cost and duration.

Note;

$$Max.Cost = \sum (TC\ Associated\ with\ the\ first\ OPtion)$$

$$Cost\ Solution = \sum (selected\ Total\ Cost\ TC)$$

$$Min.Cost = \sum (TC\ Associated\ with\ the\ third\ OPtion)$$

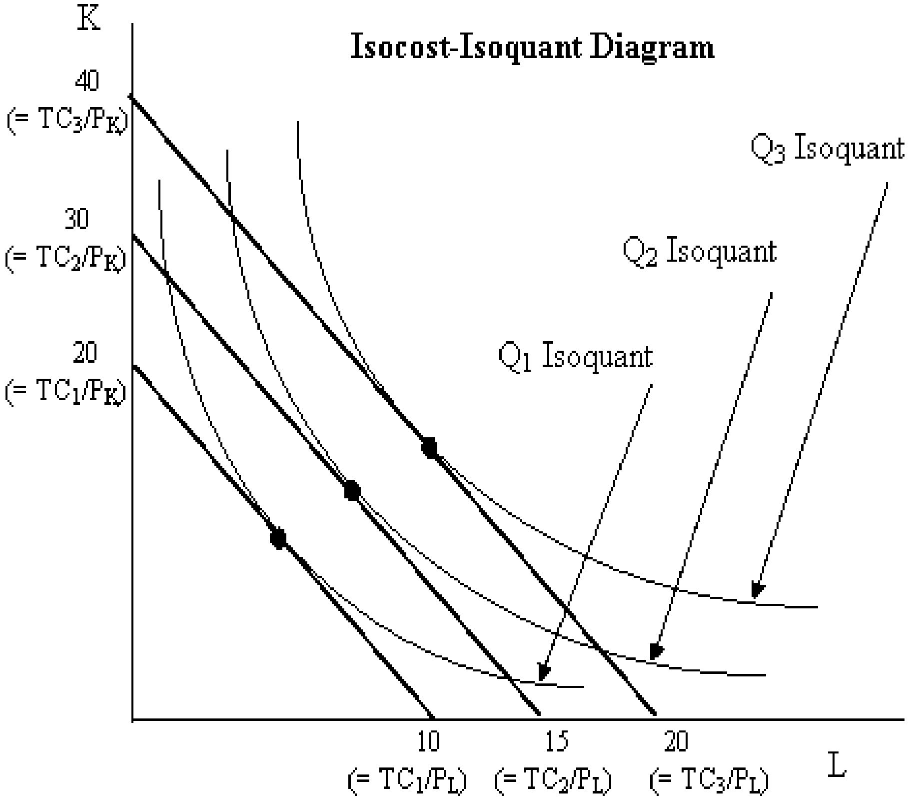
$$Max\ Time = \sum(TC\ Associated\ with\ the\ first\ OPtion)$$

$$Time\ Solution = \sum(TC\ Associated\ with\ the\ first\ OPtion)$$

$$Min.Cost. = \sum(TC\ Associated\ with\ the\ first\ OPtion)$$

3.5 Isocost and Isoquant

Isocost curve is a producer budget line while isoquant are curves that show all combinations of inputs that yield the same amount of output.



Simple example of finding minimal activity cost using Isocost and Isoquant.

The isocost lines when combined with the isoquant map is use to determine the optimal production point at any given level of output. These curves will be used to determine optimal level of combination of the parameters.

3.6 Hybrid Genetic Algorithm

Hybrid Genetic Algorithm shall be developed to analyze the best allocation of labour, equipment/capital and raw materials. We shall adopt Hassani & Ashkan (2012) method of duration and activity network, but we shall assign worked load to each activity in order to use CDPF.

3.7 System Specification

All codes are written in MATLAB 8.5 and run on a HP 520 PC with 2.0 GHz, 2 GB RAM.

CHAPTER FOUR

RESULT AND DISCUSSION

4.1 Introduction

This section contains the discussion of the result and performance of Cobb-Douglas function considering labour, equipment and raw materials in constructing time cost analysis.

We want to demonstrate the Cobb-Douglas production function; we adopted the Network used by Liu et al 1995 (which was adopted by so many). The optimal duration of project is obtained from the critical path.

Table 4.1: Optimal Solution Option

Activity	1	2	3	4	5	6	7
Option	3	1	1	3	3	3	2

For each activity, we assigned estimated work load (W) to each activity in order to use CDFF considering the three parameters Labour, equipment/capital and raw materials and also identify Unit Cost for Labour (CL) equipment/capital (CK) and Raw Materials (CR) after knowing W for each activity, associated with Q of each of the options obtained in equation 3.4

4.2 Example

Given a Cobb-Douglas production Function as

$$Min. 204.5 = L^{0.2} K^{0.4} R^{0.4}$$

$$subject\ to; 60L + 60K + 100R$$

Solution

$$Min. 204.5 = L^{0.2} K^{0.4} R^{0.4} \tag{4.1}$$

$$subject\ to; 60L + 60K + 100 \tag{4.2}$$

Forming the Langrangian function (L^*) and inroducing the multiplier (λ)

we have;

$$L^{*} = 60L + 60K + 100R + \lambda\,(204.5 - L^{0.2}K^{0.4}R^{0.4})$$

$$\frac{\partial L^{*}}{\partial L} = 60 - 0.2L^{-0.8}K^{0.4}R^{0.4}\lambda = 0$$

$$60 = 0.2L^{-0.8}K^{0.4}R^{0.4}\lambda$$

Making λ the subject

$$\lambda = \frac{60}{0.2L^{-0.8}K^{0.4}R^{0.4}} \qquad (4.3)$$

$$\frac{\partial L^{*}}{\partial K} = 60 - 0.4L^{0.2}K^{-0.6}R^{0.4}\lambda = 0$$

$$60 = 0.2L^{-0.8}K^{0.4}R^{0.4}\lambda$$

Making λ the subject

$$\lambda = \frac{60}{0.4L^{0.2}K^{-0.6}R^{0.4}} \qquad (4.4)$$

$$\frac{\partial L^{*}}{\partial R} = 100 - 0.4L^{0.2}K^{0.4}R^{-0.6}\lambda = 0$$

Making λ the subject

$$\lambda = \frac{100}{0.4L^{0.2}\,K^{0.4}R^{-0.6}} \tag{4.5}$$

$$\frac{\partial y}{\partial x} = 204.5 - L^{0.2}\,K^{0.4}\,R^{0.4} \tag{4.6}$$

equating equation 4.3 and 4.4

$$\frac{60}{0.2L^{-0.8}\,K^{0.4}R^{0.4}} = \frac{60}{0.4L^{-0.8-}K^{-0.6}R^{0.4}}$$

$$24L^{0.2}\,K^{-0.6}R^{0.4} = 12L^{-0.8}\,K^{0.4}R^{0.4}$$

$$\frac{24L^{0.2}\,K^{-0.6}R^{0.4}}{12L^{-0.8}\,K^{0.4}R^{0.4}} = 0$$

$$2L\,K^{-1}R^{0} = 1 \qquad But\ R^{0} = 1$$

$$2L\,K^{-1} = 1$$

$$\frac{2L}{K} = 1$$

$$K = 2L \tag{4.7}$$

equating equation 4.3 and 4.5

$$\frac{60}{0.2L^{-0.8}\,K^{0.4}R^{0.4}} = \frac{100}{0.4L^{0.2}\,K^{0.4}R^{-0.6}}$$

$$24K^{0.4}R^{-0.6} = \; 20L^{-0.8}\,K^{0.4}R^{0.4}$$

$$\frac{24L^{0.2}\,K^{0.4}R^{-0.6}}{20L^{-0.8}\,K^{0.4}R^{0.4}} = 0$$

$$1.2L\,K^{0}R^{-1} = 1 \qquad\qquad But\,K^{0} = 1$$

$$1.2L\,R^{-1} \; = 1$$

$$R = 1.2\,L \tag{4.8}$$

substituting equation 4.7 and 4.8 into equation 4.6

$$204.5 - \; L^{0.2}\,(2L)^{0.4}\,(1.2)^{0.4}$$

$$205.4 - 1.419L = 0$$

$$205.4 = 1.419L$$

Making L the subject. we have

$$L = \frac{205.4}{1.4193}$$

$$L = 1.4408$$

Substituting L imto equation 4.5, we have

$$K = 2(1.4408)$$

$$K = 288.16$$

substituting L into equation 4.6 , we have

$$R = 1.2(1.4408)$$

$$R = 172.90$$

we continue by substituting the next quantity which is 187.5 into equation 4.1

following the same procedure, we have

$$L = 132.11$$

$$K = 264.22$$

$$R = 158.53$$

repeating same procedure when the quantity is 160.7, we have

$$L = 113.22$$

$$K = 226.45$$

$$R = 135.86$$

Summary of the Solution

Quantity	**L**	**K**	**R**
205.4	144.08	288.16	158.53
187.5	132.11	264.22	158.53
160.7	113.32	226.45	135.86

Table 4.2**:** The Case Project Information

Activity	**Options**	**W**	**Days**	**Q**	**CL**	**CK**	**CR**
1	1	4500	22	204.5	60	60	100
1	2	4500	24	187.5	60	60	100
1	3	4500	28	160.7	60	60	100
2	1	5000	14	357.1	40	100	60
2	2	5000	20	250	40	100	60
2	3	5000	24	208.3	40	100	60
3	1	5000	15	333.3	50	70	60
3	2	5000	18	277.8	50	70	60
3	3	5000	20	250	50	70	60
4	1	4700	9	522.2	65	30	40
4	2	4700	15	313.3	65	30	40
4	3	4700	18	261.1	65	30	40
5	1	6000	15	400.1	45	80	75
5	2	6000	22	272.2	45	80	75
5	3	6000	33	181.8	45	80	75
6	1	6000	12	500	75	70	80
6	2	6000	16	375	75	70	80
6	3	6000	20	200	75	70	80
7	1	5500	14	196.4	55	20	30
7	2	5500	18	392.9	55	20	30
7	3	5000	28	305.6	55	20	30

The amount of labor L, capital K and Raw materials R. are determined according to the cost minimization function constrained to Cobb-Douglas function presented in Equation (3.17)

Table 4.3: The Quantity and Amount for L, K and R

Q	**CL**	**CK**	**CR**	**L**	**K**	**R**
204.5	60	60	100	144.1	288.2	172.9
187.5	60	60	100	132.1	264.2	185.5
160.7	60	60	100	113.2	226.5	135.9
357.1	40	100	60	348.3	278.7	463.2
250	40	100	60	242.7	194.2	322.8
208.3	40	100	60	202.2	161.8	269.9
333.3	50	70	60	235.4	336.6	392.9
277.8	50	70	60	190.3	272.1	317.2
250	50	70	60	176.6	252.5	294.8
522.2	65	30	40	181.8	781.9	590.9
313.3	65	30	40	109.2	469.4	354.9
261.1	65	30	40	261.1	391.2	295.8
400.1	45	80	75	354.8	399.2	425.8
272.2	45	80	75	241.3	271.5	425.8
181.8	45	80	75	142	159.8	170.4
500	75	70	80	286.8	613.8	537.8
375	75	70	80	215.5	412	404.1
200	75	70	80	114.9	246	215.4
196.4	55	20	30	59	324.7	216.4
392.9	55	20	30	118	648.9	431.9
305.6	55	20	30	91	504.7	336

Table 4.4: Total Labor Cost, Total Capital Cost and Total Material Cost

Options	**W**	**Days**	**Q**	**CL**	**CK**	**CR**	**L**	**K**	**R**	**TLC**	**TKC**	**TRC**	**TC**
1	4500	22	204.5	60	60	100	144.1	288.2	172.9	8646	17292	17290	43228
2	4500	24	187.5	60	60	100	132.1	264.2	185.5	7926	15852	18550	42328
3	4500	28	160.7	60	60	100	113.2	226.5	135.9	6792	13590	13590	33972
1	5000	14	357.1	40	100	60	348.3	278.7	463.2	13932	27870	27792	69594
2	5000	20	250	40	100	60	242.7	194.2	322.8	9708	19420	19368	48496
3	5000	24	208.3	40	100	60	202.2	161.8	269.9	8088	16180	16194	40462
1	5000	15	333.3	50	70	60	235.4	336.6	392.9	11770	23562	23574	58906
2	5000	18	277.8	50	70	60	190.3	272.1	317.2	9515	19047	19032	47594
3	5000	20	250	50	70	60	176.6	252.5	294.8	8830	17675	17688	44193
1	4700	9	522.2	65	30	40	181.8	781.9	590.9	11817	23457	23636	58910
2	4700	15	313.3	65	30	40	109.2	469.4	354.9	7098	14082	14196	35376
3	4700	18	261.1	65	30	40	261.1	391.2	295.8	16971.5	11736	11832	40539.5
1	6000	15	400.1	45	80	75	354.8	399.2	425.8	15966	31936	31935	79837
2	6000	22	272.2	45	80	75	241.3	271.5	425.8	10858.5	21720	31935	64513.5
3	6000	33	181.8	45	80	75	142	159.8	170.4	6390	12784	12780	31954
1	6000	12	500	75	70	80	286.8	613.8	537.8	21510	42966	43024	107500
2	6000	16	375	75	70	80	215.5	412	404.1	16162.5	28840	32328	77330.5
3	6000	20	200	75	70	80	114.9	246	215.4	8617.5	17220	17232	43069.5
1	5500	14	196.4	55	20	30	59	324.7	216.4	3245	6494	6492	16231
2	5500	18	392.9	55	20	30	118	648.9	431.9	6490	12978	12957	32425
3	5000	28	305.6	55	20	30	91	504.7	336	5005	10094	10080	25179

Table 4.5: Summary case project information

Activity	**Options**	**W**	**Days**	**Q**	**CL**	**CK**	**CR**	**L**	**K**	**R**	**TLC**	**TKC**	**TRC**	**TC**
1	1	4500	22	204.5	60	60	100	144.1	288.2	172.9	8646	17292	17290	43228
1	2	4500	24	187.5	60	60	100	132.1	264.2	185.5	7926	15852	18550	42328
1	3	4500	28	160.7	60	60	100	113.2	226.5	135.9	6792	13590	13590	33972
2	1	5000	14	357.1	40	100	60	348.3	278.7	463.2	13932	27870	27792	69594
2	2	5000	20	250	40	100	60	242.7	194.2	322.8	9708	19420	19368	48496
2	3	5000	24	208.3	40	100	60	202.2	161.8	269.9	8088	16180	16194	40462
3	1	5000	15	333.3	50	70	60	235.4	336.6	392.9	11770	23562	23574	58906
3	2	5000	18	277.8	50	70	60	190.3	272.1	317.2	9515	19047	19032	47594
3	3	5000	20	250	50	70	60	176.6	252.5	294.8	8830	17675	17688	44193
4	1	4700	9	522.2	65	30	40	181.8	781.9	590.9	11817	23457	23636	58910
4	2	4700	15	313.3	65	30	40	109.2	469.4	354.9	7098	14082	14196	35376
4	3	4700	18	261.1	65	30	40	261.1	391.2	295.8	16971.5	11736	11832	40539.5
5	1	6000	15	400.1	45	80	75	354.8	399.2	425.8	15966	31936	31935	79837
5	2	6000	22	272.2	45	80	75	241.3	271.5	425.8	10858.5	21720	31935	64513.5
5	3	6000	33	181.8	45	80	75	142	159.8	170.4	6390	12784	12780	31954
6	1	6000	12	500	75	70	80	286.8	613.8	537.8	21510	42966	43024	107500
6	2	6000	16	375	75	70	80	215.5	412	404.1	16162.5	28840	32328	77330.5
6	3	6000	20	200	75	70	80	114.9	246	215.4	8617.5	17220	17232	43069.5
7	1	5500	14	196.4	55	20	30	59	324.7	216.4	3245	6494	6492	16231
7	2	5500	18	392.9	55	20	30	118	648.9	431.9	6490	12978	12957	32425
7	3	5000	28	305.6	55	20	30	91	504.7	336	5005	10094	10080	25179
Optimum Solution			146								70963	119740	119757	310460

$$C = \frac{Max\ coxt - cost\ solution}{Max\ cost - \min cost}$$

$$= \frac{434206 - 310460}{434206 - 259369}$$

$$C = \frac{123746}{174837} = 0.708$$

$$T = \frac{maxtime - time\ solution}{\max time - \min time}$$

$$\frac{171 - 146}{171 - 101} = \frac{25}{70} = 0.357$$

The objective value of the solution is calculated using equation 3.19 and 3.20.

As stated earlier in real world problem the cost of capital and raw materials are jointly added together (in a case that we do not consider utility function)

W = 0.8 and W_2 = 0.2

Where W, goes with cost of structure

W_2 goes with time

$Z_i = W_1 C_i + W_2 T_i$

$Z_i = 0.8 \times 0.708 + 0.2 \times 0.357$

$Z_i = 0.638$

It will be observed that C_1 is approaching 1 and T is approaching 0, which means that the decision maker (manager) considers both cost and time equally. So the optimal solution identified using HGA will depends on the decision maker's preference of time or cost.

However, Note that changes of W_1 affect the objective Z value. When W_1 or W_2 becomes 1, it means that the decision maker (manager) prefer cost either than time.

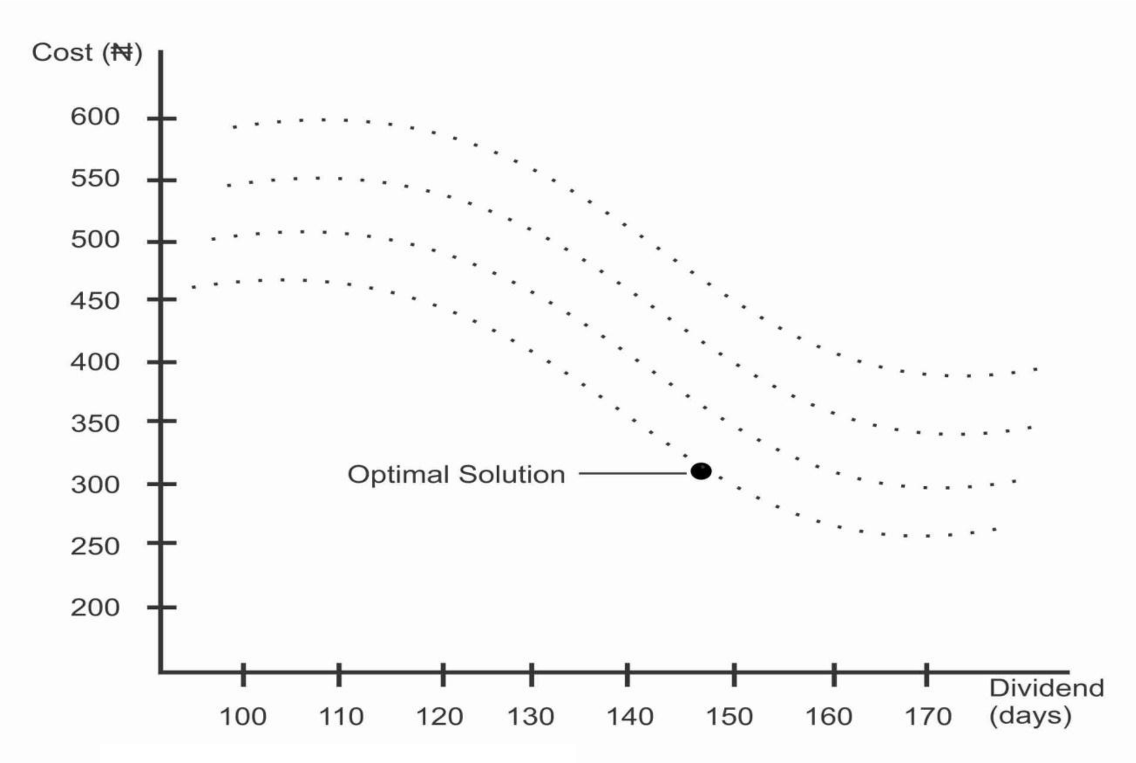


Figure 4.1 Optimal Solution Graph

4.2 Sensitivity Analysis

Duration of activities discussed as either fixed or random numbers with known characteristics, in this section, we do sensitivity analysis on project cost and time. However, activity duration can often vary depending upon the type and amount of resources that are applied. Assuming mere workers to a particular activity will normally result in higher cost and lower quality. This is to say time and cost have an impact in any project activity.

4.2.1 Activity time-cost relationship

There is a trade-off between time and cost to complete an activity. The less expensive the resources, the larger duration they take to complete an activity. Shortening the duration on an activity which will normally increase it direct cost capital and materials. It should never be assumed that the quality of resources deployed and the task duration are inversely related. This one should never be automatically assumed= that the work that can be done by one man in 20 weeks can actually be done by twenty men in one week.

A simple representation of the relationship between the duration of an activity and it direct cost appears in figure …… considering only this activity in isolation and without reference to the project completion deadline. A manager would choose duration which implies minimum direct cost called the normal duration. At the other extreme, a manager might choose to complete the activity in the minimum possible time called crashed duration but at a maximum cost.

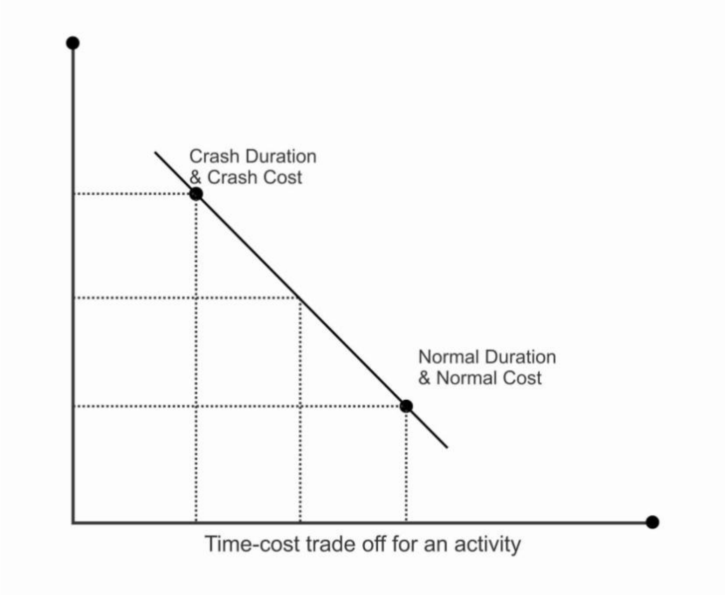


Figure 4.2 Time Cost Trade-off for an activity

4.2.2 Project Time Cost

Total project cost includes both direct cost and indirect cost of performing the activity project. Direct cost for the project includes the cost of materials, labor, equipment and sub-contractors. Indirect cost on the other hand are necessary cost of doing work which cannot be related to a particular activity and in some cases cannot be related to a specific project.

If each activity was scheduled for the duration that resulted in the minimum direct cost in this way, the time to complete the entire project might be too long and substantial penalties associated with the late project completion might be incurred. These planners perform what is called time-cost trade-off analysis to shorten the project duration. This paper focused on using CDPF associated with labor, equipment/capital and raw materials by selecting some activities on the critical path to shorten their duration and cost

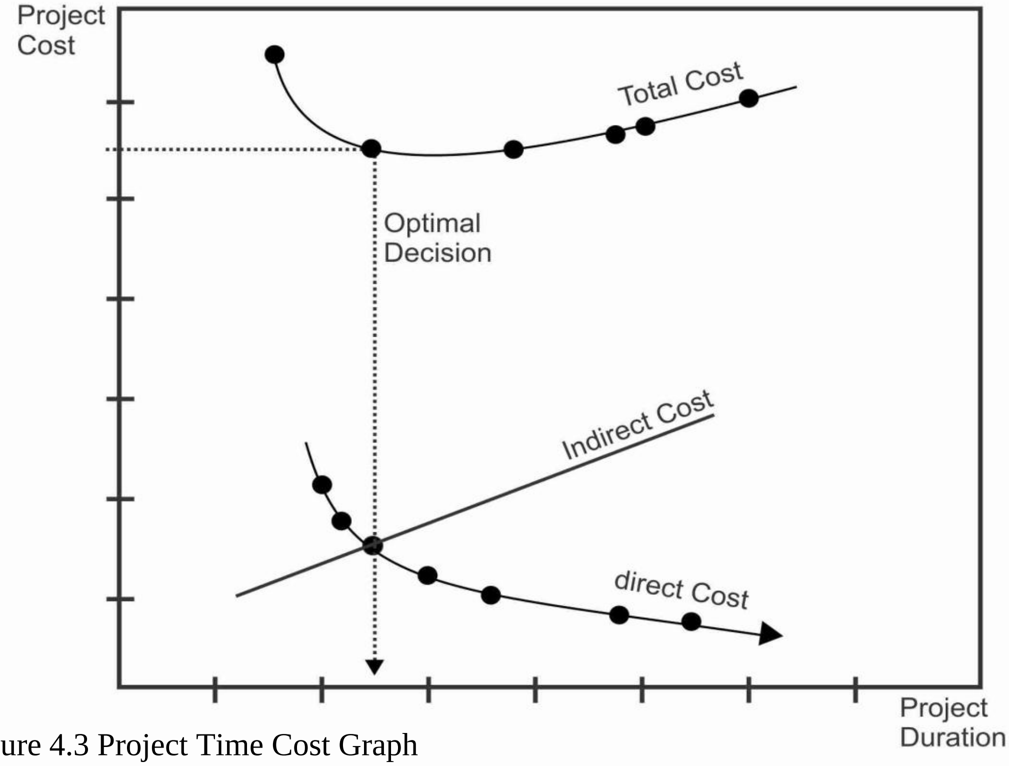


Figure 4.3 Project Time Cost Graph

4.3 Uncertainties (Market Influence)

Market can influence the cost of labour, equipment or raw material, so we consider uncertainties as a factor that can affect the result, according to equation 3.3 CL, CK and CR are considered to stochastic ones.

Based on expert point of views, we assume that there would be an inflation rate of 5 to 15% for labour, 3 to 5% for equipment/capital and 5 to 10% for raw materials.

Table 4.6: High and Low Inflation Rate

Scenario	**Total Cost**	**Labor Cost**	**Capital Cost**	**Raw material Cost**
Cost	310460	70963	119740	119757
High Inflation rate (15%)	46569	10644.45	17961	17963.55
Low inflation rate (5%)	15523	3548.15	5987	5987.85

From table 6, the total cost of each parameter is calculated, the manager or the decision maker have the choice to plan against uncertainties that may arise due to market influence. When inflation rate increases to the highest of 15% (Maximum inflation rate) which amounts to 434206 for the three parameters labor, equipment/capital and raw material. Equally, when the inflation rate increases to 5% (minimum inflation rate expected), which amounted to 259369 for the three parameters.

4.4 Internal or External Risk

The duration tend to be 146 days when $\alpha + \beta + \gamma$ goes to 1. Moreover, if the elasticity of labour, equipment/capital and raw material decreases significantly, the total cost arises irrationally.

According to the fact that there are thousands of points in each figure and it seems that the minimum of the total cost figure is 0. Let consider maximum and minimum cost under decrease return to scale and inflation rate of 5%.Table: 4.7 Minimum and Maximum Cost and Inflation

Table 4.7: Maximum and Minimum Cost

Parameter	**Minimum Cost**	**Maximum Cost**
Labor	60694	88886
Capital	99279	173577
Raw Material	99396	173743
Total	**259369**	**434206**

4.5 Isocost and Isoquant Graphs

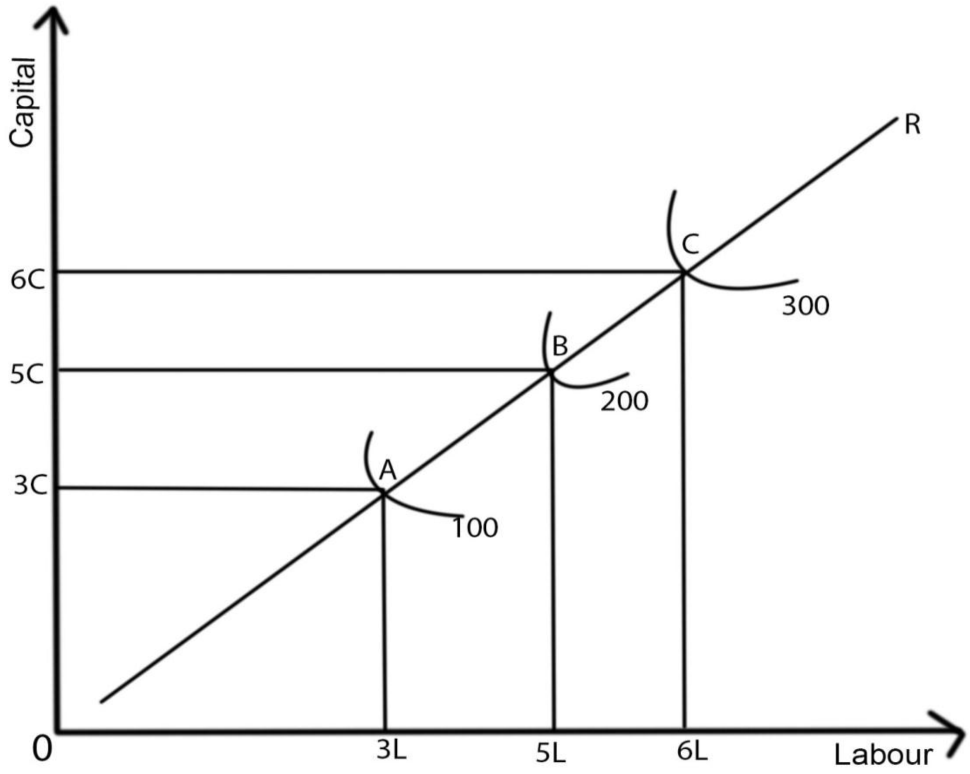


Figure 4.4 Issocost Graph

The graph above shows the case of increasing return to scale, where to get equal increases in output, lesser proportionate increase in labor, equipment/capital and raw material. In the expansion path OR, OA>AB>BC. The increase returns to scale are to the existence of indivisibles in machines, management, labor, finance etc. some equipment or some activities have a minimum size of activities and cannot be divided into small unit.

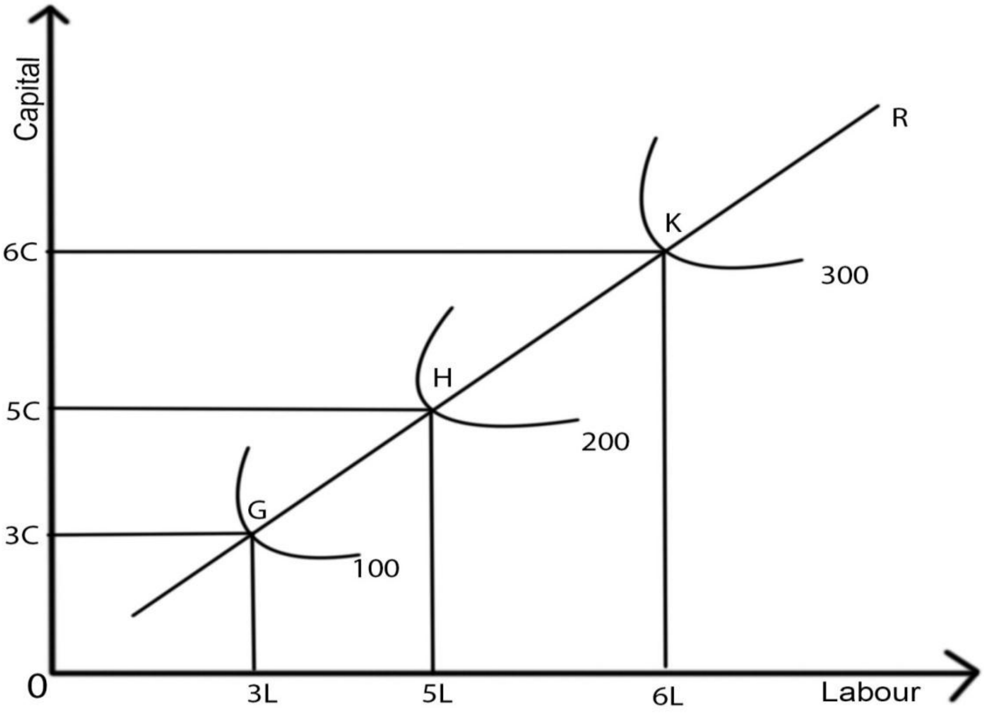


Figure 4.5 Isocost Graph

Show the case of decrease return to scale, where to get, and larger proportional increase in labour capital raw materials.

Along the expansion path OR, OG<GH<HK, but return to scale may start to diminished due to some factors like problem of management, low maintenance of machines etc.

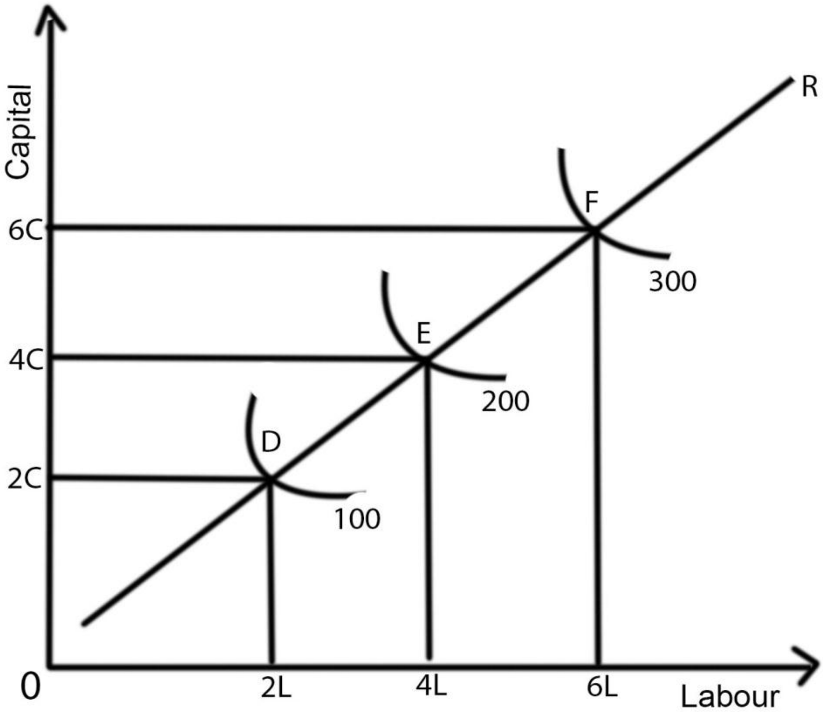


Figure 4.6 Isocost Graph (constant)

The above graph also shows the case of constant return to scale, where the distance between the isoquant 100, 200 and 300 along the expansion path OR is the same (i.e. OD=DE=EF). It means that if unit of both factors of labor and capita are double. The return to scale is constant when internal economics enjoyed by a firm are neutralized by internal diseconomies so output increases in the same proportion.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

We developed a new framework using Cobb-Douglas production function (CDPF) considering three parameters (Labour, equipment/capital and raw materials) in construction of time cost analysis by using Hybrid Genetic Algorithm to find the optimal allocation of L, K, and R sensitive analysis on activity time-cost relation, project time cost relation and uncertainties which may arise from market influence were considered after which we compared our result with other method using MATLAB 8.5.

5.2 Conclusion

In conclusion, we have developed a framework using three parameters to CDPF into time-cost trade of problems. The idea and techniques of introducing labour, capital and raw material cost in CDPF yielded or provides us with extra capacity to identify the optimal allocation of labour, equipment and raw materials during crashing. The expected approach is expected to work efficiently in large and complex applications.

5.3 Recommendations

In this project we have developed a framework for time and cost scheduling in construction using Cobb – Douglas production function based on labour, capital and raw materials. We hereby recommend this method for further research, like most studies in the literature review, Technology impact is presumed to be 1 that is no change in the level of technology. In reality, there is high significant improvement in technology day in day out many types of machines are launched to do a task in a shorter time. Therefore, technology impact should not be presumed as 1in any time nearby.

5.4 Contribution to Knowledge

This research has led to the following contribution:

i. A new framework using three parameters in Cobb-Douglas production function in crashing time cost tradeoff problem has been obtained
ii. Sensitivity analysis on uncertainties and project time cost relationship.
iii. MATLAB code is written for the implementation of the method.

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APPENDIX A
Simulation Code

```
clear
clc
```

*** According to the fact that it needs to do some optimization stuff in this problem in every iteration of the simulation, it needs to set optimization assumptions correctly regarding the problem.

```
opts = optimset('fmincon');
opts.LargeScale = 'off'; %
opts.MediumScale = 'on';
opts.TolFun = 1.e-6;
  format long
global Q
global alpha
global beta
global gama
global act
global cost
global rep
```

*** N: number of iterations

```
  N=1000;
```

*** cost and time priorities are defined.

```
  w1=0.4;
w2=0.4;
w3=1-
(w1w2);

SERI=zeros(N,14);
% alpha=0.2
% beta=0.4
     % gama = 0.4
```

*** In the new problem, according to the fact that alpha, beta, labor and equipment costs are stochastic, and they are generated randomly in their predefined ranges in each iteration, total cost is achivied after optimization calculations. So table here is different from the previous problem and the third row is empty which is set after optimization calculations.

```
Table = [1 22
```

```
1    24
1    28
2    14
2    20
2    24
3    15
3    18
3    20
4    9
4    15
4    18
5    15
5    22
5    33
6    12
6    16
6    20
7    14
7    18
7     28];
```

*** For the optimization purpose, regarding the total number of activities, initial points are defined.

```
X000=[100 400 500
    150 350 450
    200 300 400
    50 60 70
    30 40 50
    45 55 65
    65 70 80
    75 40 60
    20 30 50
    200 400 450
    300 350 500
    290 310 340
    440 520 600
    250 350 400
    270 330 390
    230 300 330
    280 180 80
    170 70 30
```

```
    125 145 155
    175 335 3385
    180 200 220];
```

*** For each activity option in case that labor and equipment costs are determined, FACTORCOST shows these costs.

```
FACTORCOST=[60 60 100
60 60 100];
      60 60 100
     40 100 60
     40 100 60
     40 100 60
     50 70 60
     50 70 60
     50 70 60
     65 30 40
     65 30 40
     65 30 40
     45 80 75
     45 80 75
     45 80 75
     75 70 80
     75 70 80
     75 70 80
     55 20 30
     55 20 30
     55 20 30];
```

*** TotalQ shows the Q rate of each activity's option which is achieved from w/t.

```
TotalQ=[204.5
187.5
160.7
357.1
250.1
```

```
208.3
333.3
277.8
250.0
522.2
313.3
261.1
400.1
272.2
181.8
500.0
575.0
200.0
196.4
392.9
305.6];

%   albetagama(rep)=1;
%     alpha(rep)=0.4;
%     beta(rep)=0.4
%     gama (rep)=albetagama(rep)-[alpha(rep)+beta(rep);
%
%
% for act=1:21
%         act;
%         x0=X00(act,:);
%          A=[ ];
%          b=[ ];
%          Aeq=[ ];
%          beq=[ ];
%           lb=[ ];
%           ub=[ ];
%         cost(act,:)=FACTORCOST(act,:);
%         Q(act)=TotalQ(act);
%         x(act,:)= fmincon(@Cobb,x0,A,b,Aeq,beq,lb,ub,@mycons);
%      %   x(act,:)= fmincon(@Cobb,x0,A,b,Aeq,beq,@mycons)
%         %x = fmincon(@(x)cost(1)*x(1)+cost(2)*x(2),x0,A,b,Aeq,beq,lb,ub,@mycons);
%         LaborCost(act)=x(act,1)*cost(act,1);
%         CapitalCost(act)=x(act,2)*cost(act,2);
%         Table(act,3)=LaborCost(act)+CapitalCost(act);
%
% end
```

*** the simulation process starts from this point.

```
 for rep=1:N
rep;

% INFL(rep,1)=rand()*0.05+1.10; %
INFL(rep,2)=rand()*0.05+1.10;

% INFL(rep,1)=rand()*0.10+1.05;
% INFL(rep,2)=1;



INFL(rep,1)=1;
INFL(rep,2)=1;



%INFL(3)=rand()*0.03+1.03;
%INFL(4)=rand()*0.03+1.04;
SERI(rep,15)=INFL(rep,1);
SERI(rep,16)=INFL(rep,2);
%A(rep)=2;
%while A(rep)>1
```

*** One of the main steps in the simulation process is to define alpha, beta and summation of those. Based on the predefined ranges for each of those, random numbers are generated here.

```
    albeta(rep)=0.4*rand()+0.6;      alpha(rep)=(0.2*rand()+0.2)*albeta(rep);
beta(rep)=albeta(rep)-alpha(rep);

if albeta(rep)>=0.6 && albeta(rep)<0.7
w1=0.1; else
   if albeta(rep)>=0.7 && albeta(rep)<0.8          w1=0.2;
else        if albeta(rep)>=0.8 && albeta(rep)<0.9
w1=0.2;        else             w1=0.4;         end     end end
            w2=1-w1;



%    albeta(rep)=1;
%    alpha(rep)=0.3;
%    beta(rep)=albeta(rep)-alpha(rep);
```

```
   SERI(rep,5)=alpha(rep);
SERI(rep,6)=beta(rep);
   SERI(rep,14)=albeta(rep);

% alpha(rep)=0.3; %
beta(rep)=0.7;
% albeta(rep)=1;
%end
```

*** In each iteration, based on the determined values for alpha, beta, labor and equipment costs, the optimum values of labor cost, equipment cost, and total cost are calculated.

```
  for act=1:21        act;
x0=X00(act,:);          A=[ ];
b=[ ];          Aeq=[ ];
beq=[ ];          lb=[ ];
ub=[ ];
      cost(act,1)=FACTORCOST(act,1)*INFL(rep,1);
cost(act,2)=FACTORCOST(act,2)*INFL(rep,2);        Q(act)=TotalQ(act);
x(act,:)= fmincon(@Cobb,x0,A,b,Aeq,beq,lb,ub,@mycons);
    %   x(act,:)= fmincon(@Cobb,x0,A,b,Aeq,beq,@mycons)
      %x = fmincon(@(x)cost(1)*x(1)+cost(2)*x(2),x0,A,b,Aeq,beq,lb,ub,@mycons);
      LaborCost(act)=x(act,1)*cost(act,1);
      CapitalCost(act)=x(act,2)*cost(act,2);
      Table(act,3)=LaborCost(act)+CapitalCost(act);
  end
Table;
kkk=1;
```

*** Based on the determined values for parameters, optimum objective value is achives via checking all possible combinations of activities' options.

```
  for i=1:3     for ii=1:3        for iii=1:3
for j=1:3               for jj=1:3
for k=1:3                     for kk=1:3
Eval(kkk,1)=(Table(i,3)+Table(ii+3,3)+Table(jj+12,3)+Table(kk+18,3));
Eval(kkk,2)=(Table(i,3)+Table(iii+6,3)+Table(jj+12,3)+Table(kk+18,3));
   Eval(kkk,3)=(Table(i,3)+Table(j+9,3)+Table(k+15,3)+Table(kk+18,3));
Eval(kkk,4)=(Table(i,2)+Table(ii+3,2)+Table(jj+12,2)+Table(kk+18,2));
Eval(kkk,5)=(Table(i,2)+Table(iii+6,2)+Table(jj+12,2)+Table(kk+18,2));
   Eval(kkk,6)=(Table(i,2)+Table(j+9,2)+Table(k+15,2)+Table(kk+18,2));
```

```
Eval(kkk,7)=Table(i,3)+Table(ii+3,3)+Table(jj+12,3)+Table(kk+18,3)+Tabl
e(iii+6,3)+Table(j+9,3)+Table(k+15,3);
    Eval(kkk,8)=max(Eval(kkk,4:6));
    X(kkk,1)=i;
    X(kkk,2)=ii;
    X(kkk,3)=iii;     X(kkk,4)=j;
    X(kkk,5)=jj;     X(kkk,6)=k;
    X(kkk,7)=kk;

kkk=kkk+1;
end
end                 end
end         end
end end
```

*** In order to calculate the objective value, according to equation 11 and 12, max and min of time and cost are needed. They are calculated in this step.

```
 maxt=max(Eval(:,8));  mint=min(Eval(:,8));
maxc=max(Eval(:,7));  minc=min(Eval(:,7));

 SERI(rep,12)=maxt;
  SERI(rep,13)=mint;
   SERI(rep,11)=minc;
    SERI(rep,10)=maxc;

    for ss=1:3^7

%     if 59<Eval(ss,8) && Eval(ss,8)<70
%         Eval(ss,7);
%      Eval(ss,7)=Eval(ss,7)*INFL(1);
%  else
%      if 69<Eval(ss,8)&& Eval(ss,8)<80    %
Eval(ss,7);
%          Eval(ss,7)=Eval(ss,7)*INFL(2);
%      else
%          if 79<Eval(ss,8)&& Eval(ss,8)<90
%                      Eval(ss,7);
%              Eval(ss,7)=Eval(ss,7)*INFL(3);
%          else
%                      Eval(ss,7);
%              Eval(ss,7)=Eval(ss,7)*INFL(4);
%          end
%      end
```

```
%    end
%
%

   Eval(ss,9)=(maxt-Eval(ss,8))/(maxt-mint);

   Eval(ss,10)=(maxc-Eval(ss,7))/(maxc-minc);

   Eval(ss,11)=w1*Eval(ss,9)+w2*Eval(ss,10);


end
```

*** Comparing all possible combination of activities' options, the optimum objective value and corresponding activities' options are chosen.

```
Best(1)=max(Eval(:,11));

[cv,cd]=find(Eval(:,11)==Best(1));
CV=cv(1); cd;
Eval(CV,9);
Eval(CV,7);
Eval(CV,8);
Eval(CV,3);
Eval(CV,10);
BestSolution=X(CV,:);

%%% SERI 1:time  2:cost   3:labor cost   4:capital cost
Eval(CV,8);

SERI(rep,1)=Eval(CV,8);
SERI(rep,2)=Eval(CV,7);
LCost=0; pop=0;
for asas=1:7

 LCost=LaborCost(3*pop+BestSolution(1,asas))+LCost;
 Pop = pop+1; end
 SERI (rep, 3) = L Cost;
 SERI (rep, 4) = SERI (rep, 2) –SERI (rep, 3);

SERI(rep,7)=Best;
SERI(rep,8)=Eval(CV,9);
```

```
SERI(rep,9)=Eval(CV,10);

%SERI(rep,10)=Eval(CV,11);
  end
100
```

INFL; Table; *** this function is one of the functions needed by MATLAB software to calculate the optimum value of total cost.

In this function constrints of the model which is the CD function are presented.

```
      function [c ceq] = mycons(x)
  global Q global alpha global beta global act global rep
c=[]; ceq=(x(1)^alpha(rep))*(x(2)^beta(rep))-Q(act);
```

*** This function defined the objective function of the model. This function is called in the optimization process to calculate the optimum value of total cost.

```
function f = Cobb(x)
  global cost
global act

f=cost(act,1)*x(1)+cost(act,2)*x(2);
```

APPENDIX B

HGA Code

```
clear clc

%%%%%% Parameters  %%%%%

*** In this section parameters are defined and


*** n: number of activities
*** option: number of activities' options
*** a: alpha
*** b: beta
*** C: gama
*** PS: population size
 n=7;
option=3;
a=0.2;
b=0.4;
b=0.4;
PS=100;

*** W: workload
 W=[4500 5000 5000 4700 6000 6000 5500];



*** Cost: in each row, it defines the labor and equipment costs respectively for that activity


Cost=[60 60 100
   40 100 100
   50 70 60
   65 30 40
   45 80 70
   75 70 80
   55 20 30];


% Table=[1  22  48523.399
% 1 24  33966.38
% 1 28  28305.316
% 2 14  3997.706
```

```
% 2 20  3331.421
% 2 24  2998.279
% 3 15  4982.924
% 3 18  3397.448
% 3 30  2264.965
% 4 9  70524.268
% 4 15  52893.201
% 4 18  40596.496
% 5 15  24056.162
% 5 22  22051.482
% 5 33  18901.27
% 6 12  23082.507
% 6 16  17953.061
% 6 20  13464.796
% 7 14  41838.75
% 7 18  25103.25
% 7 28  20919.375];
```

*** Table: it determined different options for each activity. Fits colums shows the activity number, second column it shows duration, and the last one determined the total cost

```
Table=[1    14  357.1428571
1   20  250
      1      24  208.3333333
      2      15  33.33333333
2   18  27.77777778
      2      20  25
      3      15  40
3   22  27.27273
      3      33  18.18182
      4      12  500
4   16  375
      4      20  300
      5      22  204.5455
5   24  187.5
      5      28  160.7143
      6      14  392.8571


6   18  305.5556
      6      24  229.1667
      7      9   522.2222
```

```
7   15  313.3333
7   18  261.1111];
% a=0.2
% b=0.4
% C=0.4
% maxt=171;
% mint=70;
% maxc=503682.5;
% minc=270311;

% a=0.2 % b=0.4
maxt=171;
 mint=70;
maxc=503682.5;
minc=270311;

 *** w1 and w2 determined the priorities of cost and time respectively.
  routs=3;
w1=0.2;
w2=0.4;
w3=0.4;



%%%% Initialization %%%%%%%

*** in this phase, PS chromosomes are randomly generated
  for i=1:PS
for j=1:n
        X(i,j)=floor(option*rand)+1;     end end

%%%%%%%%  Improvement  %%%%%%%%%%



*** in this stage, as stated in the cntext, 2opt procedure is applied to improve the
randomly generated population
  for k=1:PS


    Temp(1,:)=X(k,:);     opt1=floor(n*rand)+1;
opt2=floor(n*rand)+1;


Temp(2,:)=X(k,:);     temp=Temp(2,opt1);
Temp(2,opt1)=Temp(2,opt2);
```

```
   Temp(2,opt2)=temp;
Temp(3,:)=Temp(2,:);     if
opt1==1
      Temp(4,:)=Temp(2,:);         TTemp=Temp(4,1);
      Temp(4,1)=Temp(4,2);
Temp(4,2)=TTemp;     else         if
opt1==n
         Temp(4,:)=Temp(2,:);             TTemp=Temp(4,n);
         Temp(4,n)=Temp(4,n-1);
Temp(4,n-1)=TTemp;         else
         TTemp=Temp(3,opt1);             gh=opt1+1;
         Temp(3,opt1)=Temp(3,gh);             Temp(3,gh)=TTemp;
         Temp(4,:)=Temp(2,:);
TTemp=Temp(4,opt1);             jh=opt1-1;
Temp(4,opt1)=Temp(4,jh);
Temp(4,jh)=TTemp;         end     end
   Temp(5,:)=Temp(2,:);     if
opt2==1
      Temp(6,:)=Temp(2,:);         TTemp=Temp(6,1);
      Temp(6,1)=Temp(6,2);
Temp(6,2)=TTemp;     else         if
opt2==n

         Temp(6,:)=Temp(2,:);
         TTemp=Temp(6,n);
         Temp(6,n)=Temp(6,n-1);
Temp(6,n-1)=TTemp;         else
         TTemp=Temp(5,opt2);             ser=opt2+1;
         Temp(5,opt2)=Temp(5,ser);
         Temp(5,ser)=TTemp;

Temp(6,:)=Temp(2,:);
TTemp=Temp(6,opt2);             sre=opt2-1;
         Temp(6,opt2)=Temp(6,sre);
Temp(6,sre)=TTemp;         end     end
```

*** after that the 2-opt procedure is done, the new solutions (offspring) are compared with the first solution, the best one is chosen.   for tt=1:6

```
       dd1=Temp(tt,1);
dd3=Temp(tt,2)+3;
dd6=Temp(tt,3)+6;
dd9=Temp(tt,4)+9;
dd12=Temp(tt,5)+12;
```

```
dd15=Temp(tt,6)+15;
dd18=Temp(tt,7)+18;



   Eeval(tt,1)=Table(dd1,3)+Table(dd3,3)+Table(dd12,3)+Table(dd18,3);
   Eeval(tt,2)=Table(dd1,3)+Table(dd6,3)+Table(dd12,3)+Table(dd18,3);
   Eeval(tt,3)=Table(dd1,3)+Table(dd9,3)+Table(dd15,3)+Table(dd18,3);
   Eeval(tt,4)=Table(dd1,2)+Table(dd3,2)+Table(dd12,2)+Table(dd18,2);
   Eeval(tt,5)=Table(dd1,2)+Table(dd6,2)+Table(dd12,2)+Table(dd18,2);
   Eeval(tt,6)=Table(dd1,2)+Table(dd9,2)+Table(dd15,2)+Table(dd18,2);
Eeval(tt,7)=Table(dd1,3)+Table(dd3,3)+Table(dd12,3)+Table(dd18,3)+Table
(dd6,3)+Table(dd9,3)+Table(dd15,3);
Eeval(tt,8)=max(Eeval(tt,4:6)); end
% maxt=max(Eval(:,7));
% mint=min(Eval(:,7));
% maxc=max(Eval(:,8));
% minc=min(Eval(:,8));
%  for ttt=1:6
   Eeval(ttt,9)=(maxt-Eeval(ttt,8))/(maxt-mint);
   Eeval(ttt,10)=(maxc-Eeval(ttt,7))/(maxc-minc);
Eeval(ttt,11)=w1*Eeval(ttt,9)+w2*Eeval(ttt,10); end
X(k,:)=Temp(1,:);
Beste=max(Eeval(:,11));
[sv,sd]=find(Eeval(:,11)==Beste);     sv=sv(1);
sd;
   X(k,:)=Temp(sv,:);



  end
```