KWARA STATE UNIVERSITY, MALETE, NIGERIA
SCHOOL OF POSTGRADUATE STUDIES (SPGS)

Alpha decay study of superheavy nuclei with the aid of machine learning

Oluwatobiloba Kazeem AZEEZ

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ALPHA DECAY STUDY OF SUPERHEAVY NUCLEI WITH THE AID OF MACHINE LEARNING

AN M.SC. THESIS SUBMITTED AND PRESENTED

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Oluwatobiloba Kazeem AZEEZ

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DECEMBER, 2021

Declaration page

I hereby declare that this thesis titled "Alpha decay study of superheavy nuclei with the aid of machine learning" is a record of my research. It has neither been presented nor accepted in any previous application for higher degree.

16/12/2021.

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OLUWATOBILOBA KAZEEM AZEEZ   Signature and Date

Approval

This dissertation titled "Alpha decay study of superheavy nuclei with the aid of machine learning" by Oluwatobiloba Kazeem AZEEZ with matriculation number 19/57MPH/00003 has met all the requirements for the award of degree of Master of Science in Physics and Materials Science, Kwara State University, Malete, and is approved for its contribution to knowledge.

DR. W. A. YAHYA	19-12-2021
SUPERVISOR	Date
DR. N. K. OLASUNKANMI	22/12/2021
CO-SUPERVISOR	Date
PROF. A. O. AINA (20/12/21)	20-12-2021
HEAD OF DEPARTMENT	Date
DR. K. ISSA	22/12/2021
INTERNAL EXAMINER	Date
PROF. K. J. OYEWUMI	20/12/2021
EXTERNAL EXAMINER	Date
PROF. HAMZAT I. ABDULRAHEEM	22-12-21
Dean, School of Postgraduate Studies (SPGS)	Date

Dedication

I dedicate my dissertation work to my Late Father Adisa Azeez and Dr Faluyi, May their souls rest in perfect peace. Amin

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Abstract

The alpha decay half-lives of superheavy nuclei have been studied using various alpha decay formulas and machine learning algorithms. The alpha decay empirical formulas employed in studying the alpha decay half-lives are Royer, Denisov-Khudenko, Akrawy and Poenaru (AKRE), Viola-Seaborg, New Ren A, New Ren B, Universal Decay Law, and Wang. The machine learning (ML) models considered in the study of the half-lives are k-nearest neighbour (kNN), support vector regression (SVR), decision tree (DT), random forest (RF), extra trees (ET) and artificial neural network (ANN). Different ML models have been considered to determine the best ML model to predict half-lives for alpha decay. This is motivated by the "no free lunch theorem" which states that the best ML model for a particular dataset cannot be known apriori. Improved empirical formulas are introduced using data from the NUBASE2020 database with the aid of a least-square fit scheme. The improved models are then used to calculate the alpha decay half-lives of nuclei with $Z = 92 - 118$*. The results show that the improved Royer formula with a standard deviation value of 0.5411 is the best empirical formula to predict the alpha decay half-lives of the superheavy nuclei. Among the ML models considered, the Artificial neural network (ANN) model has the least deviation from experimental values, suggesting that it is the best ML model to study alpha decay of superheavy nuclei. To predict the alpha decay half-lives for unmeasured superheavy nuclei, the energy released in the decay process (called Q-values) is trained using an artificial neural network (ANN). The predicted Q-values by the ANN are then used as inputs to calculate the alpha decay half-lives for unmeasured superheavy nuclei with* $Z = 120 - 126$ *using the improved formulas and the trained ML models. It is expected that these predictions will serve as motivations for future experimental studies on superheavy nuclei.*

Chapter 1

Introduction

1.1 Background

Most heavy and superheavy nuclei undergo α-decay. α-decay, discovered by Ernest Rutherford in 1899 [1, 2], is important to understand the structure of heavy and superheavy nuclei. The decay process was first explained by Gamow, Condon, and Gurney via the quantum tunneling effect in 1928 [3]. The first law that describes α-decay half-life is the Geiger-Nuttall law [4]. There are various theoretical investigations to study and calculate the half-lives of radioactive nuclei. Some of the theoretical methods include the fission-like model, the generalized liquid drop model, the effective liquid drop model, the preformed cluster model, and so on [5].

The half-lives of an α-decay particle range from 10^{-7} s to 10^{18} s and the kinetic energies of the emitted α-decay particle is in the range 4 to 11 MeV [1]. The preformation of α-decay particle within a nucleus is an important factor. In many theoretical calculations, the decay constant deviate from experimental values until the preformation probability was taken into account [6]. Alpha decay is one of the lightest clusters to be emitted from the nuclei and is governed by the interplay between nuclear and electromagnetic forces [6]. α-decay plays quite an important role in the fundamental and also the development of nuclear physics.

Recently, experimental studies were carried out to detect the naturally long-lived alpha decay nuclides [7]. However, bismuth ^{209}Bi was believed to be the heaviest stable nucleus [8]. In 2003, de Marcillac [8] studied the experimental detection of α-particles from the radioactive decay of natural bismuth and observed that the decay usually evades observation because the nuclear structure of ^{209}Bi which gives rise to an extremely low decay probability. Indeed, experiments attempting to record the α-decay of ^{209}Bi in nuclear emulsions failed. Until 2012, the measurement of the α-decay half-lives show the fact that Lead (Pb) is to be considered as the heaviest stable element with half-life $> 10^{35}$ years, even though the alpha decay of Pb is energetically favorable from all the four naturally occurring isotopes [9].

In Nuclear physics, one of the first and most useful models of nuclear structure is the Shell model [10]. This idea began with the striking observation of magic numbers. Magic number is the number of nucleons i.e. protons and neutrons such that they are arranged into complete shells within the atomic nucleus. They consist of $2, 8, 20, 28, 50, 82$, and 126 for neutrons. The Atomic nuclei having such magic number of nucleons have a higher average binding energy per nucleon than one would expect, and are hence more stable against nuclear decay [11]. Binding energy is smallest amount of energy required to remove a particle from a system of particles [12].

The nuclear mass plays an important role in understanding the nuclear processes. However, nuclides far from the valley of stability become impractical to produce in the laboratory due to extremely short half-lives and thus the mass knowledge becomes dependent on good systematic or a reliable mass prediction model. Protons are well known for their stability in an unbound state, whereas neutrons will decay in about ten minutes. Despite this, there is a significantly higher presence of neutrons over protons in bound states of nuclei. The default response is explained by Coulomb force [13].

Stable nucleus has the right amount of neutrons and proton, that is the attractive nuclear force between the nucleons overcomes the Coulombs repulsive force that tends to pull the protons apart. While on the other hand the attractive nuclear force in an unstable nucleus does not provide the required amount of binding energy to hold the nucleus together. On a neutron-proton plot, Figure 1.1 the stable nuclei lie along the line of stability. The unstable nuclei, however, lie above and below the line of stability. Those that lie below the line of stability are said to be proton rich, while those above this line are said to be neutron rich [14].

The superheavy nuclei (SHN), are traditionally considered to be those that lie above ^{253}Rf, represents the very top end of the Periodic Table and a study of their properties is intrinsically linked to an understanding of the physics at the limit of stability in mass and charge [15]. SHN are obtained in the laboratory by colliding stable heavy atoms using two fusion evaporation approaches. Studies on the properties of superheavy elements after the predictions of magic islands in 1960 have boosted the experimental synthesis of SHN. Nuclei that exist due to the closing of proton and neutron shells are said to be magic and are relatively stable compared to the surrounding nuclei with larger or smaller nucleons [6, 16].

With the availability of intense radioactive beams, researchers are developing interest in the study of unstable heavy and superheavy nuclei. However, new facilities such as Facility for Antiproton and Ion Research (FAIR, Germany), Darmstadt, High Intensity and Energy ISOLDE (HIE-ISOLDE at CERN Switzerland/France), Systéme de Production d'Ions Radioactifs en Ligne, generation 2 (SPIRAL 2, France), Facility for Rare Isotope Beams (FRIB at MSU, USA), Radioactive Ion Beam Factory (RIBF at RIKEN Japan) have made it possible to investigate highly unstable nuclei and also to probe existing formalism trying to describe those nuclei [17].

Empirical formulae were normally used to predict the half-lives of superheavy nuclei. Geiger and Nuttal were the first to propose a formula to compute the logarithmic half-lives of α-decay [18]. Later, Viola and Seaborg constructed semi-empirical relation for alpha decay half-lives [19]. In 1992, Brown proposed an empirical formula based on the experimental variation of logarithmic half-lives [20]. Royer in 2000 formulated an empirical formula for alpha decay half-lives using the liquid drop model which includes proximity effects [21]. Also, Poenaru proposed the SemFIS formula for alpha decay half-lives of superheavy nuclei taking into account magic numbers of nucleons, the analytical super asymmetric fission model, and the universal curves [22]. With the aid of NUBASE 2012, Wang et al. in 2015 calculated the alpha decay half-lives by considering the ground state spin and parity of parent and daughter nuclei [23].

Machine learning is a subset of artificial intelligence, which build a mathematical model based on sample data known as training data in order to make predictions without being explicitly programmed to perform the task [24]. Recently, machine learning have been used in many fields most especially physics to study nuclear structures [25], such as developing nuclear mass systematic using neural networks [26], identification of impact parameters in heavy-ion collisions [27], and also in the estimation of beta decay half-lives [28]. Samuel et al. in 2018 [29] combined machine learning and physics to study the glassy systems, Decelle et al. [30] used machine learning approach to study the thermodynamics of restricted Boltzmann machines and related learning dynamics, Utama et al. in 2016 [31] studied the nuclear mass predictions for the crustal composition of neutron stars using the Bayesian neural network approach, Ubaldo in 2019 [32] predicted the α-decay half-lives of superheavy nuclei using the Q-value obtained by the Bayesian neural network.

Figure 1.1 shows the nuclear landscape. Stable nuclei are shown in black, at the bottom of Valley of Stability in the nuclear landscape. Unstable nuclei that decay predominantly via β^+ decay or eletron capture are shown in orange, and by β^- decay in blue. Spontaneous fission nuclei are shown in green and only appear at heavier masses. Neutron emission are shown in purple. Proton emission are shown in red. and α-decay are shown in yellow.

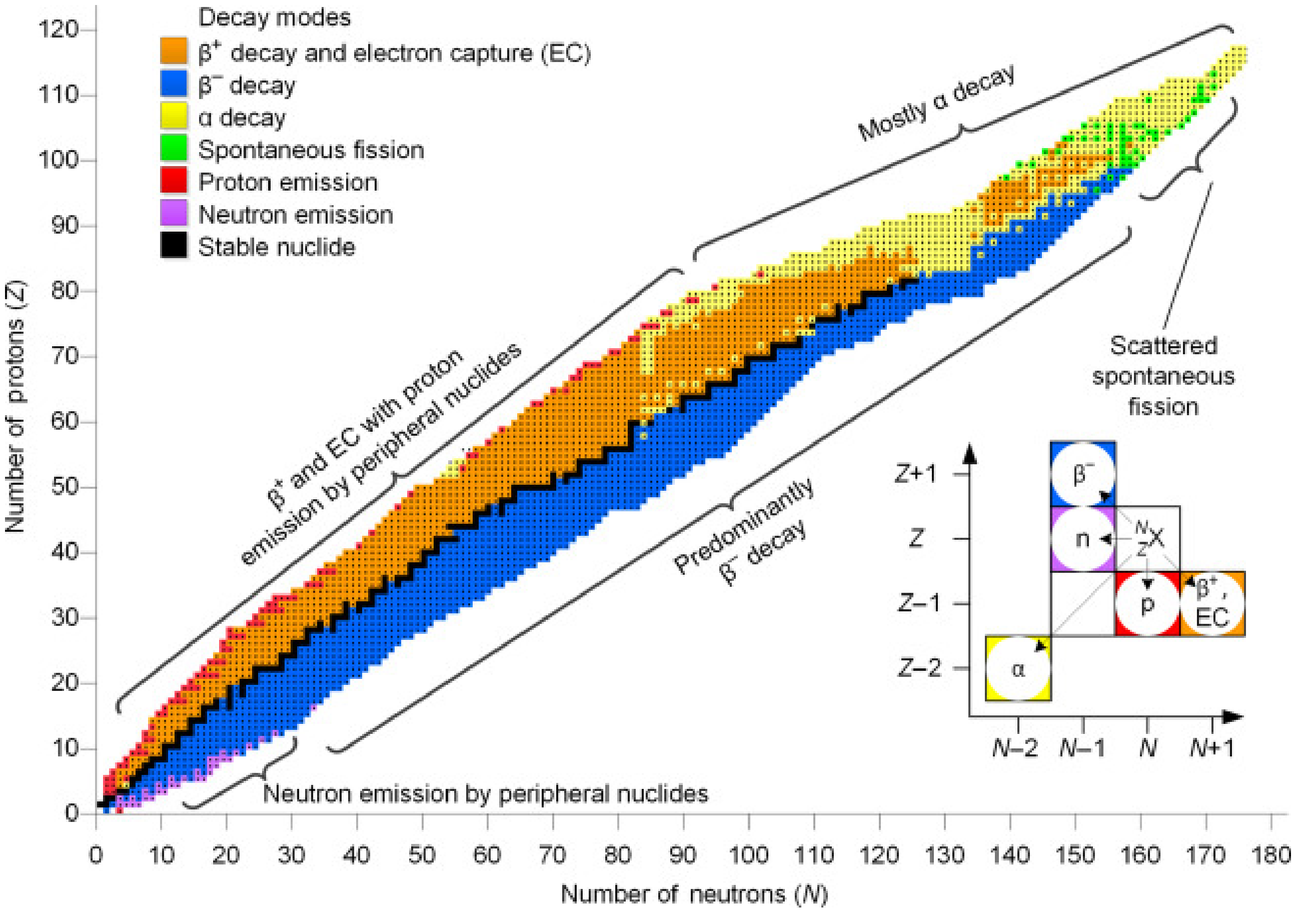

Figure 1.1: Chart of nuclides [33].

1.2 Superheavy Nuclei

Currently, there are 118 elements in the periodic table. Superheavy nuclei are nuclei with atomic numbers $Z \geq 104$. These elements are produced by fusing atomic nuclei of lighter elements using particle accelerators. Nuclear physics laboratory around the world such as The Helmholtz Center for Heavy Ion Research in Darmstadt, GSI, Germany, created elements 107 (Bohrium) through element 112 (Copernicium) by bombarding elements 82 (Lead) and 83 (Bismuth) with moderately heavy projectile nuclei $Z = 24, 26, 28$, and 30. The study of this superheavy nuclei will aid experimentalists in discovering elements with $Z > 118$. The superheavy nuclei provide opportunities to get insights into the influence of strong relativistic effects on the atomic electrons and to probe relativistically influenced chemical properties and the architecture of the periodic table at its farthest reach. In addition, they establish a test bench to challenge the validity and predictive power of modern fully relativistic quantum chemical models [34].

1.3 Statement of Problem

(i) To predict the half-lives of unmeasured superheavy nuclei, researchers often use formulas that have been obtained from the least square fit. It is known that least-square fit schemes do not extrapolate reasonably. However, machine learning algorithms learn from data and this can serve as a good tool to predict the half-lives of unmeasured superheavy nuclei.

1.4 Aim & Objectives

The aim of this research is to predict the α-decay half-lives of measured and unmeasured superheavy nuclei using empirical formulas and machine learning algorithms. The objectives of this research are:

(i) to calculate the α-decay half-lives ($T^{\alpha}_{\frac{1}{2}}$) for all α-emitters using existing empirical formulas;

(ii) to obtain improved alpha decay empirical formulas using least square fit scheme and input data from NUBASE2020 database;

(iii) to calculate $T^{\alpha}_{\frac{1}{2}}$ for nuclei with atomic number in the range $92 \leq Z \leq 118$ using the improved formulas;

(iv) to train machine learning (ML) models such as, Support Vector Regression (SVR), k-Nearest Neighbour (kNN), Decision Trees (DT), Extra Trees (ET), and Random Forest (RF) to predict half-lives of all α-emitters in the NUBASE2020 database;

(v) to predict the $T^{\alpha}_{\frac{1}{2}}$ using the trained ML models for superheavy nuclei;

(vi) to predict the decay energy (Q_{α}) using machine learning algorithms;

(vii) to predict the half lives $T^{\alpha}_{\frac{1}{2}}$ for unmeasured superheavy nuclei with Z in the range $120 \leq Z \leq 126$.

1.5 Significance of the Study

The outcome of this research will be useful for researchers in experimental nuclear physics in carrying out experiments to study alpha decay half-lives of currently unmeasured superheavy nuclei.

This thesis is organised as follows: the theoretical formalism used in this research viz. alpha decay formulas and machine learning models, are presented in Chapter 2. The methods used for the calculations are discussed in Chapter 3. The results of the calculations are presented and discussed in Chapter 4 and the conclusion of this research and recommendation for further studies are given in Chapter 5.

Chapter 2

Literature Review

2.1 The Atomic Nucleus

The atomic nucleus consists of proton and neutron at the center of an atom. The nucleus was discovered by Rutherford in 1911 based on the Geiger-Marsden gold foil experiment [35]. Proton and neutron are bound together to form the nuclear force. The proton and neutron have approximately the same mass, about 1.67×10^{-24} grams. The proton is positively charged with an half integer spin, neutron has no charge with half integer spin. The electron is the lightest particle of an atom, with a mass of 9.1×10^{-31} kg. Atomic number is the number of protons in the nucleus. The sum of the atomic number (Z) and the number of neutrons N gives the mass number (A) [36]. Nuclides that have the same number of neutrons (N) but different atomic number (Z) are called isotopes. For example, ^{16}O, ^{17}O, ^{18}O are all isotopes of oxygen. Isotones are nuclides that have the same number of neutron (N), but different atomic number (Z). They have different mass number (A) and do not correspond to the same chemical elements. Examples are ^{13}C and ^{14}N. Isobars are nuclides with the same mass number but different atomic number. ^{14}C and ^{14}N are isobars.

2.2 Radioactivity

Radioactivity is the disintegration of an atomic nucleus with the spontaneous emission of particles or radiation. Uranium, Polonium, and Radium disintegrate spontaneously and emit radiation. These elements are said to be radioactive [37]. Generally, radioactive decay is of three types [38] :

(i.) Alpha decay,

(ii.) Beta decay,

(iii.) Gamma decay.

Alpha decay

Alpha particles are fast-moving Helium nuclei (^4_2He) with a charge of +2. They have mass number of 4 and atomic number of 2. When a nuclide ^A_ZX undergoes α-decay, we have:

$$^A_ZX \rightarrow ^{A-4}_{Z-2}Y + ^4_2He, \tag{2.1}$$

where A and Z are the mass number and atomic number, respectively. In alpha (α) decay, the nuclide X loses two protons and two neutrons to form nuclide Y. The total energy available for the α-decay, Q_α is defined as:

$$Q_\alpha = E_\alpha + E_\gamma, \tag{2.2}$$

where E_α is the kinetic energy of the α particle and E_γ is the recoil energy of the daughter nucleus. α-decay can occur spontaneously only when $Q_\alpha > 0$ and the angular momentum and parity are conserved. In the study of α-decay from the $I_i^{\pi_i}$ state to $I_f^{\pi_f}$ state, the value of the α-decay angular momentum L_α carried by the α particle is obtained via these relations:

$$|I_i - I_f| \leq L_\alpha \leq I_i + I_f, \tag{2.3}$$

$$\pi_f = \pi_i \times (-1)^{L_\alpha}. \tag{2.4}$$

The alpha decay without a change of spin and parity are called allowed decays and they are most probable. However, other decay modes are also possible if the decay energy, spin, and parity are also conserved. If the energy window is small it is rare to detect alpha decay especially if other decay channels are open concurrently.

The probability of α-decay depends on the kinematics of the process and structure of the parent and daughter nuclei. In order to extract the information regarding the structure, the reduced α-decay width δ^2_α, commonly expressed in keV, can be calculated. In the Rassmusen formalism, it is given by the formula:

$$\delta^2_\alpha = \frac{h \ln 2}{T^p_{\frac{1}{2}} P}, \tag{2.5}$$

where h is the Planck's constant, P is the penetration probability, $T^p_{\frac{1}{2}}$ is the partial half-life of parent nucleus.

The penetration probabilty contains the information regarding the probability of the α-particle of a given energy penetrating Coulomb and centrifugal barriers while the partial half-life depends on the α-branching of the parent nucleus and the intensity of the decay to the given state.

α decay is fundamentally a quantum tunneling process. In 1928, Gamow solved the theory of alpha decay via quantum mechanical tunneling [1]. The alpha particle is trapped inside the nucleus by an attractive nuclear potential well and a repulsive electromagnetic potential barrier. Classically, it is impossible to escape, but according to the discovered principles of quantum mechanics, there is a tiny but non-zero probability of "tunneling" through the barrier and appearing on the other side to escape the nucleus [39]. The α-particle escapes from the nucleus not by acquiring enough energy to pass over the barrier but by tunneling through the wall [3].

β-decay

The β-decay is a type of radioactive decay in which a fast energetic electron or positron is emitted from an atomic nucleus, transforming the original nuclide to an isobar of that nuclide. The two types of beta decay are known as beta minus and beta plus. In beta minus (β^-) decay, a neutron is converted to a proton, and the process creates an electron and an electron antineutrino. A beta-minus particle is an electron [40] i.e.

$$ {}^{1}_{0}n \rightarrow {}^{1}_{1}P + {}^{0}_{-1}e^{-}. \tag{2.6} $$

In this form of decay, the original element becomes a new chemical element in a process known as a nuclear transmutation. This new element has an unchanged mass number A, but an atomic number Z that is increased by one.

γ-decay

Paul Villard, a French physicist, discovered γ radiation in 1900, when studying radiation emitted by radium. Ernest Rutherford then named it "gamma rays" because of their strong penetration of matter. They range from a few keV to approximately 8 MeV. High-energy gamma-rays in the range of 100-1000 teraelectronvolt (TeV) have been noticed from sources such as microquasars. Due to their high penetrability, they easily pass through the human body and thus pose great radiation protection challenges, which can only be avoided by using shielding made from dense materials such as lead or concrete.

Gamma rays are produced in a lot of particle physics processes. They are produced by the hottest energetic objects in the universe, such as neutron stars and pulsars. In gamma decay, a nucleus changes from a higher energy level to a lower energy level by emitting photons [41, 42]:

$$ {}^{240}_{94}\mathrm{Pu}^{*} \longrightarrow {}^{240}_{94}\mathrm{Pu} + {}^{0}_{0}\gamma. $$

2.2.1 Radioactive decay law

The probability that a radioactive nucleus will not decay at time t is given as [43]:

$$ P(t) = \exp(-\lambda t), \tag{2.7} $$

where λ is the decay constant. If N_0 is the number of atoms at $t = 0$ and N the number of atoms at t, then

$$P(t) = \frac{N}{N_0}, \tag{2.8}$$

$$N = N_0 \exp(-\lambda t). \tag{2.9}$$

The decay law for a radioactive substance is thus stated as:

$$\frac{dN}{dt} = -\lambda N_0 \exp(-\lambda t) = -N\lambda. \tag{2.10}$$

2.2.2 Mean life and half-life

Mean life is the average life time for a nucleus to decay. The mean life can be calculated using

$$\tau_{av} = \frac{\int_{\infty}^{0} t \exp(-\lambda t) dt}{\int_{\infty}^{0} \exp(-\lambda t) dt} = \frac{\frac{1}{\lambda^2}}{\frac{1}{\lambda}} = \frac{1}{\lambda} = \tau. \tag{2.11}$$

Half-life is the time it takes a radioactive nucleus to reduce by half [43]:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = 0.693\tau. \tag{2.12}$$

2.2.3 Activity

Activity (A) is the number of disintegration per unit time. Therefore

$$A = \frac{dN}{dt} = N\lambda. \tag{2.13}$$

2.2.4 Nuclear Isomer

Nuclear isomeric states are excited states which have considerable half-lives. The lower limit of the half-life is not well defined and it changes over time, from 1 ms to the average nanoseconds range [44]. There are different types of isomeric state which include the fission isomer, shape isomer and spin isomer.

The spin isomer normally occurs when the spin difference between the given state and all states of lower excitation energy is relatively large. This difference leads to the retardation of the γ-decay and consequently an isomerism. An example of these kind of isomeric states are the $2-$ and $7+$ states in the neutron-deficient thallium isotopes.

2.3 Theory of α-decay

Rutherford suggested that to study the atomic nucleus, non-Coulomb attractive force plays an important role in the study of the stability of the nucleus. But Gamow believed that the existence of these attractive forces implies that the Coulomb potential begins to break down at some position r that the total potential reaches a maximum U_0 at some smaller position $r_0 \approx 10^{-14}m$, and at the $r > r_0$. To explain how an α-particle of energy $E < U_0$ could traverse a region between $r_1 = 3.2 \times 10^{-14}$ m and $r_2 = 6.3 \times 10^{-14}$ m, which naturally would be impossible according to classical approach, Gamow went through, step by step, one tunneling problem after another. He first considered a single rectangular barrier of height U_0 and width ℓ with an alpha particle and generate a total wave function given by [45]:

$$\psi = \psi(q) \exp\left(\frac{2\pi i E}{h} t\right). \tag{2.14}$$

The solutions ψ(q) of the time-independent Schrödinger equation is given as:

$$\frac{\partial^2 \psi}{\partial q^2} + \frac{8\pi^2 m}{h^2}(E - U)\psi = 0. \tag{2.15}$$

Gamow, Gurney, and Condon [39] adopted Born's probabilistic interpretation of Schrödinger's wave function. They illustrated its use by proving that a simple harmonic oscillator in its ground state has more than a 15% chance of being outside its classically permitted amplitude. They then turned to the single-barrier problem for which the incident, reflected, and transmitted waves obey conservation property that is:

$$(\psi\bar{\psi})_{inc} = (\psi\bar{\psi})_{ref} + (\psi\bar{\psi})_{tr}. \tag{2.16}$$

Gamow et al. [39] proved that for high and broad barriers the probability of transmission is:

$$P_\alpha(E) = \frac{16E}{U_0}\left(1 - \frac{E}{U_0}\right) \exp\left[-\frac{4\pi}{h}\sqrt{2m(U_0 - E)r}\right]. \tag{2.17}$$

They assumed that the factor in the exponent can be approximated by an integral expression, to give

$$\exp\left[-\frac{4\pi}{h}\int \sqrt{2m(U - E)}dr\right]. \tag{2.18}$$

Condon and Gurney considered the case of a particle in a potential valley and presented two simple arguments for estimating the factor in front of the exponential. Since E is the energy, its velocity is represented by:

$$v = \sqrt{\frac{2E}{m}}. \tag{2.19}$$

Hence, the time it must spend in a region of length r before standing a chance of escaping is

$$T = r\sqrt{\frac{m}{2E}}, \tag{2.20}$$

which is directly proportional to the decay constant λ. The decay period is [45]

$$T = \frac{1}{\lambda} = r\sqrt{\frac{m}{2E}} \exp\left[\frac{4\pi}{h}\int \sqrt{2m(U-E)}dr\right]. \tag{2.21}$$

2.3.1 Gamow-like model (GLM)

A variation of the Gamow model was proposed by Zdeb et al. [1], where they applied a Gamow-like model to study α-decay half-lives of nuclei. In this model, the α-decay half-lives of the decaying nucleus is calculated using [1]:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\nu S_\alpha P}, \tag{2.22}$$

where ν is the number of collective assaults per time unit of the emitted object on the barrier with a value of $10^{20}s^{-1}$ and S_α is the preformation probability of the α or cluster at the surface, A value of $S_\alpha = 1$ was found to give the best descriptions of the alpha decay.

The penetration probability of the α particle through a potential barrier is obtained within the Wentzel-Kramers-Brillouin (WKB) approximation as [46, 47]:

$$P = \exp\left[\frac{-2}{h}\int_R^b \sqrt{2\mu(V(r)-Q)}dr\right], \tag{2.23}$$

where h is the hindrance factor and μ is the reduced mass of the emitted α-particle which can be calculated as:

$$\mu = \frac{mA_1A_2}{(A_1+A_2)}. \tag{2.24}$$

Here A_1 is mass number of parent nucleus and A_2 is mass number of daughter nucleus, $m = 931.5MeV/c^2$ is the nuclear mass unit. The spherical square well radius R is calculated from

$$R = r_0\left(A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}}\right), \tag{2.25}$$

while the turning point b, is obtained using :

$$b = \frac{Z_1Z_2e^2}{Q}. \tag{2.26}$$

The radius constant r_0 is 1.2 fm, Z_1 and Z_2 are the atomic number of the emitted cluster and daughter nucleus, respectively. In this model, the interaction potential $V(r)$ is given in the form

$$V(r) = \begin{cases} -V_0, & 0 \le r \le R \\ \frac{Z_1Z_2e^2}{r} & r > R, \end{cases} \tag{2.27}$$

where the depth of the potential well $V_0 = 25A_1$.

2.3.2 Modified Gamow-Like Model (MGLM)

Cheng et al. [46] introduced the modified Gamow-like model. In the modified Gamow-Like model, the interaction potential between the alpha decay particle and daughter nucleus is given as [46, 48] :

$$V(r) = \begin{cases} -V_0, & 0 \leq r \leq R \\ V_H(r) + V_\ell(r), & r \geq R \end{cases}, \tag{2.28}$$

where the Hulthen type of screened electrostatic Coulomb potential

$$V_H(r) = \frac{aZ_1Z_2e^2}{e^{ar} - 1}, \tag{2.29}$$

and the centrifugal potential is given as:

$$V_\ell(r) = \frac{(\ell + \frac{1}{2})^2\hbar^2}{2\mu r^2}. \tag{2.30}$$

V_0 is the depth of the square well, Z_1 and Z_2 are the atomic numbers of the α-particle and daughter nucleus, respectively, ℓ is the orbital angular momentum that the α-particle takes away, and a is the screening parameter. The radius of the spherical square well is calculated by summing the radii of both the daughter nucleus (A_2) and the α-particle (A_1) using:

$$R = r_0(A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}}), \tag{2.31}$$

where r_0 is a constant of an adjustable parameter. The α-decay half-life can be calculated using [48]:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}10^h, \tag{2.32}$$

where h is the decay hindrance factor. The value is zero for even-even nuclei. The values of the three parameters (a, r_0, and h) in the model were determined in Ref [46] to be

$$a = 7.8 \times 10^{-4}, r_0 = 1.14fm, h = 0.3455 \tag{2.33}$$

For odd-odd nuclei, $h_{np} = 2h$. The decay constant λ is calculated using:

$$\lambda = \nu P, \tag{2.34}$$

where the penetration probability is P is given as:

$$P = \exp\left[-\frac{2}{\hbar}\int_R^b \sqrt{2\mu(V(r) - E_k)}\right] dr. \tag{2.35}$$

The kinetic energy of the emitted particle is denoted by $E_k = \frac{Q_\alpha(A-4)}{A}$. The classical turning point b is obtained through the condition $V(b) = E_k$.

In MGLM, the assault frequency on the potential barrier and the radius of the parent nucleus are calculated using [46]:

$$\nu = \frac{(G+\frac{3}{2})\hbar}{1.2\pi\mu R_0^2}, \tag{2.36}$$

and

$$R_0 = 1.28A^{\frac{1}{3}} - 0.76 + 0.8A^{-\frac{1}{3}}, \tag{2.37}$$

respectively. The main quantum number G is calculated using:

$$G = \begin{cases} 22 & N > 126 \\ 20 & 82 < N \leq 126, \\ 18 & N \leq 82 \end{cases} \tag{2.38}$$

where N is the neutron number [48].

2.4 α-Decay Empirical Formula

Empirical formulas should be considered as an effective method of calculating the α-decay half-lives of radioactive nuclei because they are based on different microscopic, phenomenological models or different adaptation of the Gamow's theory.

2.4.1 Geiger-Nuttall Rules

After Rutherford and his co-workers in 1907 discovered that the α-particle emitted from short-lived isotopes have high penetrating power as a result of high energy, his co-workers in 1912 (Geiger and Nuttall) established the relation between the particle range R_α and the emitter half-lives given as:

$$\log_{10} T_{1/2}(s) = -57.5 \log_{10} R_\alpha + c, \tag{2.39}$$

where c is a constant. By adding the mean decay rate λ of an isotope, the Geiger–Nuttall formula becomes

$$\log_{10} \lambda = 57.5 \log_{10} R_\alpha - C', \tag{2.40}$$

where the $C' \approx 41$. In 1921, Geiger and Nuttall observed the trend of alpha decay half-lives and its Q-value and proposed the first empirical relation which state:

$$\log_{10} T_{1/2}(s) = \alpha Q^{-1/2} + b. \tag{2.41}$$

The dependence of the parameters α and b of the isotopic chain came out to be a disadvantage to this formula [49].

2.4.2 Viola-Seaborg Semi-emperical formula (VSS)

Viola and Seaborg predicted a formula which was based on Gamow model in 1966. This formula produces logarithmic half-lives of alpha decay by including the intercept parameter of linear dependence on the charge number of daughter nucleus to produce a formula given as:

$$\log_{10} T_{1/2}^{VSS}(s) = (aZ + b)Q_\alpha^{-1/2} + cZ + d + h_i, \tag{2.42}$$

where Z is the proton number, Q_α is the alpha decay energy of parent nucleus, while the quantity a, b, c and h_i are the adjustable parameters; $i = p, n, pn$ are the average hindrance factors for odd-even, even-odd and odd-odd nuclei, respectively. For even-even nuclei, h_i=0 [19].

2.4.3 Universal Curve (UNIV)

Poenaru et al. [22] proposed the universal curve by extending a fission theory to the large mass asymmetry. The Universal curve for α-decay and cluster radioactivities can be derived by plotting the sum of the decimal logarithms half-lives and the cluster preformation probability versus the decimal logarithms of the probability of external barrier [22]. This formula is known as the UNIV formula for α-decay half lives which is given as:

$$\log_{10} T_{1/2}^{UNIV}(s) = -\log P_s - 22.169 + 0.598(A_\alpha - 1), \tag{2.43}$$

where

$$-\log P_s = c_{AZ}[\arccos\sqrt{r} - \sqrt{r(1-r)}], \tag{2.44}$$

$$c_{AZ} = 0.22873(\mu_A Z_d Z_\alpha R_b)^{1/2}, \tag{2.45}$$

$$r = \frac{R_t}{R_b}, \quad R_t = 1.2249(A_d^{1/3} + A_\alpha^{1/3})\text{fm}, \quad R_b = 1.43998\frac{Z_d Z_\alpha \text{fm}}{Q_\alpha}, \tag{2.46}$$

and $\mu_A = \frac{A_d A_\alpha}{A}$. R_t and R_b are the classic turning points [50].

2.4.4 Universal decay law (UDL)

Qi et al. [51] proposed a universal decay law to describe α-decay and cluster decay modes starting from α-like R-matrix theory and the microscopic mechanism of the charged particle emission. This relates the half-lives of monopole radioactive decays with the Q_α values as well as its masses and charge. The formula of universal decay law is giving as:

$$\log_{10} T_{1/2}^{UDL}(s) = a\chi' + b\rho' + c, \tag{2.47}$$

where

$$\chi' = aZ_\alpha Z_d\sqrt{\frac{\mu}{Q_\alpha}}, \tag{2.48}$$

$$\rho' = \sqrt{AZ_\alpha Z_d(A_d^{\frac{1}{3}}) + A_\alpha^{\frac{1}{3}}}. \tag{2.49}$$

The parameters $a = 0.4314, b = -0.4087, c = -25.7725$ are determined by fitting to experimental alpha and cluster decay half-lives.

2.4.5 Denisov-Khudenko formula (DK)

The relationship between the half-life of alpha transition between the ground state of parent and daughter nuclei of heavy and light nuclei was proposed by Denisov and Khudenko. The relation expresses the logarithm of half-life as a function of the decay energy Q with parameters for even-even, even-odd,odd-even, and odd-odd nuclei with $\ell = 0$ for even-even alpha emitters. The formula is given thus [52]:

$$\log_{10} T_{1/2}^{DK}(s) = a + b\frac{A^{\frac{1}{6}}\sqrt{Z}}{\mu} + \frac{cZ}{\sqrt{Q}} + \frac{d\sqrt{\ell(\ell+1)}}{QA^{\frac{-1}{6}}} + e((-1)^\ell - 1), \tag{2.50}$$

where A is the mass number of parent nucleus, Z is the proton number of the parent nucleus, and ℓ is the orbital momentum of the emitted α-particle. Reduced nucleus mass can be calculated using:

$$\mu = \left[\frac{A}{(A-4)}\right]^{\frac{1}{6}}, \tag{2.51}$$

where a, b, c, d, e are the fitting parameters. The values obtained for e-e, e-o, o-e and o-o for heavy and light nuclei are given in Tables 2.1 and 2.2. By considering 334 nuclei where 200 are light nuclei and 114 are heavy nuclei with $A - Z > 126$ and $Z > 82$.

Table 2.1: The coefficients for the Denisov- Khudenko (DK) empirical formula for heavy nuclei.

Heavy nuclei	a	b	c	d	e
even-even	-27.9238	-1.0521	1.5847	0	0
even-odd	-34.9988	-0.8552	1.6822	0.2278	-0.6763
odd-even	-33.5438	-0.9627	1.7077	0.1583	-0.5200
odd-odd	-38.8157	-0.5200	1.5645	0.5175	0.0287

Table 2.2: The coefficients for the Denisov- Khudenko (DK) empirical formula for light nuclei.

Light nuclei	a	b	c	d	e
even-even	-29.2230	-1.0347	1.6290	0	0
even-odd	-29.3760	-1.0835	1.6711	0.3324	-6.2873
odd-even	-28.7300	-1.1068	1.6652	0.1377	-0.6153
odd-odd	-31.5090	-1.0626	1.7298	0.1675	0.1080

2.4.6 Royer formula (Royer)

Royer in 2000 [53], proposed analytical formula for alpha decay logarithmic half-lives of alpha emitters which depends on charge numbers of parent nuclei, mass number of the parent nuclei and the energy released during the reaction Q, and is given as:

$$\log_{10} T_{1/2}^{Royer} = a + bA^{\frac{1}{6}}\sqrt{Z} + c\frac{Z}{\sqrt{Q}}, \tag{2.52}$$

where a, b, c are parameters obtained by fitting the experimental data [21]. Ten years later, Royer modified this alpha decay empirical formula by adding angular momentum ℓ using the experimental data of 334 nuclei [54]. The modified Royer formula is given as:

$$\log_{10} T_{1/2}^{Royer} = a + bA^{\frac{1}{6}}\sqrt{Z} + c\frac{Z}{\sqrt{Q}} + \frac{dANZ}{Q}(\ell(\ell+1))^{\frac{1}{4}} + eA[1-(-1)^{\ell}], \tag{2.53}$$

where a, b, c, d, e are adjustable parameters. The values obtained for e-e, e-o, o-e and o-o are given Table 2.3.

Table 2.3: The coefficient of the empirical formulae for Royer [54]

	a	b	c	d	e
even-even	-27.690	-1.0441	1.5702	0	0
even-odd	-27.750	-1.1138	1.6378	1.7383×10^{-06}	0.0025
odd-even	-27.915	-1.1292	1.6531	8.9785×10^{-07}	0.0025
odd-odd	-26.448	-1.1023	1.5967	1.6961×10^{-06}	0.0010

2.4.7 AKRE formula

Akrawy and Poenaru [55] postulated a new formula for calculating the alpha decay half life by adding new parameter 'I' called isospin asymmetry to the Royer [21] relation in equation (2.52). The new formula proposed is given as:

$$\log_{10} T_{1/2}^{AK2} = a + bA^{\frac{1}{6}}\sqrt{Z} + \frac{cZ}{\sqrt{Q}} + dI + eI^2. \tag{2.54}$$

The Isospin asymmetry is given as:

$$I = \frac{N-Z}{A}. \tag{2.55}$$

The fitting parameters a, b, c, d and e are obtained by fitting to experimental data. The values of a, b, c, d and e for e-e, e-o, o-e and o-o are given in Table 2.4.

Table 2.4: The coefficients for the AKRE semi-empirical formula [55]

	a	b	c	d	e
even-even	-26.3228	-1.1599	1.5923	12.0606	-41.6633
even-odd	-24.4072	-1.2320	1.6549	-31.8629	159.7768
odd-even	-31.7925	-1.0764	1.7535	-2.2263	-0.3938
odd-odd	26.2790	-1.2013	1.6591	-10.0841	67.5973

2.4.8 Ren A formula

To generalize the Viola-Seaborg semi-emperical formula Ren et al in 2004 gave a formula for RenA for α-decay and cluster radioactivity half-life which state thus:

$$\log_{10} T_{\frac{1}{2}}^{RenA} = aZ_1Z_2Q^{-\frac{1}{2}} + cZ_1Z_2 + d + h, \tag{2.56}$$

where a, c, and d are the constants to be determined for even-even cluster emitters and h represents a blocking factor of an odd nucleon in odd-A nuclei. Z_1 and Z_2 are the atomic number of the daughter and cluster nuclei, respectively [56]. The coefficients obtained are $a = 1.5180$,$c = -0.0534$, $d = -92.9114$, $h = 1.402$.

2.4.9 Ren B formula

Ren et al. [57] proposed a new formula to calculate the half-lives of α and cluster decay by modifying the Ren A formula given above. They added another term to the formula, which is the reduced mass μ. The Ren B formula is given as:

$$\log_{10} T_{\frac{1}{2}}^{RenB} = a\sqrt{\mu}Z_1Z_2Q^{-\frac{1}{2}} + b\sqrt{\mu Z_1Z_2} + c, \tag{2.57}$$

where a, b, and c are free parameter coefficient obtained by least square fitting for both α-decay and cluster radioactivity data. μ is the reduced mass, defined as:

$$\mu = \frac{A_1A_2}{(A_1 + A_2)}, \tag{2.58}$$

where A_1 and A_2 are the mass number of both daughter nuclei and cluster nuclei, respectively [57]. The values of a, b, and c were determined using the experimental data of 71 even-even data with $Z = 84$–118, $N = 128$–176 nuclei. The values of a, b and c are given as 0.3996,-1.3101,-17.0070, respectively.

2.4.10 New Ren A

The Ren B formula was modified by Akrawy et al. [58] to give the New Ren A formula by adding the nuclear isospin asymmetry term. The new formula for Ren A is given as:

$$\log_{10} T_{\frac{1}{2}}^{NRA} = a\sqrt{\mu} Z_1 Z_2 Q^{-\frac{1}{2}} + b\sqrt{\mu Z_1 Z_2} + c + dI + eI^2, \tag{2.59}$$

where I is the nuclear isospin asymmetry $I = \frac{N-Z}{A}$, and a, b, c, d, e are the free parameter coefficient obtained by least square fitting to α-decay data. The values of a, b, c, d and e for e-e, e-o, o-e and o-o are given in Table 2.5 [58].

Table 2.5: The coefficients for the New Ren A empirical formula [58]

heavy nuclei	a	b	c	d	e
even-even	0.41107	-1.44914	-14.87085	13.38618	-61.47107
even-odd	0.42795	-1.51802	-12.85355	-29.91325	134.38603
odd-even	0.45247	-1.36567	-20.43095	-2.84063	-12.84442
odd-odd	0.42803	-1.49781	-14.45798	-8.51460	45.74770

2.4.11 New Ren B

Akrawy et al. [58] modified the Ren B formula by including nuclear isospin asymmetry and angular momentum. The term $\frac{\hbar}{2\mu} l(l+1)$ is known as the centrifugal potential which increases the height of the potential barrier for even-odd, odd-even and odd-odd nuclei while for even-even nuclei it is zero. The formula for New Ren B is given as:

$$\log_{10} T_{\frac{1}{2}}^{NRB} = a\sqrt{\mu} Z_1 Z_2 Q^{-\frac{1}{2}} + b\sqrt{\mu Z_1 Z_2} + c + dI + eI^2 + f(\ell(\ell+1)), \tag{2.60}$$

where I is the asymmetry term. The term is related to the total angular momentum that has been added to the centrifugal potential through the $\ell(\ell+1)$ dependence. The values of $a, b, c, d, e,$ and f for e-e, e-o, o-e and o-o are given in Table 2.6.

Table 2.6: The coefficients for the New Ren B empirical formula [56]

Heavy Nuclei	a	b	c	d	e	f
even-even	0.41107	-1.44914	-14.87085	13.38618	-61.47107	0
even-odd	0.44145	-1.42068	-16.59713	-27.68464	91.70405	0.07470
odd-even	0.44660	-1.32208	-21.09761	-1.64226	-17.02692	0.07767
odd-odd	0.43323	-1.40527	-17.13866	-7.66291	22.26925	0.06902

2.4.12 Dong formula

Dong et al. [5] proposed the empirical formula for alpha decay half-lives by combining Royer's formula and WKB approximation. The formula is then denoted as:

$$\log_{10} T_{\frac{1}{2}} = a + bA^{\frac{1}{6}}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} + \frac{\ell(\ell+1)}{\sqrt{(A-4)(Z-2)A^{\frac{-2}{3}}}}, \tag{2.61}$$

where a, b, c are the fitting parameters which are fixed to a set of experimental data [5].

2.4.13 Wang formula

Wang et al. [59] modified the Dong formula by adding the fitting coefficient $d^{1-(-1)^\ell}$ and a correction factor S which they set as 0.5

$$\log_{10} T_{\frac{1}{2}}^{Wang} = aZQ^{-\frac{1}{2}} + bA^{-\frac{1}{6}}Z^{1/2} + c + \frac{d^{1-(-1)^\ell}\ell(\ell+1)}{\sqrt{(A-4)(Z-2)A^{-2/3}}} + S. \tag{2.62}$$

where the fitting parameters a, b, c and d of the above equation are fixed by fitting to a set of experimental data. The values of a, b, c, d for even-even, even-odd, odd-even, odd-odd are given in Table 2.7

Table 2.7: The coefficients for the Wang empirical formula [59]

	a	b	c	d
even-even	-25.432	-1.146	1.577	-
even-odd	-26.591	-1.171	1.639	1.123
odd-even	-27.747	-1.093	1.620	0.829
odd-odd	-28.460	-0.984	1.573	0.970

2.5 Machine Learning

Machine learning is a branch of Artificial Intelligence (AI). The basic concept of this field is to enable machines to learn from data without explicitly programming and make predictions on a set of unknown data. Machine learning has been applied to global statistical modeling and prediction of nuclear properties since the early 1990's [60]. For example, Machine Learning systems can be trained on an automatic speech recognition system to convert acoustic information in a sequence of speech data into semantic structure expressed in the form of a string of words [61]. Learning is the process of converting experience into expertise or knowledge. The input to a learning algorithm is training data, representing experience [62].

2.5.1 Types of Machine Learning

There are different types of machine learning algorithms [63]:

(i.) Supervised learning algorithm

(ii.) Unsupervised learning algorithm

(iii.) Semi-supervised learning algorithm

(iv.) Reinforcement learning algorithm

2.5.2 Supervised Learning

In supervised learning, the target is to infer a function or mapping from training data that is labeled. The training data consist of input vector X and output vector Y of labels or tags. A label or tag from vector Y is the explanation of its respective input example from input vector X. Together they form a training example, i.e. training data comprises training examples. If the labeling does not exist for input vector X, then X is unlabeled data [64].

2.5.3 Unsupervised Learning

Unsupervised learning involves *unlabeled* data, which is typically the case for clustering algorithms. In unsupervised learning, the training data is unlabeled i.e The system tries to learn without a teacher. The goal of unsupervised learning algorithm is to create a model that takes a feature vector x as input and either transforms it into another vector or into a value that can be used to solve a practical problem [65].

2.5.4 Semi-supervised Learning

Semi-supervised learning algorithms are combinations of unsupervised and supervised learning. For example, *deep belief networks* (DBNs) are based on supervised components called *restricted Boltzmann machines* (RBMs) stacked on top of one another. RBMs are trained sequentially in an unsupervised manner, and then the whole system is fine-tuned using supervised learning techniques [63]. In semi-supervised learning, the quantity of unlabeled examples is much higher than the number of labeled ones. The goal of a semi-supervised learning is the same as that of supervised learning algorithm. The hope is that using many unlabeled examples can help the learning algorithm to produce a better model [65].

2.5.5 Reinforcement Learning

Reinforcement learning is a sub-field of machine learning where an agent learns by interacting with an environment and changing its behavior to maximize its reward. For example, a robot can be trained to navigate in a complex environment by assigning a high reward to actions that help the robot reach a desired destination [66]. The machine can execute actions in every state. Different actions bring different rewards and could also move the machine to another state of the environment. The goal of a reinforcement learning algorithm is to learn a policy [65].

2.6 Linear and Non-Linear Machine Learning Models

Algorithms are the most obvious part of machine learning i.e. any problem can be solved differently. The method of the machine learning algorithm we choose to use affects the precision, performance, and size of the final model. The machine learning models can be classified into two which includes the linear machine learning models and non-linear machine learning models.

2.6.1 Linear machine learning models

The linear model assumes that the population of a given data can be adequately modeled by a probability distribution that has a fixed set of parameters. The machine learning algorithms that fall under this category include :

1. Linear regression
2. Logistic regression

Linear regression

This is one of the supervised machine learning algorithm where the predicted output is continuous and has a constant slope. It is normally used to predict values within a continuous range, rather than trying to classify them into categories [63]. The linear regression model prediction is giving by:

$$\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + ... + \theta_n x_n. \tag{2.63}$$

$\hat{y}$ is the predicted value, n is the number of features, x_i is the i^{th} feature value , θ_j is the jth model parameter (which include the bias term θ_0 and the feature weights $\theta_1, \theta_2, \theta_3.....\theta_n$). The Mean Square Error (MSE) of a linear regression hypothesis h_θ on a training set $\mathbf{X}$ and is calculated using [63]:

$$MSE(\mathbf{X}, h_\theta) = \frac{1}{m}\sum_{i=1}^{m}(\boldsymbol{\theta^T x^i} - y^i)^2. \tag{2.64}$$

To calculate the value of θ which minimizes the cost function, there is a closed-form solution i.e a mathematical equation that gives the result directly which is regarded as the normal equation and it is given by:

$$\hat{\boldsymbol{\theta}} = (X^T X)^{-1} X^T \mathbf{y}. \tag{2.65}$$

$\hat{\boldsymbol{\theta}}$ is the value of θ that minimizes the cost function, $\mathbf{y}$ is the vector of values containing $y^{(i)}$ to $y^{(m)}$

Ridge Regression

Ridge Regression is also known as Tikhonov regularization. Here, the regularization term is added to the cost function to force the learning algorithm to not fix the data but to keep the model weights as small as possible. The regularization term added to the cost function is given as $\alpha \sum_{i=1}^{n} \theta_i^2$, where α, in this case, is known as hyperparameter that controls how much the model is regularized [63]. The Ridge regression formula is given as:

$$J(\theta) = MSE + \alpha \frac{1}{2} \sum_{i=1}^{n} \theta_i^2. \tag{2.66}$$

MSE is the Mean Square Error (MSE) and θ is the minimizing cost function.

Lasso Regression

Lasso Regression is otherwise known as Least Absolute Shrinkage and Selection Operator Regression. This is another regularization version of linear regression which is similar to Ridge Regression. It adds a regularization term to the cost function, but it uses the ℓ_1 norm of the weight vector instead of half the square of the ℓ_2 norm. The formula for the Lasso regression is given as:

$$J(\theta) = MSE + \alpha \sum_{i=1}^{n} |\theta_i|. \tag{2.67}$$

One of the important characteristics of Lasso Regression is that it normally eliminates the least important features [63].

Elastic Net

The middle ground between Ridge and Lasso regression is the Elastic Net. The Regularization term is a mixture of both Ridge and Lasso's regularization terms. The formula is given as

$$J(\theta) = MSE(\theta) + r\alpha \sum_{i=1}^{n} |\theta_i| + \frac{1-r}{2}\alpha \sum_{i=1}^{n} \theta_i^2, \quad (2.68)$$

where r is the mix ratio. When $r = 0$, the elastic net is equivalent to the ridge regression, and when $r = 1$ the elastic net is equivalent to Lasso Regression [63]

2.6.2 Logistic Regression

This predicts the output of a categorical dependent variable. The outcome of logistic regression must be a categorical or discrete value, i.e Yes or No, 0 or 1, True or False, etc. Instead of giving the exact value as 0 or 1, it gives the probabilistic values which lie between 0 or 1 [67].

2.6.3 Decision Trees

A Decision Tree is an acyclic graph that is used to make decisions i.e. A decision tree can learn from data. In each branching node of the graph, a specific feature j of the feature vector is examined. If the value of the feature is below a specific threshold, then the left branch is followed; otherwise, the right branch is followed. As the leaf node is reached, the decision is made about the class to which the example belongs. The formulations of the $ID3$ decision tree learning algorithm is given as:

$$\frac{1}{N} \sum_{i=1}^{N} y_i \ln f_{ID3}(x_i) + (1 - y_i) \ln(1 - f_{ID3}(x_i)). \quad (2.69)$$

where $f_{ID3}(x_i)$ is a decision tree [65].

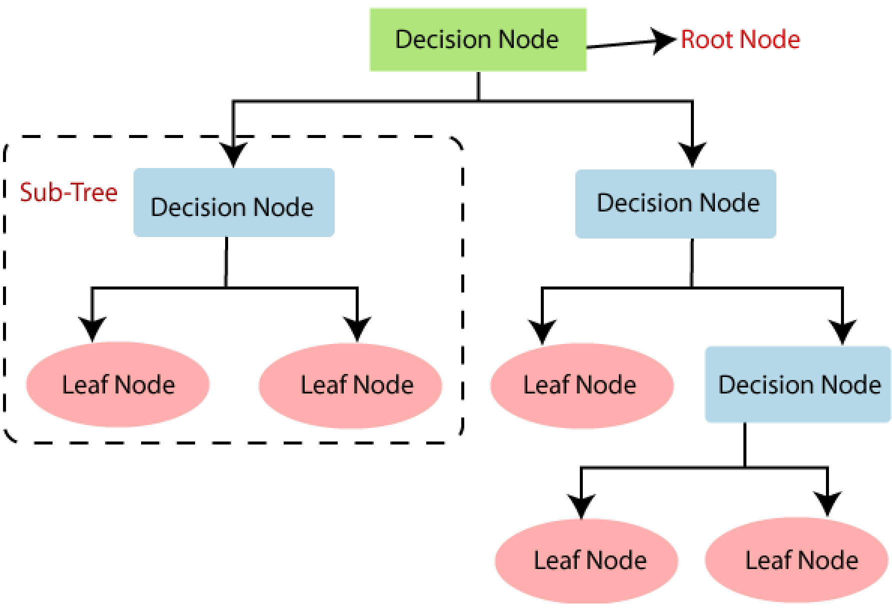

Figure 2.1: Decision Tree classification algorithms

2.6.4 Random Forest

Random forest is an ensemble learning method for classification, regression, and other tasks. It operates by constructing a multitude of decision trees at a training time [63]. Random forest uses a modified tree learning algorithm that inspects, at each split in the learning process, a random subset of the features. The reason for doing this is to avoid the correlation of the trees: if one or a few features are very strong predictors for the target, these features will be selected to split examples into many trees. This would result in many correlated trees in the "forest". Correlated predictors cannot help in improving the accuracy of prediction. Giving a training database, a random sample S_b is created for each $(b = 1, 2, ..., B)$ of the training set and build a decision tree model f_b using each sample S_b as the training set. To sample S_b for some b, we do the sampling with a replacement that is, we pick examples at random until $|S_b| = N$. After training, we have B decision trees. The prediction for a new example x is obtained as the average of B predictions is given as [65]:

$$y \leftarrow \hat{f}(x) \stackrel{def}{=} \frac{1}{B}\sum_{b=1}^{B} f_b(x). \tag{2.70}$$

In the case of regression, or by taking the majority vote in the case of classification [65].

2.7 Non-Linear Machine learning model

Non-linear Machine learning models make no assumptions about some probability distribution when modeling the data. The Machine learning model that falls under this category includes

1. k Nearest neighbour (kNN) Classifier
2. Support Vector Machine
3. Neural Networks.

2.8 k-Nearest neighbour (kNN)

The k-nearest neighbour (kNN) algorithm is a simple, easy to implement supervised machine learning algorithm which can be used to solve both classification and regression problems [67]. kNN is one of the non-linear machine learning algorithms that keeps all training examples in memory. That is, when a new example of X comes in, the kNN algorithm finds k training example that is closest to X then returns the majority label [65]. The most common distance is the euclidean and negative cosine similarity.

The Euclidean distance is used to check the closeness between two examples and the equation is given as:

$$d(x_i, x_k) \stackrel{def}{=} \sqrt{(x_i^{(1)} - x_k^{(1)})^2 + (x_i^{(2)} - x_k^{(2)})^2 + ... + (x_i^{(N)} - x_k^{(N)})^2}, \tag{2.71}$$

or

$$d(x_i, x_k) \stackrel{def}{=} \sqrt{\sum_{j=1}^{n}(x_i^{(j)} - x_k^{(j)})}, \tag{2.72}$$

while the negative cosine similarity distance function is defined as [65]:

$$s(x_i, x_k) \stackrel{def}{=} \frac{\sum_{j=1}^{n} x_i^{(j)} x_k^{(j)}}{\sqrt{\sum_{j=1}^{n}(x_i^{(j)})^2}\sqrt{\sum_{j=1}^{n}(x_k^{(j)})^2}}. \tag{2.73}$$

KNN (Classification)

KneighboursClassifier implements learning based on the k nearest neighbour of each query point, where k is an integral value. The simplest nearest neighbour classification uses uniform weight i.e the value assigned to a query point is computed from a simple majority vote to the nearest neighbour [67]. The k-nearest neighbour classifier can be visualized by assigning the k nearest neighbour a weight $1/k$ and all others 0. This generalizes to weighted nearest neighbour classifiers. where the ith nearest neighbour is assigned a weight w_{ni}, with $\sum_{i=1}^{n} w_{ni} = 1$ [68].

KNN (Regression)

The neighbours-based regression can be used when the data labels are continuous. The label assigned to the query point is dependent on the mean of its nearest neighbours. Basic nearest neighbour regression uses uniform weight i.e. each point in the local neighbourhood contributes uniformly to the classification of a query point [67].

2.8.1 Support Vector Machine

Support Vector Machine (SVM) is a machine learning algorithm model which performs linear and non-linear classification, Outlier Detection, and Regression. SVM is one of the important and popular models in machine learning which falls under the supervised learning method. Support vector machine can be used to find a hyperplane in an N-dimensional space i.e. the number of features that distinctly classify the dataset. SVM can be used for both classification and regression problems [65].

Support Vector Classification (SVC)

SVC performs multi-class and binary classification on dataset by locating the hyperplane with the greatest margin between the predicted classes in the training data. The SVC is denoted by the equation

$$P(Y_i|X_i) = b_0 + \sum a_j K(x_i, x_j). \tag{2.74}$$

K is the kernel function that compares the similarities of (x_i, x_j) observations and a_j is the learning parameter. Given training vectors $x_i \epsilon \mathbb{R}^p$, $i = 1, ..., n$ in two classes, and a vector

$y\epsilon(1,-1)^n$, to get $w\epsilon\mathbb{R}^p$ and $b\epsilon\mathbb{R}$ such that the prediction is given by

$$sign(w^T\phi(x)+b). \tag{2.75}$$

SVC can be used to solve the primal problem using

$$\min \frac{1}{2}w^T w + c\sum_{i=1}^{n}\zeta, \tag{2.76}$$

subject to

$$y_i(w^T\phi(x_i)+b) \geq 1-\zeta_i, \tag{2.77}$$

where $\zeta_i \geq 0$ and $i = 1......n$ [67].

Support Vector Regression

The Support Vector Regression (SVR) are used to solve regression problems. The SVR uses a kernel to transform data and then find an optimal boundary between the possible results [61]. The SVR models depend on a subset of the training data because the cost function ignores samples whose prediction is close to their target [67].

2.9 Artificial Neural Network (ANN)

The mathematical model that mimics the human brain is known as the Artificial Neural Network (ANN). ANN is a machine learning model which is inspired by the network of biological neurons. These neurons are regarded as the processing unit. The neurons are connected via adaptive synaptic weights [69]. ANN is composed of three layers. The first layer is regarded as the input layer, the intermediate layer is called the hidden layer, and the last layer is the output layer as shown in Figure 2.2 [25].

The simplest form of ANN is known as the perceptron invented in 1957 by Frank Rosenblatt. Perceptron uses the principle of Threshold logic unit (TLU), that is the input and output are numbers instead of binary on and off values. In Perceptron, each input connection is associated with a weight. The weighted sum of its input is given as:

$$z = w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n = X^TW. \tag{2.78}$$

Then step function will be applied to the sum which gives an output of

$$h_w(x) = step(z), \tag{2.79}$$

where $z = X^TW$. x_1,x_2, x_3......x_m are the input data while y is the output. To compute the fully connected layers that is when all the neurons in a layer are connected to every neuron in

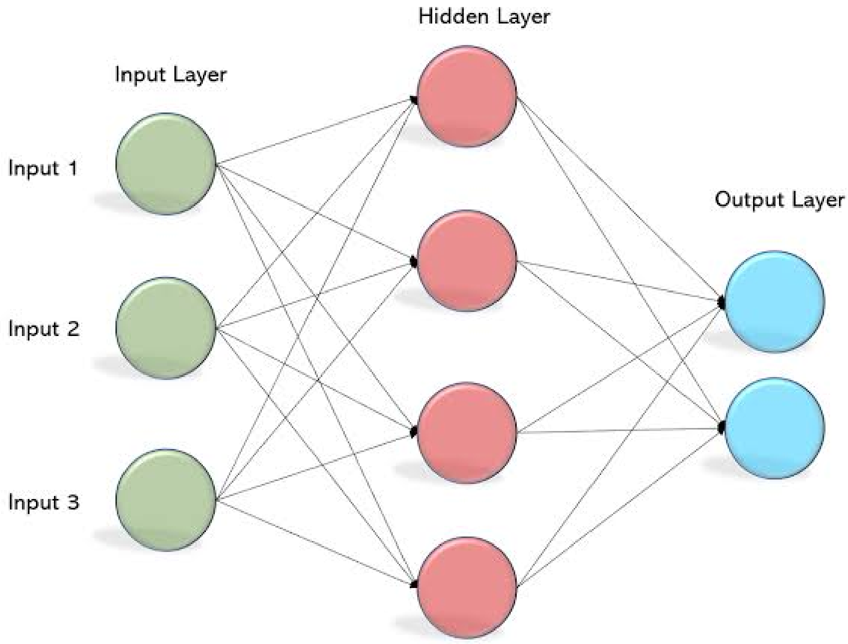

Figure 2.2: Multi-layer Perceptron [70].

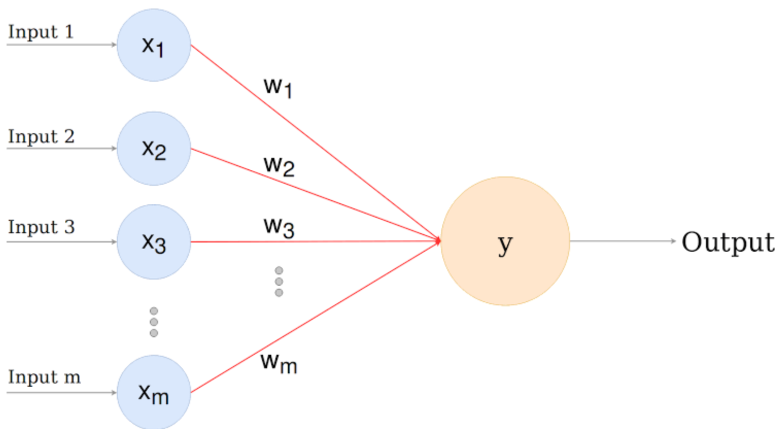

Figure 2.3: Perceptron Model [71]

the previous layer is given as [63]:

$$h_{W,b}(X) = \phi(XW + b), \tag{2.80}$$

where X represents the matrix of input features, having one row per instance and one column per feature. The weight matrix W contains the connection weights except for the ones from the bias neuron. It has one row per input neuron and one column per artificial neuron in the layer. The bias vector b contains all the connection weights between the bias neuron and the artificial neurons. It has one bias term per artificial neuron. The function ϕ is called the activation function [63].

Chapter 3

Methodology

3.1 Computational details

Here, the description of the computation is presented. The programming language used for the computations is Python 3.8. Jupyterlab, a web-based IDE was utilized. Jupyterlab is an extensive environment for interaction and reproducible computing based on the architecture. Python is a high-level programming language that is utilized because of its English-like syntax and also, a lesser line of code is needed to perform the same tasks compared to other programming languages such as Java, MatLab, C, C++, and so on. In python, debugging is easy because it displays an error at a time. Assignment of data, declaration of a variable is normally done automatically. The python modules used include Numpy, pandas, Matplotlib, Scikitlearn and Tensorflow [63].

This study involves three main calculation stages. In the first stage, new coefficients for the α-decay empirical formulas are obtained using a least square fit scheme. The empirical formulas considered are AKRE, Royer, Denisov-Khudenko, New Ren A, New Ren B, Wang, and Viola-Seaborg formulas. The python module used for the fitting procedure is Scipy.

The second stage involves the training of the machine learning models to predict the alpha decay half-lives of all alpha emitters in the NUBASE2020 database. A total of 549 alpha emitters from the NUBASE2020 database were considered in the computation. Machine learning models such as support vector regression, k nearest neighbour, decision trees, random forest, extra trees, and artificial neural network were employed in the calculation. The Scikitlearn (SKLearn) python module was used in training the machine learning models, while TensorFlow was used for artificial neural network.

The third stage involves the training of the Q_α values using artificial neural network. The Q_α values are required as inputs to predict the alpha decay half-lives of unmeasured superheavy nuclei. Since these nuclei to be predicted have not been measured in the laboratory, this study employs the use of artificial neural network to predict the Q_α values. The feed-forward neural network implemented in TensorFlow (in conjunction with scikitlearn) was used.

There are some steps to take when employing machine learning models to make predictions. They are:

1. Collection of data,
2. Preparation of data,
3. Choosing of the model,
4. Training of the machine learning model,
5. Evaluation of the model,
6. Tuning of the hyper-parameters,
7. Making predictions.

3.1.1 Collection and Preparation of Data

The data used in this study viz. the mass number (A), atomic number (Z), experimental Q_α values, orbital angular momentum carried by the emitted α-particle are all taken or derived from the NUBASE2020 database [72, 73, 74]. NUBASE2020 is an evaluated nuclear data that contains the recommended values of nuclear physics properties such as excitation energies, masses, decay modes, and their intensities for all nuclei with experimental values in ground, excited, and isomeric ($T_{\frac{1}{2}} \geq 100ns$) states. The NUBASE2020 database also contains information of unobserved nuclei whose properties were derived by following the systematic trends in neighbouring nuclei [73]. The unobserved nuclei were not considered in this study. Only experimental values were extracted. The data retrieved include atomic number(Z), mass number(A), spin and parity, half-lives, Q_α-value, decay modes, and their intensities.

The data for 554 alpha emitters were derived from the database. The data include the mass number, atomic number, spin and parity, mass excess, and half-lives. But it was observed that some nuclide do not have spin and parity values, and some do not have defined decay modes; this leaves the space blank (or inclusion of NaN). So a simple line of code was written to remove the “Not a Number” (NaN). This is necessary because some machine learning models do not work with missing features. After data cleaning, a total of 549 nuclei were obtained. The decay energy Q_α was computed using [72]:

$$Q_\alpha = \Delta M_P - \Delta M_D - \Delta M_\alpha, \tag{3.1}$$

where ΔM_P and ΔM_D are the mass excesses of parent and daughter nuclei, respectively and ΔM_α is the excess mass of the alpha particle [52, 72].

Alpha-particle emission obeys the spin-parity selection rule. When the spin and parity values are different, the alpha emitter carries a non-zero angular momentum ℓ [59]. The minimum value of angular momentum ℓ_{min} at the α-transition between states with j_p, π_p, j_d and π_d is given as [48, 52]:

$$\ell = \begin{cases} \Delta_j \text{ for even } \Delta_j \text{ and } \pi_p = \pi_d \\ \Delta_j + 1 \text{ for odd } \Delta_j \text{ and } \pi_p = \pi_d \\ \Delta_j \text{ for odd } \Delta_j \text{ and } \pi_p \neq \pi_d \\ \Delta_j + 1 \text{ for even } \Delta_j \text{and } \pi_p \neq \pi_d \end{cases}, \tag{3.2}$$

where

$$\Delta_j = |j_p - j_d|, \tag{3.3}$$

and j_p, π_p, j_d and π_d are the spin and parity values of the parent and daughter nuclei, respectively. The spin and parity values extracted from the NUBASE2020 database were used to compute the angular momentum.

3.1.2 Choosing of the models

The machine learning models employed in this study are:

1. k Nearest neighbour (kNN),
2. Support Vector Regression,
3. Decision Tree,
4. Random Forest,
5. Extra Tree,
6. Artificial Neural Network.

Machine learning models such as linear regression, ridge regression, and LASSO regression were not considered because of their linear nature. Moreover, a quick calculation involving the training of all the machine learning models including linear models show that the linear models have higher standard deviation values than the non-linear models.

3.1.3 Training and Evaluation of the Machine learning models

The training of the machine learning models is done via regression. Before the machine learning models are trained, feature scaling is carried out. Most machine learning algorithms do not perform satisfactorily when the input attributes have very different scales. Only the input features are scaled, while the output features are not scaled. However, it is a good practice to ensure that the scales of the output features are not on very different scales. In practice, one employs either min-max scaling or standardization to ensure that all input features have the same scale. In this research, standardization was employed. The normalisation is done using the scikit-learn module MinMaxScalar. Normalisation is the process by which a data is rescaled from the original range so that all the values are in range of 0 and 1. The min-max scaling is defined with default hyper-parameters. Once defined, the dataset can be called using the fit_transform() function which create transformed version of the data. Standardization is the process of scaling each input variable separately by subtracting the mean and dividing by the standard deviation to shift the distribution to a mean of zero and a standard deviation of one [75].

The data is then split into two, called the training set and the test set. Mostly, the data is split in the ratio of 80 : 20, that is 80% of the data for training, and 20% of the data for testing/validation. Scikit-learn can be used to split data into multiple subset using the function **train_test_split()** [63]. The machine learning algorithms are trained using the training set. They are then validated using the test set. This study is interested in machine learning algorithms that can make the best predictions of the test set.

When a machine learning algorithm performs well on the training dataset but does not generalize well (on the test data for example), it is said to overfit. If the algorithm does not perform well on both the testing and training dataset, it is said to be underfitting. This study ensures that the algorithms do not overfit. Some techniques to avoid overfitting are:

(i) Cross-validation: This allows the tuning of hyperparameters with only the training dataset, where the test dataset is kept unseen to select the best machine learning model. Cross-validation is used to measure the skill of a machine learning model. It is used to compare a model for a given predictive modeling problem. This process has a single parameter K that is referred to the numbers of group that a given dataset can be split into known as the K-fold cross-validation where a specific K is chosen for example $K = 7$ means 7-fold cross-validation. In this research k-fold cross-validation was used.

(ii) Reduce Complexity or Data Simplification: Overfitting can occur due to the complexity of a model, such that, even with large volumes of data, the model still manages to overfit the training dataset. Simplifying the model can also make the model lighter and run faster.

(iii) Regularization: Regularization refers to a broad range of techniques for artificially forcing your model to be simpler [67].

3.1.4 Parameter Tuning

Many machine learning algorithms include parameters that cannot be directly estimated from the data. They are called hyperparameters; these hyperparamaters can be tuned to improve the performance of the algorithm. The tuning parameters control the complexity of the model and also affect any variance-base trade-off that can be made. The tuning of the hyperparameters was used in this study to obtain the best performance of a particular machine learning algorithm.

3.1.5 Prediction

After successful training and testing of the machine learning algorithms, the trained models can then be used to make predictions. In this study, the trained machine learning models are used to predict the α-decay half-lives of unmeasured superheavy nuclei.

3.1.6 Performance Measure

The performance measure employed for the regression algorithms is the root mean square error, also called the standard deviation. The standard deviation is calculated using the formula [4, 49]:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[(\log_{10} T_{\frac{1}{2}}^{theory} - \log_{10} T_{\frac{1}{2}}^{expt})^2\right]}, \tag{3.4}$$

where $\log_{10} T_{\frac{1}{2}}^{theory}$ is the theoretical α-decay half-lives and $\log_{10} T_{\frac{1}{2}}^{expt}$ is the experimental half-lives from the NUBASE2020 database [76].

Chapter 4

Results and Discussion

This chapter presents the results of the calculations described in the previous chapter, and using the theoretical frameworks described in chapter 2. As explained previously, the input parameters such as the atomic number, mass number, orbital angular momentum, and Q_α values have been taken or derived from the NUBASE2020 database. The results of the calculations are presented in five stages. In the first stage, the empirical formulas described in chapter 2 are used to study the half-lives of some alpha emitters. New coefficients are obtained for the empirical formulas with the aid of least square fit scheme and using half-lives from the NUBASE2020 database. The improved formulas are then used to calculate the alpha decay half-lives for the 549 alpha emitters.

In the second stage of the calculations, the machine learning algorithms are trained as described in chapter 3. The trained machine learning models are then used to predict the alpha decay half-lives of the test dataset. The third stage involves the training of artificial neural network (ANN) to predict Q_α values. The trained ANN will then be used to predict the Q_α values for unmeasured superheavy nuclei. In the fourth stage, the Q_α values predicted by ANN (denoted Q_{ANN}) are used as inputs to calculate the alpha decay half-lives using the improved empirical formulas. In the last stage, the alpha decay half-lives of unmeasured superheavy nuclei are predicted using the empirical formulas and the trained machine learning models, with Q_{ANN} as input.

4.1 α-decay half-lives using improved empirical formulas

This research begins with the calculations of the alpha decay half-lives of the α-emitters in the NUBASE2020 database using the empirical formulas. The existing coefficients of the formulas were obtained using old NUBASE databases. Since the NUBASE2020 has been published (in March 2021), this study used input values from the database. A least square fit scheme was used to obtain the new coefficients for the empirical formulas. The empirical formulas used are AKRE formula, Royer formula, Viola-Seaborg Semi-empirical formula (VSS), Denisov-Khudenko formula (DK), Universal Decay Law (UDL), New Ren A (NRA), New Ren B (NRB) and Wang formulas. The new coefficients obtained are shown in Table 4.1 for AKRE formula, Table 4.2 for Royer formula, Table 4.3 for DK, Table 4.4 for UDL, Table 4.5 for VSS, Table 4.6 for NRA formula, Table 4.7 for NRB formula, and Table 4.8 for Wang formula.

Table 4.1: New Coefficient of AKRE empirical formula.

	Even-Even	Even-Odd	Odd-Even	Odd-Odd
a	-26.31197	-25.88866	-26.37629	-25.53476
b	-1.12636	-1.07649	-1.12706	-1.16056
c	1.58222	1.57300	1.63619	1.59501
d	6.79950	-12.51646	-10.56125	1.42134
e	-27.47641	58.13543	28.38162	20.05239

Table 4.2: New Coefficients of Royer empirical formula .

	Even-Even	Even-Odd	Odd-Even	Odd-Odd
a	-25.59937	-25.00305	-24.30631	-25.95298
b	-1.13616	-1.13267	-1.18607	-1.10881
c	1.57714	1.56228	1.57867	1.58483
d	0	6.9116×10^{-07}	9.7862×10^{-07}	9.7423×10^{-07}
e	0	0.00172	6.3120×10^{-05}	-9.6530×10^{-05}

Table 4.3: New Coefficients of Denisov-Khudenko empirical formula.

	Even-Even	Even-Odd	Odd-Even	Odd-Odd
a	-25.76959	-26.21963	-26.62950	-28.25889
b	-1.33049	-1.10154	-1.10950	-1.04583
c	1.57760	1.57809	1.59559	1.61437
d	0	0.23867	0.39786	0.24588
e	0	-0.44987	0.00991	-0.02741

Table 4.4: New Coefficients of UDL empirical formula.

	Even-Even	Even-Odd	Odd-Even	Odd-Odd
a	0.40777	0.41198	0.42107	0.421626
b	-0.418080	-0.38596	-0.43146	-0.39697
c	-21.89697	-24.16290	-22.19507	-24.26627

Table 4.5: New Coefficients of Viola-Seaborg Semi-empirical formula.

a	1.51469
b	4.57594
c	-0.18333
d	-35.08098
h_{e-o}	0.52757
h_{o-e}	0.48138
h_{o-o}	0.84940

Table 4.6: New Coefficients of New Ren A empirical formula.

	Even-Even	Even-Odd	Odd-Even	Odd-Odd
a	0.40945	0.40635	0.42309	0.41175
b	-1.41110	-1.34494	-1.41382	-1.43931
c	-15.22596	-15.29620	-15.23777	-14.28919
d	8.66869	-10.65773	-8.29247	2.21427
e	-49.59890	36.49591	5.39787	1.35486

Table 4.7: New Coefficients of New Ren B empirical formula.

	Even-Even	Even-Odd	Odd-Even	Odd-Odd
a	0.40945	0.40946	0.42030	0.41347
b	-1.41110	-1.38150	-1.40927	-1.43801
c	-15.22596	-14.85695	-15.39430	-14.53357
d	8.66869	-9.51163	-7.25867	1.70485
e	-49.59890	27.50118	7.13575	1.27281
f	0	0.03226	0.02983	0.0086

Table 4.8: New Coefficients of Wang empirical formula.

	Even-Even	Even-Odd	Odd-Even	Odd-Odd
a	-26.09938	-27.44251	-27.66732	-29.16054
b	-1.13616	-1.09889	-1.12399	-1.06569
c	1.57714	1.60178	1.62393	1.64075
d	0	0.99693	0.83934	0.63339

The standard deviation values obtained using the eight empirical formulas and data from 549 alpha emitters are shown in Table 4.9. For even-even nuclei, the AKRE formula gives the lowest standard deviation value of 0.37866. Royer gives the lowest standard deviation value (0.51962) for even-odd nuclei while for odd-even nuclei, New Ren B gives the lowest standard deviation (0.55537). For odd-odd nuclei, Royer formula gives the lowest standard deviation value (0.75933). With a standard deviation value of 0.541078, the Royer formula gives the best descriptions of the alpha decay half-lives of the alpha emitters. The standard deviation value given in the seventh column of the Table is the value obtained using existing coefficients in the empirical formulas, but with Q_α values from NUBASE2020 database.

Table 4.9: Standard deviations (σ) for the improved empirical formulae.

Empirical Formula	Even-Even	Even-Odd	Odd-Even	Odd-Odd	All Nuclei	Old Coefft.
Denisov	0.38850	0.53959	0.55712	0.77752	0.547775	0.95642
AKRE	0.37866	0.62536	0.64147	0.77815	0.588615	0.75221
NRA	0.48731	0.62310	0.64305	0.77892	0.589183	0.74672
NRB	0.38173	0.57433	0.55537	0.77079	0.553762	0.86885
Royer	0.38849	0.51962	0.57056	0.75933	0.541078	0.86846
UDL	0.39699	0.65166	0.64837	0.79755	0.606310	0.80785
WANG	0.38849	0.59973	0.58111	0.85959	0.589789	0.77004
VSS	0.41714	0.65212	0.67399	0.77973	0.629456	1.02196

The logarithms of the α-decay half-lives for nuclei with atomic number in the range of $Z = 92 - 118$ have been calculated using the improved formulas. The results are shown in Table 4.10. This range of Z contains both heavy and superheavy nuclei. The first five columns of the Table shows the atomic number (Z), mass number (A), experimental Q_α value, orbital angular momentum (ℓ) and the experimental half-lives. The columns sixth to thirteenth are the empirical formulas used to calculate the α-decay half-lives.

Table 4.10: Computed α-decay half-lives, $\log\left[T_{1/2}(s)\right]$, for nuclei with $Z = 92 - 188$ using the improved empirical formulas.

				$\log\left[T_{1/2}(s)\right]$								
Z	A	Q_α	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
92	216	8.531	0	-2.161	-2.615	-2.615	-2.533	-2.669	-2.544	-2.544	-2.615	-2.642
92	218	8.775	0	-3.451	-3.352	-3.352	-3.284	-3.360	-3.294	-3.294	-3.352	-3.376
92	219	9.950	5	-4.222	-4.939	-4.830	-5.875	-5.837	-5.874	-5.133	-4.810	-5.824
92	221	9.889	0	-6.180	-6.011	-5.952	-5.725	-5.698	-5.723	-5.963	-5.999	-5.715
92	222	9.481	0	-5.328	-5.293	-5.295	-5.263	-5.205	-5.279	-5.279	-5.293	-5.312
92	223	9.158	2	-4.187	-3.943	-4.026	-3.904	-3.905	-3.902	-3.949	-3.942	-3.916
92	224	8.628	0	-3.402	-3.059	-3.061	-3.042	-2.949	-3.053	-3.053	-3.059	-3.067
92	225	8.007	2	-1.208	-0.631	-0.711	-0.562	-0.603	-0.558	-0.594	-0.600	-0.576
92	226	7.701	0	-0.570	-0.209	-0.210	-0.206	-0.082	-0.211	-0.211	-0.209	-0.205
92	227	7.235	2	1.820	2.022	1.939	2.127	2.044	2.130	2.098	2.070	2.095
92	228	6.800	0	2.748	3.109	3.108	3.097	3.248	3.099	3.099	3.109	3.128
92	229	6.476	0	4.239	4.607	4.777	5.227	5.094	5.229	5.010	4.897	5.179
92	230	5.992	0	6.243	6.699	6.698	6.671	6.848	6.679	6.679	6.699	6.733
92	231	5.576	2	9.958	9.525	9.427	9.678	9.484	9.681	9.673	9.611	9.632

Table 4.10 (contd)

Z	A	Q_α	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
92	232	5.414	0	9.337	9.748	9.748	9.700	9.911	9.711	9.711	9.748	9.796
92	233	4.909	0	12.701	12.920	13.176	13.755	13.496	13.757	13.568	13.425	13.701
92	234	4.858	0	12.889	13.182	13.183	13.114	13.356	13.128	13.128	13.182	13.247
92	235	4.678	1	16.346	15.815	15.813	15.392	15.078	15.386	15.256	15.092	15.287
92	236	4.573	0	14.869	15.161	15.162	15.066	15.357	15.077	15.077	15.161	15.237
92	238	4.270	0	17.149	17.490	17.491	17.366	17.705	17.375	17.375	17.490	17.579
93	219	9.207	0	-3.244	-4.004	-3.915	-3.861	-3.672	-3.874	-4.045	-4.008	-3.900
93	220	10.226	5	-7.538	-5.599	-5.636	-5.949	-5.800	-5.954	-5.795	-5.677	-5.946
93	222	10.200	8	-6.319	-5.375	-5.489	-5.872	-5.742	-5.874	-5.359	-3.077	-5.919
93	223	9.650	0	-5.602	-5.212	-5.131	-5.130	-4.785	-5.140	-5.279	-5.244	-5.136
93	224	9.329	2	-4.319	-3.603	-3.725	-3.732	-3.663	-3.730	-3.775	-3.771	-3.779
93	225	8.818	0	-2.187	-3.075	-2.968	-2.918	-2.625	-2.921	-3.063	-3.042	-2.918
93	227	7.816	3	-0.292	0.644	0.514	0.224	0.419	0.228	0.435	0.442	0.225
93	229	7.020	1	2.548	3.324	3.168	3.189	3.291	3.199	3.100	3.084	3.190
93	230	6.778	3	3.964	4.931	4.883	4.744	4.583	4.754	4.785	4.716	4.747
93	231	6.368	1	5.166	6.110	5.945	6.018	6.031	6.032	5.926	5.889	6.016
93	233	5.627	0	8.492	9.214	9.462	9.835	9.711	9.855	9.676	9.612	9.826
93	235	5.194	1	12.119	12.426	12.229	12.424	12.216	12.445	12.320	12.234	12.408
93	237	4.957	1	13.831	13.955	13.739	13.970	13.720	13.987	13.863	13.759	13.947
94	228	7.940	0	0.322	-0.214	-0.215	-0.175	-0.162	-0.175	-0.175	-0.214	-0.228
94	229	7.598	2	2.260	1.525	1.448	1.560	1.481	1.572	1.551	1.584	1.612
94	230	7.178	0	2.021	2.467	2.467	2.494	2.536	2.502	2.502	2.467	2.467
94	231	6.839	0	3.599	3.952	4.128	4.485	4.363	4.500	4.292	4.243	4.528
94	232	6.716	0	4.005	4.301	4.301	4.310	4.393	4.322	4.322	4.301	4.312
94	233	6.416	2	6.019	6.245	6.153	6.345	6.183	6.359	6.343	6.324	6.358
94	234	6.310	0	5.723	6.073	6.074	6.062	6.188	6.076	6.076	6.073	6.096
94	235	5.951	0	7.734	7.914	8.132	8.616	8.406	8.629	8.420	8.310	8.601
94	236	5.867	0	7.955	8.222	8.222	8.189	8.356	8.205	8.205	8.222	8.257
94	237	5.748	1	10.973	10.206	10.209	9.720	9.465	9.727	9.574	9.441	9.653
94	238	5.593	0	9.442	9.664	9.664	9.606	9.824	9.620	9.620	9.664	9.710

Table 4.10 (contd)

Z	A	Q_α	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
94	239	5.245	3	11.881	13.389	13.318	12.653	12.338	12.659	12.833	12.767	12.562
94	240	5.256	0	11.316	11.606	11.607	11.523	11.788	11.536	11.536	11.606	11.665
94	241	5.140	2	13.266	13.077	12.926	13.353	12.987	13.349	13.317	13.144	13.195
94	242	4.984	0	13.072	13.307	13.307	13.195	13.511	13.205	13.205	13.307	13.377
94	244	4.666	0	15.410	15.499	15.500	15.359	15.722	15.367	15.367	15.499	15.584
95	229	8.137	2	0.255	0.222	0.135	-0.034	0.078	-0.029	-0.018	0.070	-0.072
95	235	6.576	1	5.189	6.042	5.887	5.967	5.928	5.989	5.876	5.838	5.940
95	236	6.256	0	6.732	7.375	7.789	7.964	7.711	7.986	7.928	7.836	8.055
95	238	6.042	2	9.769	9.212	9.080	9.054	8.756	9.075	9.068	9.148	9.117
95	239	5.922	1	8.632	9.194	9.010	9.157	9.039	9.179	9.068	8.993	9.125
95	240	5.707	4	10.983	11.322	11.079	10.861	10.501	10.881	10.998	11.531	10.914
95	241	5.638	1	10.135	10.738	10.537	10.720	10.560	10.740	10.630	10.536	10.682
95	243	5.439	1	11.365	11.883	11.665	11.878	11.692	11.893	11.787	11.674	11.832
96	233	7.473	0	2.130	2.329	2.502	2.783	2.665	2.802	2.595	2.598	2.885
96	234	7.365	0	2.285	2.555	2.556	2.596	2.585	2.608	2.608	2.555	2.543
96	236	7.067	0	3.355	3.681	3.682	3.704	3.740	3.718	3.718	3.681	3.679
96	238	6.670	0	5.314	5.311	5.312	5.315	5.393	5.333	5.333	5.311	5.321
96	239	6.540	1	8.162	7.235	7.239	6.673	6.451	6.691	6.532	6.449	6.668
96	240	6.398	0	6.419	6.508	6.509	6.489	6.617	6.508	6.508	6.508	6.528
96	241	6.185	3	8.452	9.146	9.080	8.377	8.109	8.393	8.554	8.525	8.335
96	242	6.216	0	7.148	7.341	7.341	7.296	7.480	7.311	7.311	7.341	7.369
96	243	6.169	2	8.963	8.260	8.127	8.496	8.189	8.505	8.455	8.323	8.387
96	244	5.902	0	8.757	8.897	8.898	8.827	9.059	8.842	8.842	8.897	8.937
96	245	5.624	2	11.415	11.150	10.996	11.413	11.043	11.422	11.377	11.212	11.280
96	246	5.475	0	11.173	11.241	11.242	11.146	11.419	11.164	11.164	11.241	11.297
96	247	5.354	1	14.692	13.419	13.353	13.051	12.624	13.055	12.875	12.629	12.869
96	248	5.162	0	13.079	13.138	13.139	13.016	13.336	13.033	13.033	13.138	13.208
97	234	8.099	2	1.398	1.543	1.456	1.286	1.222	1.300	1.280	1.460	1.415
97	244	6.779	2	8.479	6.575	6.431	6.449	6.174	6.470	6.453	6.478	6.448
97	247	5.890	2	10.639	10.467	10.263	10.257	10.091	10.281	10.297	10.259	10.194

Table
4.10
(contd)

Z	A	Q_α	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
97	249	5.521	2	12.290	12.583	12.349	12.379	12.141	12.404	12.417	12.353	12.308
98	237	8.220	2	0.058	0.929	0.848	0.943	0.805	0.959	0.930	0.995	1.041
98	239	7.763	0	1.634	2.013	2.196	2.519	2.351	2.540	2.315	2.296	2.595
98	240	7.711	0	1.612	2.022	2.023	2.054	2.057	2.065	2.065	2.022	2.000
98	242	7.517	0	2.536	2.697	2.698	2.708	2.764	2.720	2.720	2.697	2.684
98	244	7.329	0	3.193	3.377	3.377	3.365	3.474	3.376	3.376	3.377	3.371
98	245	7.258	0	3.884	3.777	3.981	4.494	4.230	4.508	4.252	4.110	4.427
98	246	6.862	0	5.109	5.251	5.251	5.218	5.368	5.233	5.233	5.251	5.259
98	247	6.503	2	7.505	7.551	7.413	7.764	7.440	7.784	7.730	7.613	7.689
98	248	6.361	0	7.460	7.489	7.490	7.436	7.622	7.456	7.456	7.489	7.514
98	249	6.293	1	10.044	9.243	9.185	8.801	8.430	8.816	8.624	8.427	8.674
98	250	6.129	0	8.616	8.604	8.605	8.523	8.763	8.542	8.542	8.604	8.639
98	251	6.177	5	10.452	10.289	10.123	9.420	9.001	9.428	10.126	10.117	9.230
98	252	6.217	0	7.935	8.121	8.121	8.006	8.322	8.013	8.013	8.121	8.158
98	253	6.126	4	8.696	9.573	9.352	9.724	9.257	9.722	10.083	9.940	9.462
98	254	5.927	0	9.227	9.583	9.583	9.439	9.805	9.443	9.443	9.583	9.632
99	240	8.259	1	0.933	1.564	1.646	1.488	1.391	1.507	1.454	1.461	1.621
99	241	8.259	0	0.708	0.638	0.875	1.034	1.070	1.045	0.881	0.877	0.946
99	242	8.160	2	1.495	2.071	1.970	1.838	1.715	1.859	1.838	1.979	1.929
99	251	6.597	0	7.376	6.902	7.219	7.559	7.461	7.584	7.431	7.337	7.474
99	252	6.739	2	7.718	7.624	7.450	7.498	7.148	7.526	7.510	7.508	7.475
99	253	6.739	0	6.248	6.218	6.530	6.863	6.823	6.876	6.739	6.636	6.771
99	254	6.617	5	7.377	8.570	8.320	8.078	7.691	8.102	8.291	8.290	8.015
99	255	6.436	3	7.633	8.726	8.428	8.290	8.208	8.300	8.523	8.384	8.190
100	243	8.689	1	-0.595	0.868	0.872	0.188	0.005	0.204	0.025	0.027	0.250
100	246	8.379	0	0.218	0.444	0.445	0.464	0.493	0.467	0.467	0.444	0.406
100	247	8.258	4	1.685	1.634	1.504	1.609	1.370	1.626	2.009	2.086	1.590
100	248	7.995	0	1.538	1.701	1.702	1.700	1.774	1.708	1.708	1.701	1.674
100	249	7.709	4	2.464	3.558	3.410	3.552	3.269	3.572	3.955	3.997	3.508
100	250	7.557	0	3.270	3.255	3.256	3.235	3.350	3.247	3.247	3.255	3.242

Table
4.10
(contd)

Z	A	Q_α	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
100	251	7.424	1	6.025	5.170	5.118	4.660	4.336	4.679	4.474	4.324	4.573
100	252	7.154	0	4.961	4.812	4.813	4.769	4.928	4.786	4.786	4.812	4.812
100	253	7.198	5	6.334	6.485	6.328	5.599	5.231	5.614	6.303	6.332	5.461
100	254	7.307	0	4.067	4.152	4.152	4.078	4.312	4.083	4.083	4.152	4.153
100	255	7.241	4	4.859	5.331	5.133	5.466	5.059	5.472	5.820	5.721	5.254
100	256	7.025	0	5.066	5.274	5.274	5.173	5.459	5.176	5.176	5.274	5.286
100	257	6.864	2	6.940	6.705	6.528	7.093	6.630	7.097	6.988	6.731	6.837
101	244	8.947	1	-0.444	0.114	0.203	0.006	-0.073	0.023	-0.033	0.001	0.146
101	247	8.764	1	0.076	0.209	0.093	0.090	0.152	0.099	0.005	-0.003	-0.025
101	250	8.155	2	2.887	2.832	2.712	2.614	2.425	2.643	2.622	2.729	2.674
101	251	7.963	1	3.402	2.845	2.706	2.756	2.764	2.775	2.686	2.640	2.648
101	253	7.573	1	5.012	4.285	4.131	4.213	4.185	4.236	4.148	4.082	4.105
101	257	7.557	1	5.122	4.295	4.119	4.217	4.245	4.228	4.161	4.071	4.100
102	251	8.752	0	-0.016	0.132	0.325	0.735	0.473	0.755	0.485	0.411	0.731
102	252	8.549	0	0.562	0.582	0.583	0.594	0.626	0.601	0.601	0.582	0.536
102	253	8.415	1	2.234	2.415	2.367	1.837	1.539	1.858	1.646	1.549	1.796
102	254	8.226	1	1.755	1.612	1.613	1.603	1.682	1.613	1.613	1.612	1.576
102	255	8.428	5	2.848	2.723	2.575	1.822	1.495	1.838	2.514	2.581	1.720
102	256	8.582	0	0.466	0.401	0.401	0.362	0.522	0.359	0.359	0.401	0.362
102	257	8.477	2	1.460	1.404	1.263	1.697	1.338	1.706	1.591	1.430	1.529
102	259	7.854	2	3.667	3.548	3.390	3.877	3.463	3.889	3.775	3.575	3.682
103	254	8.822	3	1.224	1.450	1.382	1.121	0.953	1.152	1.179	1.212	1.191
103	255	8.556	4	1.494	1.938	1.800	1.454	1.466	1.473	1.922	2.088	1.316
103	256	8.855	1	1.516	1.027	1.099	1.038	0.853	1.068	1.005	0.911	1.054
104	255	9.055	1	0.490	1.145	1.100	0.498	0.203	0.523	0.315	0.274	0.518
104	256	8.926	0	0.328	0.105	0.107	0.131	0.112	0.140	0.140	0.105	0.042
104	257	9.083	5	0.748	1.398	1.249	0.439	0.122	0.461	1.142	1.255	0.404
104	258	9.196	0	-0.593	-0.746	-0.744	-0.746	-0.692	-0.745	-0.745	-0.746	-0.810
105	256	9.336	2	0.385	0.541	0.447	0.213	0.068	0.245	0.225	0.442	0.355
105	259	9.619	5	-0.292	-0.515	-0.687	-1.123	-1.047	-1.115	-0.356	-0.436	-1.296

Table 4.10 (contd)

Z	A	Q_α	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
106	259	9.765	2	-0.396	-0.986	-1.097	-0.870	-1.188	-0.846	-0.937	-0.938	-0.849
106	260	9.901	0	-1.768	-2.021	-2.019	-1.988	-2.033	-1.989	-1.989	-2.021	-2.114
106	261	9.714	2	-0.729	-0.872	-0.992	-0.709	-1.049	-0.686	-0.789	-0.832	-0.736
107	261	10.500	3	-1.893	-2.414	-2.568	-2.802	-2.741	-2.801	-2.585	-2.607	-3.015
108	263	10.733	5	-3.046	-1.727	-1.892	-2.732	-3.071	-2.714	-2.046	-1.893	-2.724
108	265	10.470	0	-2.708	-2.692	-2.487	-2.067	-2.429	-2.045	-2.353	-2.423	-2.094
108	266	10.346	0	-2.404	-2.587	-2.584	-2.563	-2.599	-2.564	-2.564	-2.587	-2.695
108	270	9.070	0	0.954	0.941	0.943	0.930	0.957	0.954	0.954	0.941	0.867
110	267	11.777	0	-5.000	-5.071	-4.879	-4.520	-4.881	-4.509	-4.824	-4.846	-4.527
110	270	11.117	0	-3.688	-3.862	-3.859	-3.829	-3.903	-3.835	-3.835	-3.862	-3.996
114	286	10.355	0	-0.657	-0.865	-0.860	-0.879	-0.898	-0.847	-0.847	-0.865	-0.990
114	288	10.076	0	-0.185	-0.134	-0.130	-0.171	-0.142	-0.134	-0.134	-0.134	-0.249
114	290	9.856	0	1.903	0.460	0.464	0.399	0.479	0.437	0.437	0.460	0.354
116	290	10.997	0	-2.046	-1.913	-1.908	-1.915	-1.983	-1.883	-1.883	-1.913	-2.064
116	292	10.791	0	-1.796	-1.426	-1.421	-1.450	-1.467	-1.414	-1.414	-1.426	-1.569
118	294	11.867	0	-3.155	-3.402	-3.397	-3.394	-3.502	-3.367	-3.367	-3.402	-3.584

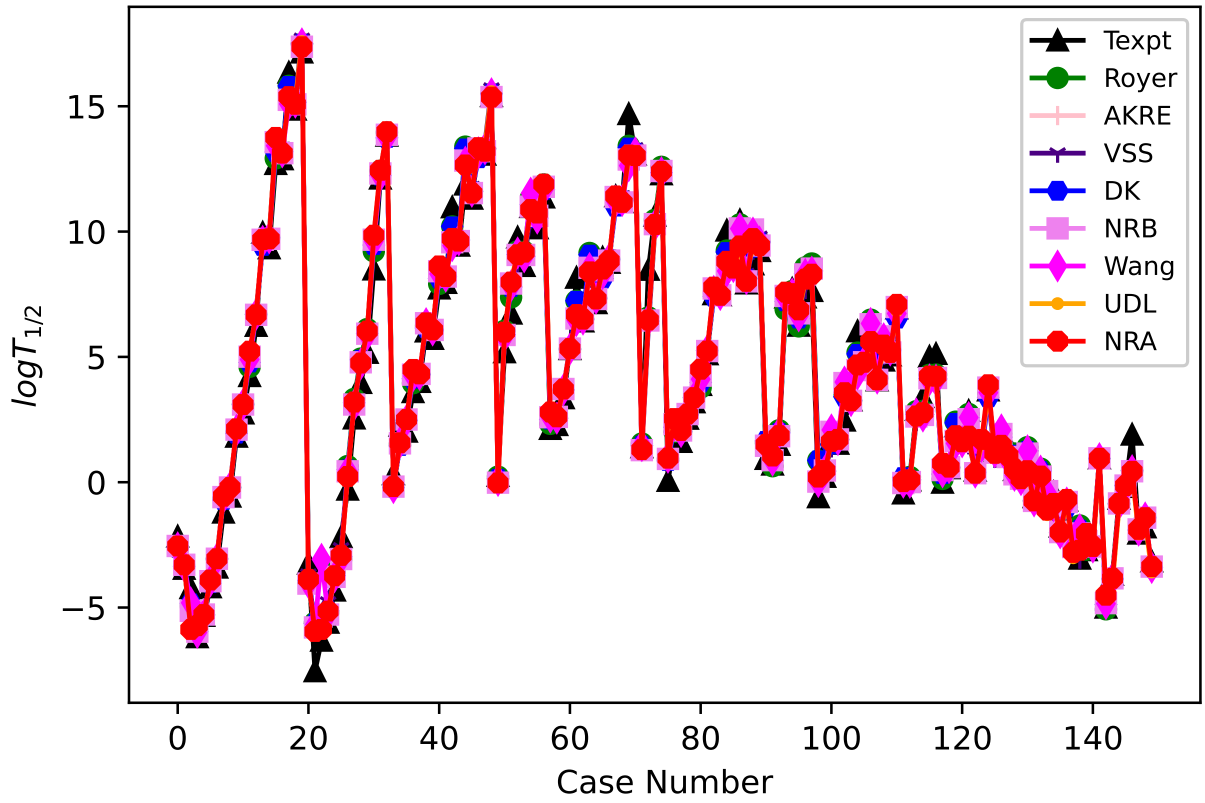

Figure 4.1: Comparison of the calculated α-decay half-lives for $Z = 92 - 118$ between empirical formulas and experiment

Figure 4.1 shows the graphical display of the results shown in Table 4.10. One can observe that the models give very good descriptions of the alpha decay half-lives. The performance of each model cannot, however, be easily deduced from the Figure. This is why the standard deviation values are calculated for all the improved empirical formulas. The calculated standard deviation values are shown in Table 4.11. The Table shows that the Royer formula still gives the best descriptions of the alpha decay half-lives of the heavy and superheavy nuclei considered, with a standard deviation of 0.562103.

Table 4.11: The standard deviation error between the experimental half-lives and α-decay empirical formula for Z= $92 - 118$.

Empirical Formula	σ
Royer	0.562103
DK	0.637016
Akrawy	0.615581
VSS	0.664104
NRA	0.659282
NRB	0.612868
UDL	0.695521
Wang	0.675344

Furthermore, the difference between the experimental and theoretically calculated α-decay half-lives is calculated using [48, 58].

$$\Delta T_{1/2} = \log_{10}\left[T_{1/2}^{\text{theory}}\right] - \log_{10}\left[T_{1/2}^{\text{expt}}\right]. \tag{4.1}$$

The results are plotted in Figure 4.2. A careful observation of the Figure indicates that the Royer formula gives $\Delta T_{1/2}$ values closer to zero than the other empirical formulas.

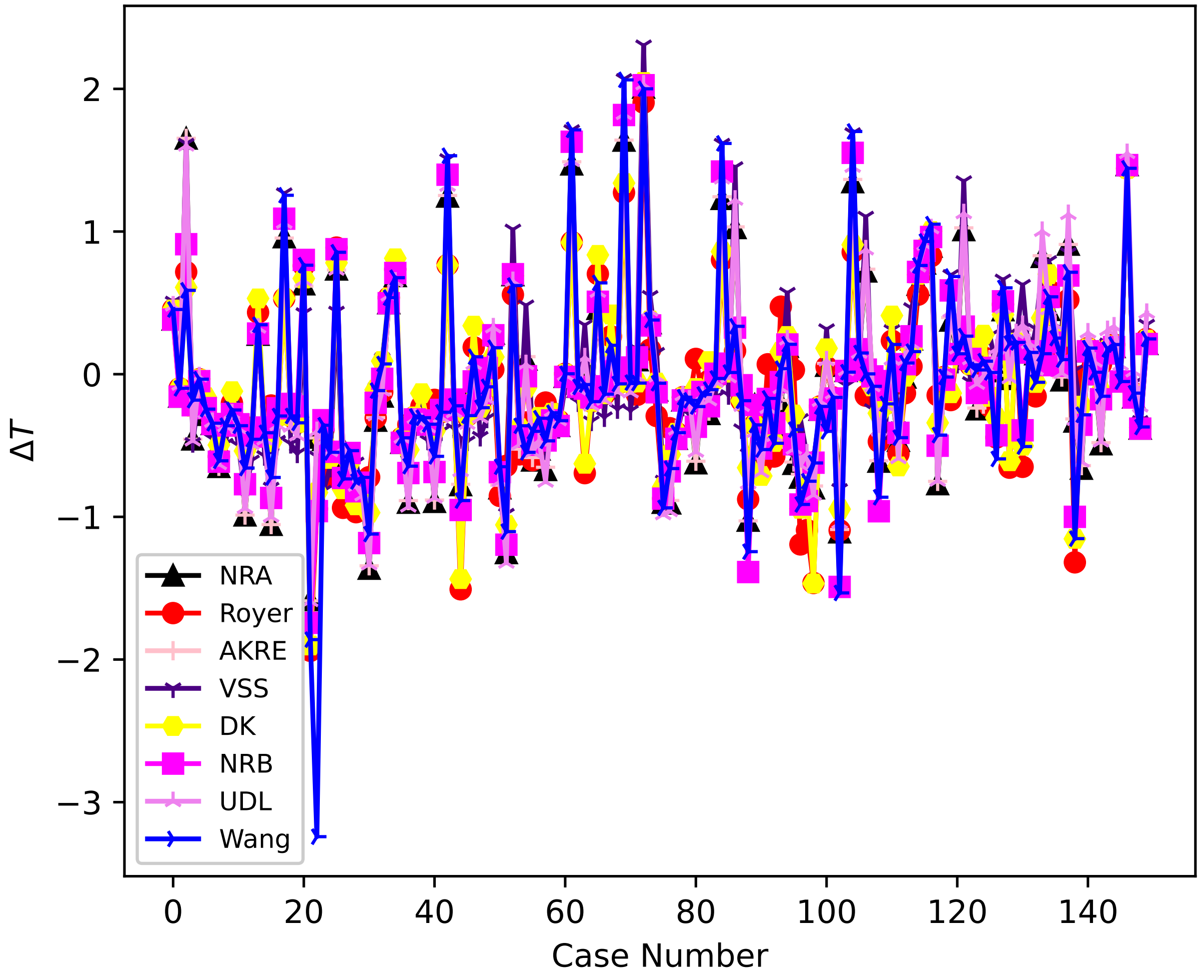

Figure 4.2: The difference between the empirical formulas and experimental half-lives

4.2 α-decay half-lives using machine learning models

Here the results of the alpha decay half-lives using the trained machine learning (ML) models are presented and discussed. The machine learning algorithms used for the study are k Nearest neighbour (kNN), Support vector regression (SVR), Decision trees (DT), Random forest (RF), Extra trees (ET) and Artificial neural network (ANN). It should be noted that random forest and extra trees are ensemble learning algorithms. As stated in the previous chapter, during training of the ML algorithms, the dataset (containing 549 nuclei) was split into training (80%) set and test (20%) set. A 10-fold cross-validation scheme was used to avoid overfitting. After training, each model is used to predict the alpha decay half-lives using the data in the test set. After splitting of the data into training and test set, the test set contains 111 nuclei. The machine learning models are used to predict the alpha decay half-lives of nuclei in the test set. The calculated standard deviation values are shown in Table 4.12. It can be observed that with $\sigma = 0.620593$, the artificial neural network model has the least deviation followed by support vector machine model, with a standard deviation value of 0.634572. So we conclude that the artificial neural network model is the best machine learning model to predict the alpha decay half-lives of nuclei.

Table 4.12: The standard deviation values for the machine learning models on the test set of 111 nuclei

Machine Learning Models	Error on Test
k Nearest neighbour (kNN)	0.828725
Support Vector Regression (SVR)	0.634572
Decision Tree (DT)	1.230638
Random Forest (RF)	0.955991
Extra Tree (ET)	0.745626
ANN	0.620593

Table 4.13: α-decay half-lives, log $\left[T_{1/2}(s)\right]$, of nuclei in the test dataset using the ML models

				log $\left[T_{1/2}(s)\right]$						
Z	A	Q_α	ℓ	Expt.	ANN	kNN	SVR	DT	RF	ET
84	201	5.799	0	4.918	4.974	4.469	4.995	4.081	5.092	5.110
96	242	6.216	0	7.148	7.060	7.437	6.851	7.935	8.016	8.052
90	217	9.435	5	-3.606	-5.057	-3.319	-4.813	-2.778	-5.108	-4.422
116	290	10.997	0	-2.046	-1.696	-1.560	-2.385	-3.948	-3.161	-2.259
72	161	4.679	0	3.802	4.226	4.372	4.318	3.040	3.370	3.856
79	179	5.981	0	1.509	1.520	1.682	1.545	1.263	1.655	1.568
89	221	7.791	0	-1.284	-1.653	-1.534	-1.696	-0.942	-1.084	-1.241
73	159	5.681	6	0.486	0.695	1.494	-0.524	0.033	0.428	0.652
52	108	3.420	0	0.632	1.200	2.071	0.978	-3.529	-0.807	1.419
85	202	6.354	0	3.186	2.672	2.457	2.947	2.489	2.535	2.540
97	244	6.779	2	8.479	5.965	7.517	5.981	4.005	5.180	5.937
78	186	4.320	0	9.728	9.543	9.765	9.982	7.873	9.175	9.281

Table 4.13 (contd.)

Z	A	Q_α	ℓ	Expt.	ANN	kNN	SVR	DT	RF	ET
85	213	9.254	0	-6.903	-6.686	-6.342	-6.528	-6.159	-6.375	-6.232
98	245	7.258	0	3.884	3.399	3.176	3.481	3.028	3.460	3.515
76	170	5.537	0	1.890	1.931	1.910	1.933	1.316	2.110	2.027
86	216	8.198	0	-4.538	-4.077	-3.998	-3.960	-4.432	-3.315	-3.480
97	247	5.890	2	10.639	10.976	10.079	10.827	8.686	9.190	9.238
84	212	8.954	0	-6.532	-6.287	-5.963	-6.177	-6.159	-5.677	-5.863
86	217	7.887	0	-3.227	-3.050	-3.319	-3.123	-3.388	-2.868	-2.903
82	181	7.240	2	-1.409	-1.210	-0.859	-1.043	-1.475	-1.354	-1.213
85	219	6.342	0	1.777	1.809	2.236	2.034	2.489	2.604	2.604
89	216	9.241	5	-3.357	-4.409	-2.944	-4.218	-2.778	-5.192	-4.375
87	205	7.055	0	0.598	0.697	0.606	0.767	-0.345	0.242	0.400
85	212	7.817	5	-0.503	-0.653	-0.909	-0.985	-1.571	-2.041	-1.635
71	156	5.596	0	-0.306	-0.333	0.115	-0.255	0.033	0.323	0.242
76	174	4.871	0	5.263	5.239	3.859	5.225	4.244	4.000	4.827
94	229	7.598	2	2.260	1.404	0.035	1.201	2.230	2.011	0.933
105	259	9.619	5	-0.292	-0.484	0.303	-0.879	-0.562	-0.050	-0.324
102	251	8.752	0	-0.016	0.097	0.545	0.289	0.466	0.821	0.844
75	169	5.014	5	5.210	4.402	3.354	3.377	4.244	3.980	4.528
92	219	9.950	5	-4.222	-6.404	-5.420	-6.306	-3.967	-5.601	-5.553
70	155	5.339	0	0.304	0.249	0.316	0.472	0.510	0.708	0.504
88	224	5.789	0	5.497	5.676	5.274	5.724	5.964	5.979	6.342
100	255	7.241	4	4.859	5.226	4.716	4.060	6.637	5.794	5.808
84	204	5.485	0	6.277	6.499	6.893	6.446	5.519	6.241	6.371
104	258	9.196	0	-0.593	-0.804	0.017	-0.511	0.488	0.422	0.165
90	210	8.069	0	-1.796	-1.642	-1.627	-1.727	-1.453	-1.473	-1.631
87	216	9.174	0	-6.155	-6.097	-6.197	-5.957	-6.159	-5.786	-6.035
77	172	5.991	5	2.342	1.387	1.887	1.286	1.263	1.057	1.161
63	147	2.991	0	10.976	11.184	12.320	11.528	14.354	12.676	12.463
90	224	7.299	0	0.017	0.428	0.153	0.230	0.475	0.137	0.390
79	173	6.836	0	-1.528	-1.674	-1.369	-1.782	-0.602	-1.336	-1.429

Table 4.13 (contd.)

Z	A	Q_α	ℓ	Expt.	ANN	kNN	SVR	DT	RF	ET
92	231	5.576	2	9.958	9.851	9.361	9.743	7.727	9.262	9.310
85	206	5.887	2	5.310	5.177	6.474	5.373	5.002	5.231	5.146
96	248	5.162	0	13.079	13.061	11.899	12.951	14.692	13.444	13.516
85	195	7.344	5	-0.538	0.189	0.161	0.296	-0.225	-0.835	-0.671
54	112	3.330	0	2.352	2.671	2.698	2.839	2.753	2.853	2.949
82	191	5.402	0	4.194	5.591	4.327	5.738	5.200	6.247	5.848
86	198	7.349	0	-1.160	-0.831	-1.107	-0.762	-1.475	-1.038	-1.063
71	155	5.802	0	-1.122	-1.002	-0.508	-1.044	0.033	-0.310	-0.209
64	151	2.652	0	14.988	15.282	14.814	15.806	14.354	16.766	15.637
87	207	6.889	0	1.193	1.331	1.760	1.395	1.122	1.387	1.253
97	249	5.521	2	12.290	12.852	10.656	13.121	11.199	11.281	12.348
91	231	5.150	0	12.013	10.335	10.616	10.296	9.337	11.788	11.313
91	224	7.694	2	-0.074	-0.015	-0.345	-0.098	-0.942	-0.374	-0.409
78	182	4.951	0	5.625	5.948	5.218	5.974	4.244	4.656	5.896
76	165	6.335	0	-1.103	-0.976	-1.004	-1.198	-1.909	-0.996	-1.142
76	167	5.985	0	0.216	0.215	0.205	0.072	-0.114	0.332	0.157
89	217	9.832	0	-7.161	-7.007	-6.644	-6.799	-6.905	-6.249	-6.133
79	180	5.831	2	3.134	2.744	2.784	2.758	1.263	1.995	2.629
91	214	8.271	0	-1.770	-1.854	-2.064	-1.974	-2.124	-2.015	-2.045
74	167	4.751	0	4.697	4.833	4.879	4.828	4.244	3.905	4.668
63	148	2.694	0	14.700	14.042	15.001	14.613	14.354	15.176	15.109
96	246	5.475	0	11.173	10.934	10.100	10.933	10.554	10.977	11.931
68	156	3.481	0	9.989	9.774	8.428	10.167	9.088	7.568	9.619
88	209	7.143	0	0.673	0.770	0.643	0.731	0.475	0.429	0.403
85	191	7.822	5	-2.678	-1.135	-0.665	-1.537	-1.571	-2.319	-1.766
102	255	8.428	5	2.848	1.920	3.104	1.913	2.292	1.724	1.621
96	239	6.540	1	8.162	6.704	6.177	6.497	5.223	5.725	5.962
108	265	10.470	0	-2.708	-2.462	-2.507	-3.134	-2.031	-2.218	-2.371
88	205	7.486	0	-0.658	-0.524	-0.900	-0.500	-1.389	-0.630	-0.796
92	227	7.235	2	1.820	1.776	0.992	1.706	2.230	2.144	1.176

Table 4.13 (contd.)

Z	A	Q_α	ℓ	Expt.	ANN	kNN	SVR	DT	RF	ET
66	153	3.559	0	8.389	8.386	8.701	8.645	9.088	6.913	8.541
114	286	10.355	0	-0.657	-0.696	-0.574	-0.915	-2.031	-2.148	-0.753
86	212	6.385	0	3.157	2.490	3.340	2.855	2.489	2.716	2.569
82	192	5.222	0	6.551	6.607	6.392	6.589	5.200	7.370	6.957
100	252	7.154	0	4.961	4.868	4.636	5.020	5.087	5.186	4.921
87	204	7.170	0	0.261	0.225	-0.285	0.318	-0.345	-0.165	-0.124
86	205	6.386	2	2.840	3.060	3.026	3.532	2.489	2.672	2.591
84	213	8.536	0	-5.431	-5.479	-4.962	-5.378	-4.571	-4.331	-4.453
103	255	8.556	4	1.494	1.598	2.150	1.322	1.400	1.213	1.337
80	177	6.736	2	-0.932	0.206	0.521	-0.034	-0.331	-0.582	-0.174
72	156	6.026	0	-1.638	-1.322	-1.056	-1.508	-1.069	-0.728	-0.826
102	252	8.549	0	0.562	0.825	0.559	0.921	0.466	0.955	0.961
86	208	6.261	0	3.372	3.309	3.499	3.628	3.289	3.357	3.330
84	219	5.914	0	3.341	3.523	4.274	3.629	5.964	5.343	4.597
67	152	4.507	0	3.130	2.828	3.462	3.268	4.897	4.279	3.705
92	229	6.476	0	4.239	4.072	3.705	4.064	3.554	4.122	4.148
85	220	6.077	4	3.444	5.224	5.518	3.558	4.035	3.828	4.221
93	233	5.627	0	8.492	8.629	7.971	8.483	7.727	8.989	9.355
84	188	8.082	0	-3.569	-3.768	-3.138	-4.033	-2.750	-3.455	-3.347
80	185	5.773	0	2.913	2.792	2.861	3.051	4.081	3.894	3.304
87	199	7.817	0	-2.180	-1.974	-2.178	-2.078	-2.750	-2.226	-2.289
78	174	6.183	0	0.061	0.375	0.070	0.189	0.812	0.490	0.229
87	212	6.529	2	3.446	2.710	1.896	3.085	2.459	2.368	2.354
98	251	6.177	5	10.452	9.041	7.414	9.928	8.686	8.685	8.661
94	238	5.593	0	9.442	9.213	9.000	9.061	10.554	10.376	10.227
90	218	9.849	0	-6.914	-6.798	-6.365	-6.556	-5.925	-6.074	-6.043
78	183	4.822	0	6.609	6.687	6.436	6.727	4.244	5.141	6.827
71	158	4.790	0	3.066	3.109	3.071	3.352	3.040	2.612	2.781
88	208	7.273	0	0.106	0.268	0.206	0.249	0.475	0.154	0.026
101	247	8.764	1	0.076	0.264	0.435	0.159	1.400	0.420	0.214

Table 4.13 (contd.)

Z	A	Q_α	ℓ	Expt.	ANN	kNN	SVR	DT	RF	ET
80	186	5.204	0	5.714	5.541	4.474	5.641	5.200	7.348	5.569
91	222	8.789	0	-2.420	-3.405	-3.821	-3.691	-3.203	-3.382	-3.699
96	240	6.398	0	6.419	6.270	6.583	6.040	5.223	6.595	6.800
87	219	7.449	0	-1.648	-1.455	-1.023	-1.388	-0.620	-1.083	-0.933
108	270	9.070	0	0.954	1.498	-0.583	1.451	0.488	-0.224	0.012
88	221	6.880	3	1.398	2.040	2.451	1.027	1.122	1.673	1.667
92	233	4.909	0	12.701	12.154	12.777	12.226	9.337	12.345	12.672
91	228	6.265	3	6.632	5.756	6.779	5.208	6.394	5.683	5.340

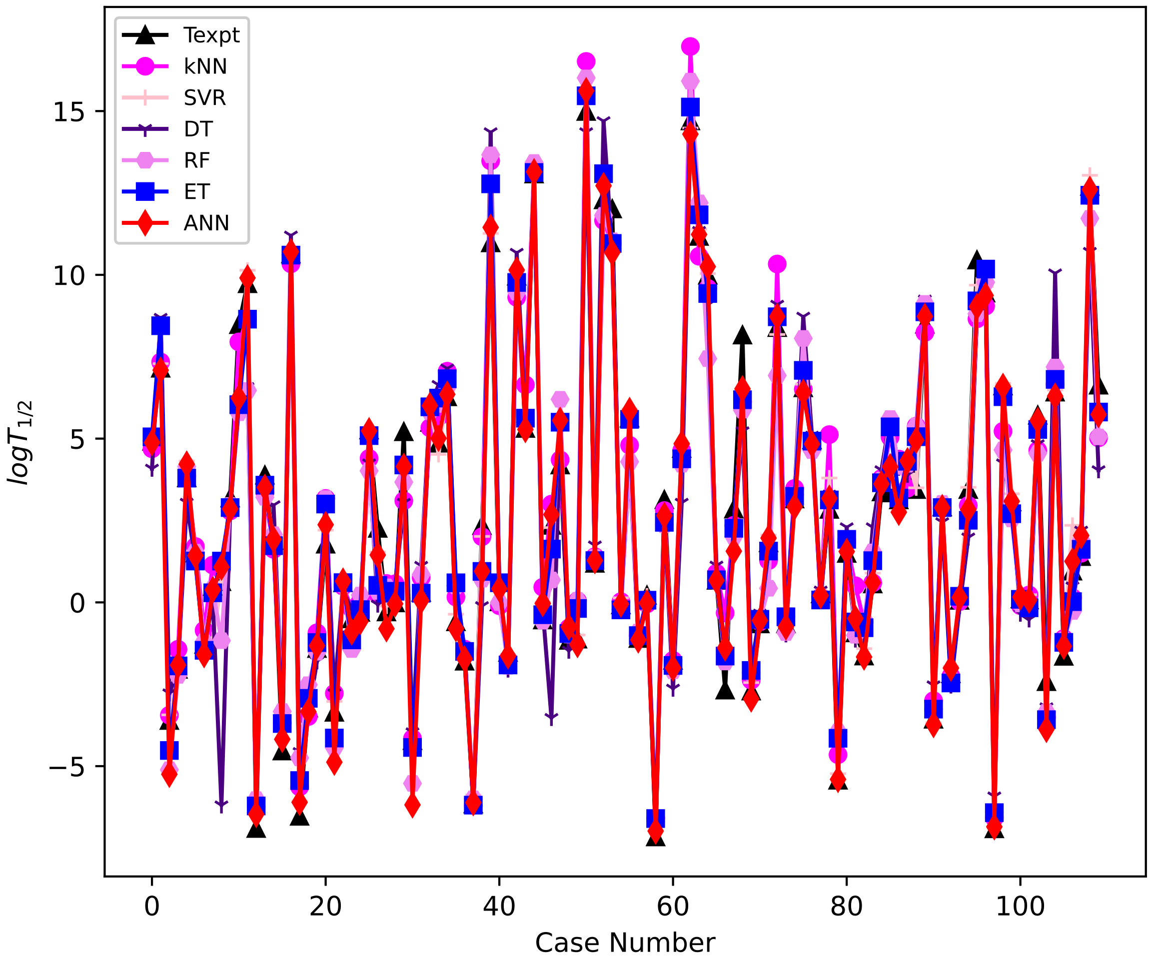

Figure 4.3: Plot of the predictions of alpha decay half-lives of nuclei in the test dataset using the ML models.

Table 4.13 shows the half-lives predicted by the machine learning models using nuclei in the test set. This is visualised in Figure 4.3. One can observe that all the models give very good values of the half-lives when compared with the experimental values.

4.3 Training of ML algorithms to predict Q_α values

To predict the half-lives of unmeasured superheavy nuclei, Q_α-values are required. These unmeasured superheavy nuclei have no experimental Q_α -values. Machine learning models can be used to predict the Q_α-values of measured and unmeasured nuclei. In order to achieve this about 1080 Q_α-values of measured nuclei in the NUBASE2020 database have been trained using the Artificial Neural Network (ANN). The 1080 Q_α-values were split into train and test datasets using the scikit-learn "train-test-split", a library under the python programming language. The full datasets are randomly split into 80% of training set and 20% of test set. The model selection was carried by 10-fold cross validation, the number of epoch was 700, the batch size of 32 was applied and the standard deviation value σ on the training set is 0.14629 and the standard deviation on test set is 0.17807. The standard deviation is computed using equation (4.2)

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[(Q_{ANN}^{theory} - Q_{\alpha}^{expt})^2\right]} \tag{4.2}$$

The results of the predicted Q_α values using the ANN model are shown in Table 4.14 and Figure 4.4. The Table and Figure show that the ANN model can successfully be used to predict the Q_α values of nuclei. This means that one can use the trained ANN model to predict the Q_α values for unmeasured superheavy nuclei.

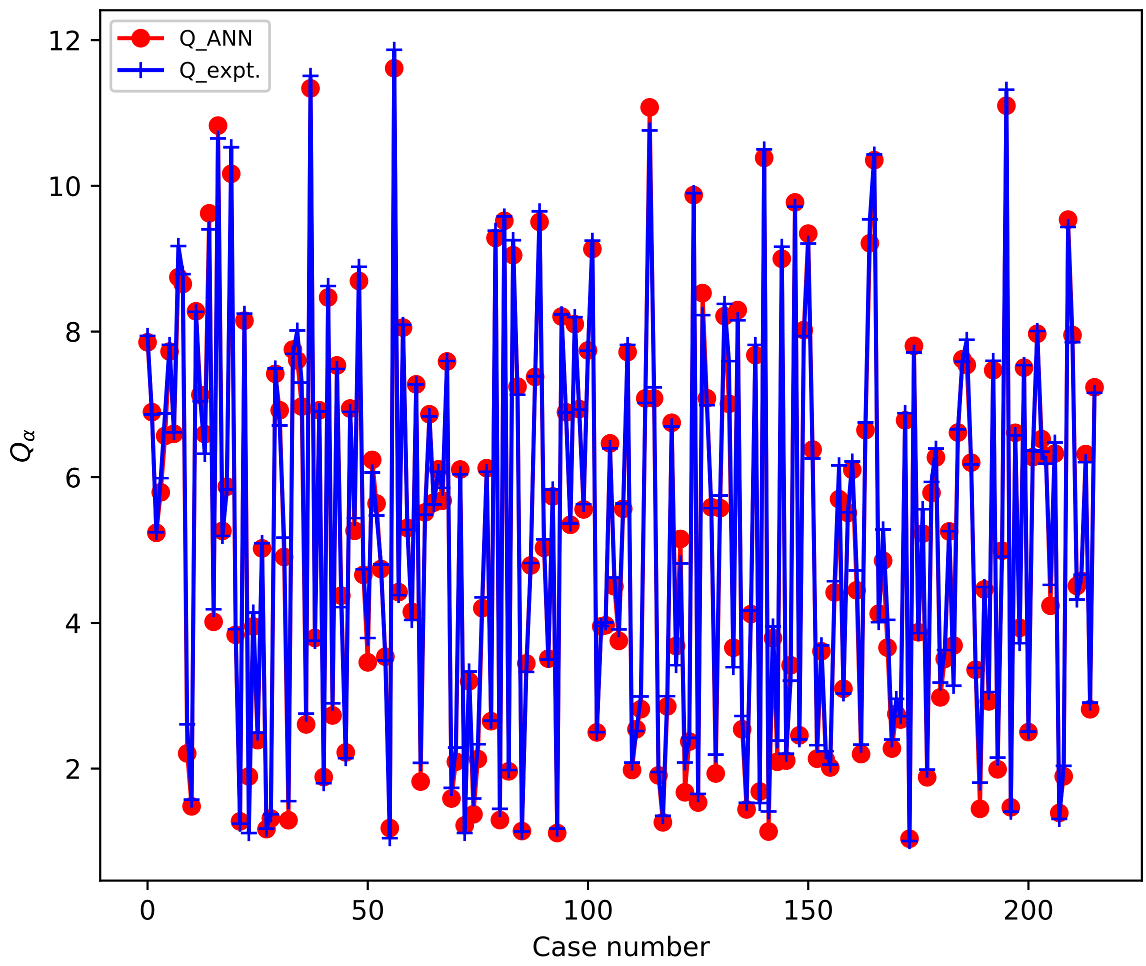

Figure 4.4: Comparison of the predicted Q_{ANN} and the experimental Q_α values for nuclei in the test dataset

Table 4.14: Comparison of the experimental Q_α values and that predicted by ANN (Q_{ANN}) for nuclei in the test dataset

Z	A	Q_{ANN}	Q_α
94	228	7.852	7.940
83	190	6.894	6.862
94	239	5.236	5.245
83	213	5.793	5.988
85	218	6.568	6.876
85	212	7.726	7.817
99	254	6.597	6.617
87	216	8.746	9.174
91	222	8.653	8.789
66	155	2.208	2.608
60	145	1.482	1.574
91	214	8.278	8.271
86	200	7.132	7.043
81	181	6.592	6.322
106	263	9.623	9.403
76	179	4.015	4.188
115	288	10.827	10.650
93	235	5.263	5.194
79	180	5.871	5.831
111	279	10.166	10.530
79	190	3.836	3.914
69	168	1.275	1.243
86	213	8.150	8.245
62	145	1.893	1.115
87	226	3.952	4.143
72	174	2.391	2.494
69	154	5.022	5.094
81	202	1.169	1.175
59	130	1.315	1.373
84	191	7.419	7.493

Table 4.14 (contd.)

Z	A	Q_{ANN}	Q_{α}
81	179	6.919	6.709
96	250	4.905	5.170
53	117	1.291	1.553
91	224	7.752	7.694
87	218	7.604	8.014
90	224	6.970	7.299
67	156	2.606	2.754
110	269	11.338	11.510
91	236	3.793	3.755
87	213	6.919	6.905
60	130	1.883	1.799
90	221	8.468	8.625
67	146	2.730	2.896
88	205	7.534	7.486
54	109	4.374	4.217
76	188	2.222	2.143
80	176	6.943	6.897
95	243	5.264	5.439
101	246	8.694	8.889
82	194	4.659	4.738
82	210	3.458	3.792
74	160	6.238	6.066
70	154	5.639	5.474
68	153	4.740	4.802
55	113	3.536	3.483
56	122	1.184	1.045
118	294	11.614	11.867
77	181	4.422	4.381
89	205	8.054	8.093
74	164	5.303	5.278
75	174	4.151	4.040
87	203	7.275	7.275

Table 4.14 (contd.)

Z	A	Q_{ANN}	Q_α
75	186	1.822	2.078
84	204	5.516	5.485
79	173	6.864	6.836
93	233	5.652	5.627
97	246	6.108	6.074
87	222	5.679	5.853
82	179	7.589	7.596
70	170	1.589	1.735
71	171	2.095	2.290
95	238	6.106	6.042
74	186	1.221	1.116
90	236	3.198	3.333
74	185	1.373	1.590
82	203	2.131	2.335
66	150	4.203	4.351
84	199	6.124	6.074
65	148	2.651	2.657
89	218	9.285	9.384
71	177	1.294	1.447
109	278	9.520	9.580
66	146	1.963	1.980
85	213	9.048	9.254
86	199	7.244	7.132
61	149	1.142	1.137
79	191	3.444	3.327
78	183	4.789	4.822
87	202	7.377	7.385
93	223	9.505	9.650
96	249	5.030	5.148
65	151	3.509	3.496
83	195	5.736	5.832
78	195	1.116	1.176

Table 4.14 (contd.)

Z	A	Q_{ANN}	Q_{α}
91	215	8.207	8.236
87	207	6.892	6.889
83	197	5.347	5.365
86	216	8.101	8.198
82	183	6.935	6.928
85	210	5.558	5.631
88	203	7.741	7.736
91	221	9.133	9.248
52	111	2.497	2.500
74	171	3.951	3.957
75	175	3.967	4.007
96	240	6.464	6.398
70	157	4.502	4.622
78	189	3.753	3.912
95	241	5.566	5.638
87	199	7.718	7.817
81	199	1.981	2.083
74	180	2.541	2.515
63	147	2.816	2.991
81	178	7.082	7.020
115	287	11.077	10.760
92	227	7.082	7.235
67	148	1.907	1.952
60	134	1.262	1.352
73	175	2.854	2.995
75	160	6.748	6.698
52	108	3.682	3.420
81	189	5.153	4.817
72	178	1.674	2.084
71	169	2.371	2.423
106	260	9.873	9.901
75	187	1.534	1.652

Table 4.14 (contd.)

Z	A	Q_{ANN}	Q_{α}
102	254	8.528	8.226
78	168	7.084	6.990
92	231	5.588	5.576
65	144	1.933	2.193
94	237	5.575	5.748
100	246	8.212	8.379
84	211	7.007	7.595
67	150	3.659	3.393
101	250	8.294	8.155
76	187	2.540	2.722
60	133	1.439	1.530
70	158	4.122	4.170
93	227	7.676	7.816
52	114	1.685	1.527
107	261	10.384	10.500
73	184	1.136	1.412
70	159	3.791	3.951
53	114	2.094	2.386
103	252	8.999	9.164
64	152	2.110	2.204
76	183	3.418	3.207
106	261	9.773	9.714
63	149	2.456	2.401
90	211	8.019	7.937
93	219	9.344	9.207
80	180	6.380	6.258
61	145	2.136	2.322
82	198	3.611	3.692
63	139	2.122	2.239
67	157	2.015	2.056
92	236	4.418	4.573
86	221	5.699	6.163

Table 4.14 (contd.)

Z	A	Q_{ANN}	Q_{α}
71	165	3.096	3.032
74	163	5.510	5.519
96	242	6.101	6.216
89	228	4.448	4.721
67	149	2.200	2.329
83	211	6.646	6.750
87	215	9.212	9.540
112	281	10.353	10.430
52	107	4.127	4.010
86	223	4.854	5.283
67	154	3.661	4.041
72	175	2.275	2.400
53	112	2.749	2.957
54	114	2.674	2.719
88	221	6.781	6.880
67	162	1.037	1.005
98	240	7.804	7.711
76	180	3.871	3.860
87	223	5.226	5.561
61	134	1.881	1.987
89	225	5.786	5.935
82	187	6.272	6.393
65	141	2.978	3.180
91	238	3.506	3.628
77	176	5.256	5.260
83	209	3.690	3.137
84	196	6.613	6.658
89	210	7.619	7.586
86	217	7.541	7.887
98	251	6.200	6.177
73	171	3.357	3.381
72	179	1.448	1.808

Table 4.14 (contd.)

Z	A	Q_{ANN}	Q_{α}
71	159	4.460	4.492
83	208	2.923	3.051
94	229	7.469	7.598
71	172	1.990	2.151
92	233	4.992	4.909
117	293	11.100	11.320
62	139	1.468	1.408
95	235	6.610	6.576
71	161	3.932	3.722
89	212	7.504	7.540
69	161	2.501	2.509
93	231	6.276	6.368
92	225	7.970	8.007
80	179	6.523	6.351
96	241	6.285	6.185
83	217	4.236	4.521
92	229	6.325	6.476
70	172	1.391	1.309
80	196	1.894	2.038
90	217	9.537	9.435
102	259	7.952	7.854
81	191	4.507	4.321
77	180	4.578	4.660
83	212	6.317	6.207
88	231	2.814	2.906
78	167	7.234	7.158

4.4 Calculations of the α-decay half-lives for nuclei with Z=92-118 using the empirical formulas and the Q_{ANN} predict.

Here, the Q_α values predicted by ANN was used to calculate the α-decay half-lives for $Z = 92 - 118$ using the improved empirical formula. The empirical formulas used are shown in Section 4.1 and the results are shown in Table 4.15. The first five columns shows the atomic number Z, mass number A, the Q_α values obtained from the machine learning model (ANN) Q_{ANN}, orbital angular momentum ℓ and the experimental α-decay half-lives. The standard deviation values obtained from the calculation in Table 4.16. The Table shows that DK with a standard deviation of 0.842077, has the least deviation from experimental values followed by Royer formula with standard deviation value of 0.850026.

Table 4.15: Computed α-decay half-lives, $\log\left[T_{1/2}(s)\right]$, for nuclei with $Z = 92 - 118$ using the improved empirical formulas and Q_{ANN}.

				$\log\left[T_{1/2}(s)\right]$								
Z	A	Q_{ANN}	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
92	216	8.612	0	-2.161	-2.851	-2.851	-2.770	-2.904	-2.782	-2.782	-2.879	-2.851
92	218	8.931	0	-3.451	-3.783	-3.784	-3.717	-3.788	-3.728	-3.728	-3.808	-3.783
92	219	10.084	5	-4.222	-5.248	-5.142	-6.182	-6.141	-6.180	-5.442	-6.134	-5.121
92	221	9.842	0	-6.180	-5.902	-5.842	-5.615	-5.589	-5.613	-5.852	-5.603	-5.887
92	222	9.344	0	-5.328	-4.950	-4.952	-4.919	-4.864	-4.933	-4.933	-4.968	-4.950
92	223	8.871	2	-4.187	-3.171	-3.251	-3.137	-3.142	-3.134	-3.175	-3.137	-3.161
92	224	8.417	0	-3.402	-2.443	-2.445	-2.424	-2.338	-2.433	-2.433	-2.450	-2.443
92	225	7.970	2	-1.208	-0.510	-0.590	-0.442	-0.483	-0.438	-0.473	-0.454	-0.478
92	226	7.521	0	-0.570	0.413	0.411	0.418	0.535	0.415	0.415	0.419	0.413
92	227	7.082	2	1.820	2.604	2.522	2.704	2.618	2.708	2.681	2.681	2.658
92	228	6.738	0	2.748	3.361	3.360	3.350	3.498	3.352	3.352	3.380	3.361
92	229	6.325	0	4.239	5.276	5.453	5.900	5.764	5.904	5.690	5.863	5.582
92	230	5.903	0	6.243	7.145	7.145	7.119	7.290	7.128	7.128	7.180	7.145
92	231	5.588	2	9.958	9.460	9.362	9.615	9.420	9.617	9.608	9.567	9.546
92	232	5.276	0	9.337	10.556	10.556	10.511	10.713	10.525	10.525	10.607	10.556
92	233	4.992	0	12.701	12.379	12.629	13.210	12.954	13.210	13.018	13.147	12.869
92	234	4.795	0	12.889	13.612	13.613	13.545	13.783	13.561	13.561	13.678	13.612
92	235	4.602	1	16.346	16.372	16.369	15.943	15.627	15.939	15.813	15.847	15.654

Table 4.15 (contd)

Z	A	Q_{ANN}	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
92	236	4.418	0	14.869	16.341	16.342	16.249	16.527	16.265	16.265	16.421	16.341
92	238	4.108	0	17.149	18.856	18.857	18.737	19.060	18.751	18.751	18.950	18.856
93	219	9.344	0	-3.244	-4.359	-4.274	-4.229	-4.023	-4.243	-4.412	-4.267	-4.373
93	220	10.313	5	-7.538	-5.797	-5.836	-6.143	-5.991	-6.149	-5.990	-6.146	-5.877
93	222	10.023	8	-6.319	-4.955	-5.065	-5.462	-5.340	-5.464	-4.947	-5.499	-2.655
93	223	9.505	0	-5.602	-4.852	-4.767	-4.756	-4.427	-4.765	-4.907	-4.763	-4.873
93	224	9.045	2	-4.319	-2.838	-2.955	-2.976	-2.922	-2.973	-3.015	-3.004	-2.994
93	225	8.599	0	-2.187	-2.448	-2.334	-2.268	-2.004	-2.270	-2.416	-2.270	-2.397
93	227	7.676	3	-0.292	1.136	1.006	0.720	0.894	0.726	0.930	0.721	0.935
93	229	6.924	1	2.548	3.711	3.556	3.584	3.668	3.595	3.493	3.584	3.475
93	230	6.587	3	3.964	5.770	5.723	5.565	5.389	5.577	5.610	5.589	5.561
93	231	6.276	1	5.166	6.543	6.378	6.459	6.452	6.475	6.365	6.456	6.327
93	233	5.652	0	8.492	9.077	9.324	9.693	9.576	9.713	9.534	9.685	9.471
93	235	5.263	1	12.119	11.994	11.797	11.985	11.796	12.005	11.883	11.970	11.799
93	237	4.933	1	13.831	14.120	13.903	14.137	13.880	14.154	14.029	14.113	13.924
94	228	7.852	0	0.322	0.080	0.080	0.120	0.130	0.121	0.121	0.067	0.080
94	229	7.469	2	2.260	1.990	1.914	2.022	1.940	2.035	2.017	2.081	2.054
94	230	7.094	0	2.021	2.794	2.794	2.822	2.860	2.831	2.831	2.795	2.794
94	231	6.871	0	3.599	3.818	3.993	4.350	4.230	4.365	4.156	4.391	4.106
94	232	6.593	0	4.005	4.832	4.833	4.843	4.920	4.857	4.857	4.845	4.832
94	233	6.381	2	6.019	6.407	6.315	6.506	6.343	6.521	6.506	6.521	6.488
94	234	6.157	0	5.723	6.804	6.804	6.795	6.912	6.812	6.812	6.829	6.804
94	235	5.948	0	7.734	7.931	8.149	8.633	8.424	8.646	8.437	8.618	8.327
94	236	5.753	0	7.955	8.827	8.827	8.796	8.956	8.815	8.815	8.864	8.827
94	237	5.575	1	10.973	11.162	11.164	10.668	10.407	10.678	10.532	10.617	10.407
94	238	5.406	0	9.442	10.741	10.742	10.687	10.892	10.706	10.706	10.791	10.741
94	239	5.236	3	11.881	13.440	13.368	12.703	12.388	12.709	12.883	12.612	12.818
94	240	5.093	0	11.316	12.632	12.633	12.552	12.805	12.570	12.570	12.695	12.632
94	241	4.962	2	13.266	14.254	14.100	14.512	14.139	14.512	14.489	14.374	14.325
94	242	4.783	0	13.072	14.688	14.689	14.580	14.880	14.597	14.597	14.764	14.688

Table 4.15 (contd)

Z	A	Q_{ANN}	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
94	244	4.458	0	15.410	17.081	17.082	16.946	17.290	16.962	16.962	17.172	17.081
95	229	8.058	2	0.255	0.486	0.399	0.234	0.334	0.240	0.249	0.196	0.336
95	235	6.610	1	5.189	5.890	5.735	5.813	5.780	5.833	5.721	5.785	5.684
95	236	6.464	0	6.732	6.400	6.796	6.984	6.750	7.003	6.940	7.048	6.827
95	238	6.106	2	9.769	8.882	8.749	8.730	8.438	8.750	8.742	8.785	8.815
95	239	5.918	1	8.632	9.215	9.031	9.178	9.060	9.200	9.089	9.146	9.014
95	240	5.742	4	10.983	11.123	10.880	10.667	10.312	10.687	10.803	10.716	11.333
95	241	5.566	1	10.135	11.151	10.950	11.139	10.961	11.161	11.048	11.101	10.952
95	243	5.264	1	11.365	12.965	12.743	12.974	12.739	12.994	12.880	12.928	12.762
96	233	7.589	0	2.130	1.909	2.077	2.360	2.245	2.377	2.167	2.454	2.167
96	234	7.435	0	2.285	2.295	2.296	2.335	2.327	2.346	2.346	2.282	2.295
96	236	7.118	0	3.355	3.476	3.476	3.498	3.536	3.511	3.511	3.473	3.476
96	238	6.798	0	5.314	4.757	4.758	4.759	4.844	4.775	4.775	4.765	4.757
96	239	6.642	1	8.162	6.773	6.777	6.215	5.996	6.231	6.069	6.202	5.982
96	240	6.464	0	6.419	6.199	6.200	6.179	6.311	6.196	6.196	6.218	6.199
96	241	6.285	3	8.452	8.654	8.589	7.892	7.628	7.907	8.064	7.842	8.032
96	242	6.101	0	7.148	7.907	7.908	7.864	8.041	7.883	7.883	7.938	7.907
96	243	5.926	2	8.963	9.511	9.375	9.731	9.415	9.744	9.704	9.642	9.580
96	244	5.749	0	8.757	9.721	9.722	9.654	9.876	9.673	9.673	9.765	9.721
96	245	5.581	2	11.415	11.403	11.249	11.663	11.291	11.673	11.630	11.534	11.467
96	246	5.449	0	11.173	11.398	11.399	11.303	11.574	11.322	11.322	11.454	11.398
96	247	5.318	1	14.692	13.639	13.571	13.268	12.839	13.272	13.095	13.090	12.850
96	248	5.183	0	13.079	13.003	13.004	12.880	13.201	12.896	12.896	13.071	13.003
97	234	8.023	2	1.398	1.804	1.718	1.543	1.473	1.557	1.538	1.679	1.724
97	244	6.456	2	8.479	8.071	7.926	7.915	7.610	7.941	7.930	7.954	7.987
97	247	5.932	2	10.639	10.234	10.032	10.023	9.867	10.046	10.064	9.960	10.027
97	249	5.706	2	12.290	11.488	11.260	11.277	11.089	11.297	11.317	11.206	11.260
98	237	8.160	2	0.058	1.128	1.047	1.140	1.000	1.157	1.129	1.241	1.195
98	239	7.915	0	1.634	1.486	1.664	1.989	1.824	2.007	1.778	2.055	1.755
98	240	7.804	0	1.612	1.691	1.692	1.722	1.729	1.731	1.731	1.668	1.691

Table 4.15 (contd)

Z	A	Q_{ANN}	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
98	242	7.560	0	2.536	2.537	2.538	2.548	2.605	2.558	2.558	2.523	2.537
98	244	7.228	0	3.193	3.773	3.774	3.763	3.867	3.775	3.775	3.769	3.773
98	245	7.030	0	3.884	4.692	4.905	5.415	5.144	5.433	5.184	5.365	5.048
98	246	6.808	0	5.109	5.481	5.482	5.449	5.595	5.466	5.466	5.490	5.481
98	247	6.627	2	7.505	6.974	6.838	7.195	6.875	7.212	7.154	7.110	7.034
98	248	6.453	0	7.460	7.050	7.051	6.995	7.187	7.013	7.013	7.072	7.050
98	249	6.336	1	10.044	9.034	8.976	8.593	8.224	8.607	8.414	8.462	8.216
98	250	6.237	0	8.616	8.061	8.061	7.978	8.225	7.994	7.994	8.093	8.061
98	251	6.200	5	10.452	10.171	10.006	9.305	8.887	9.312	10.010	9.113	10.000
98	252	6.139	0	7.935	8.514	8.515	8.401	8.711	8.409	8.409	8.554	8.514
98	253	6.049	4	8.696	9.975	9.752	10.117	9.647	10.117	10.481	9.863	10.340
98	254	5.951	0	9.227	9.453	9.453	9.309	9.677	9.312	9.312	9.502	9.453
99	240	8.350	1	0.933	1.261	1.342	1.189	1.098	1.207	1.152	1.313	1.153
99	241	8.236	0	0.708	0.713	0.951	1.111	1.143	1.123	0.958	1.023	0.954
99	242	8.119	2	1.495	2.214	2.113	1.979	1.852	2.000	1.979	2.073	2.123
99	251	6.810	0	7.376	5.942	6.249	6.564	6.512	6.583	6.437	6.478	6.350
99	252	6.780	2	7.718	7.434	7.261	7.312	6.966	7.339	7.323	7.284	7.317
99	253	6.685	0	6.248	6.463	6.778	7.118	7.066	7.132	6.993	7.026	6.889
99	254	6.597	5	7.377	8.669	8.418	8.174	7.785	8.198	8.386	8.113	8.388
99	255	6.496	3	7.633	8.435	8.138	7.998	7.930	8.007	8.231	7.898	8.094
100	243	8.535	1	-0.595	1.352	1.356	0.669	0.482	0.687	0.512	0.740	0.517
100	246	8.212	0	0.218	0.998	0.999	1.019	1.041	1.026	1.026	0.963	0.998
100	247	8.101	4	1.685	2.170	2.038	2.136	1.893	2.155	2.542	2.126	2.622
100	248	7.984	0	1.538	1.738	1.739	1.738	1.811	1.746	1.746	1.712	1.738
100	249	7.828	4	2.464	3.119	2.972	3.121	2.841	3.139	3.518	3.069	3.558
100	250	7.653	0	3.270	2.895	2.896	2.873	2.994	2.884	2.884	2.880	2.895
100	251	7.509	1	6.025	4.843	4.792	4.336	4.015	4.354	4.145	4.243	3.994
100	252	7.414	0	4.961	3.769	3.770	3.723	3.896	3.732	3.732	3.763	3.769
100	253	7.346	5	6.334	5.878	5.724	5.005	4.642	5.018	5.702	4.856	5.728
100	254	7.231	0	4.067	4.458	4.458	4.385	4.615	4.391	4.391	4.460	4.458

Table 4.15 (contd)

Z	A	Q_{ANN}	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
100	255	7.124	4	4.859	5.816	5.615	5.940	5.530	5.949	6.300	5.738	6.205
100	256	7.020	0	5.066	5.296	5.297	5.195	5.481	5.199	5.199	5.308	5.296
100	257	6.922	2	6.940	6.450	6.274	6.841	6.380	6.844	6.733	6.581	6.475
101	244	8.915	1	-0.444	0.211	0.300	0.101	0.021	0.119	0.064	0.244	0.100
101	247	8.593	1	0.076	0.754	0.637	0.644	0.680	0.657	0.559	0.530	0.547
101	250	8.294	2	2.887	2.350	2.230	2.140	1.961	2.167	2.144	2.187	2.241
101	251	8.195	1	3.402	2.026	1.888	1.923	1.970	1.937	1.853	1.814	1.814
101	253	8.069	1	5.012	2.439	2.291	2.338	2.397	2.351	2.275	2.228	2.221
101	257	7.549	1	5.122	4.326	4.150	4.248	4.275	4.259	4.192	4.131	4.101
102	251	8.648	0	-0.016	0.455	0.651	1.061	0.795	1.082	0.815	1.063	0.742
102	252	8.565	0	0.562	0.529	0.530	0.541	0.573	0.547	0.547	0.482	0.529
102	253	8.533	1	2.234	2.027	1.979	1.451	1.157	1.471	1.256	1.404	1.157
102	254	8.528	1	1.755	0.612	0.613	0.600	0.693	0.603	0.603	0.570	0.612
102	255	8.503	5	2.848	2.474	2.326	1.577	1.253	1.592	2.267	1.471	2.332
102	256	8.405	0	0.466	0.974	0.975	0.938	1.089	0.938	0.938	0.939	0.974
102	257	8.282	2	1.460	2.057	1.914	2.342	1.977	2.354	2.244	2.185	2.086
102	259	7.952	2	3.667	3.190	3.033	3.524	3.113	3.534	3.417	3.322	3.216
103	254	8.839	3	1.224	1.395	1.327	1.067	0.900	1.098	1.125	1.135	1.156
103	255	8.854	4	1.494	0.959	0.826	0.474	0.532	0.487	0.942	0.334	1.115
103	256	8.863	1	1.516	1.002	1.074	1.013	0.829	1.043	0.980	1.028	0.885
104	255	9.173	1	0.490	0.794	0.749	0.149	-0.142	0.172	-0.038	0.163	-0.081
104	256	9.168	0	0.328	-0.625	-0.624	-0.602	-0.610	-0.598	-0.598	-0.694	-0.625
104	257	9.176	5	0.748	1.117	0.969	0.163	-0.151	0.184	0.863	0.124	0.975
104	258	9.151	0	-0.593	-0.613	-0.612	-0.613	-0.561	-0.611	-0.611	-0.677	-0.613
105	256	9.597	2	0.385	-0.222	-0.316	-0.537	-0.665	-0.508	-0.532	-0.417	-0.329
105	259	9.493	5	-0.292	-0.148	-0.322	-0.756	-0.698	-0.745	0.011	-0.929	-0.072
106	259	9.941	2	-0.396	-1.465	-1.575	-1.344	-1.658	-1.323	-1.417	-1.333	-1.421
106	260	9.873	0	-1.768	-1.946	-1.944	-1.913	-1.959	-1.914	-1.914	-2.038	-1.946
106	261	9.773	2	-0.729	-1.035	-1.155	-0.871	-1.209	-0.849	-0.953	-0.902	-0.997
107	261	10.384	3	-1.893	-2.114	-2.270	-2.500	-2.454	-2.496	-2.282	-2.711	-2.307

Table 4.15 (contd)

Z	A	Q_{ANN}	ℓ	Expt.	Royer	DK	AKRE	VSS	NRA	NRB	UDL	Wang
108	263	10.826	5	-3.046	-1.952	-2.117	-2.954	-3.290	-2.937	-2.270	-2.950	-2.119
108	265	10.521	0	-2.708	-2.818	-2.614	-2.194	-2.555	-2.173	-2.482	-2.224	-2.552
108	266	10.375	0	-2.404	-2.662	-2.659	-2.638	-2.673	-2.640	-2.640	-2.771	-2.662
108	270	9.859	0	0.954	-1.371	-1.369	-1.389	-1.325	-1.385	-1.385	-1.463	-1.371
110	267	11.675	0	-5.000	-4.853	-4.659	-4.301	-4.664	-4.288	-4.601	-4.303	-4.622
110	270	11.189	0	-3.688	-4.030	-4.027	-3.997	-4.069	-4.005	-4.005	-4.166	-4.030
114	286	10.583	0	-0.657	-1.470	-1.466	-1.486	-1.495	-1.460	-1.460	-1.601	-1.470
114	288	10.150	0	-0.185	-0.339	-0.335	-0.377	-0.344	-0.342	-0.342	-0.456	-0.339
114	290	9.787	0	1.903	0.661	0.665	0.601	0.677	0.642	0.642	0.557	0.662
116	290	11.083	0	-2.046	-2.128	-2.124	-2.131	-2.195	-2.101	-2.101	-2.281	-2.128
116	292	10.614	0	-1.796	-0.962	-0.958	-0.985	-1.011	-0.945	-0.945	-1.101	-0.962
118	294	11.614	0	-3.155	-2.816	-2.811	-2.806	-2.924	-2.772	-2.772	-2.992	-2.816

Table 4.16: The standard deviation error σ between the experimental and the calculated α-decay half-lives for nuclei with $Z = 92 - 118$ using Q_{ANN}.

Empirical Formula	σ
Royer	0.850026
DK	0.842077
Akre	0.895041
VSS	0.945275
NRA	0.896574
NRB	0.886292
UDL	0.923910
Wang	0.935863

Figure 4.5 shows the graphical representation of the result shown in Table 4.15. It is observed that the empirical formula give a good description of the α-decay half-lives. This means that the machine learning model (ANN) was able to predict the experimental Q_α value in a practicable manner.

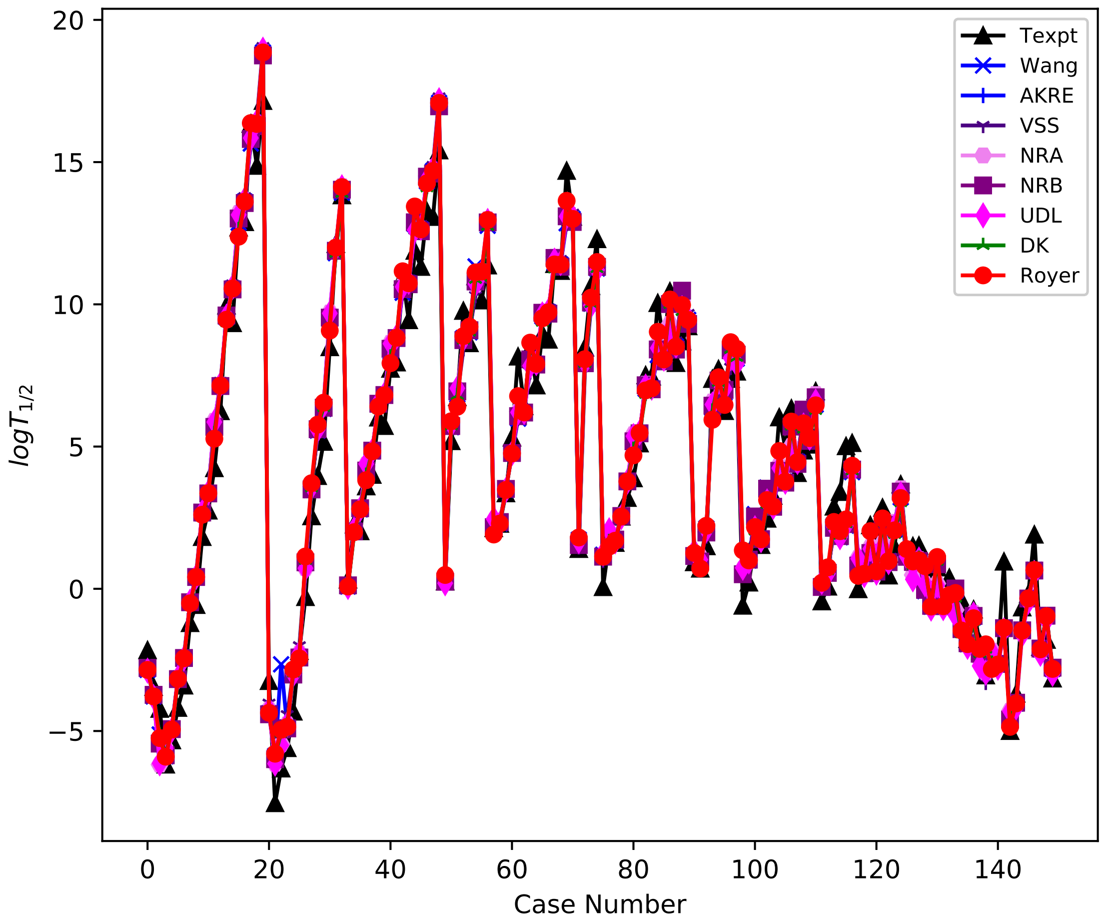

Figure 4.5: Comparison of the calculated α-decay half-lives for $Z = 92-118$ using the empirical formulas and Q_{ANN} .

4.5 Predictions of the α-decay half-lives for unmeasured superheavy nuclei using the empirical formulas and ANN model.

Since the Q_α values predicted with our ANN model were in agreement with the experimental data as shown in Table 4.14, this motivated us to calculate the α-decay half-lives for unmeasured superheavy nuclei within the range ($Z = 120 - 126, 284 \leq A \geq 350$) using the Royer formula, Denisov-khudenko formula, AKRE, NRB, UDL, and Wang formula. The artificial neural network (ANN) was used to predict the α-decay half-lives which are given in Table 4.17. The first four columns denote the atomic no Z, mass number A, the predicted Q_α value which in this case is called the Q_{ANN} and ℓ, for even-even nuclei $\ell = 0$. The fifth to tenth are the empirical formulas used while the last column is the α-decay half-lives predicted using the ANN model.

Table 4.17: Predicted α-decay half-lives ($\log\left[T_{1/2}(s)\right]$) for nuclei with $Z = 120 - 126$.

				$\log\left[T_{1/2}(s)\right]$						
Z	A	Q_{ANN}	ℓ	Royer	DK	AKRE	NRB	UDL	WANG	ANN
120	284	14.966	0	-8.588	-8.581	-8.473	-8.520	-8.863	-8.588	-8.538
120	286	14.630	0	-8.066	-8.060	-7.961	-7.994	-8.333	-8.066	-7.641
120	288	14.293	0	-7.524	-7.518	-7.429	-7.451	-7.782	-7.524	-6.956
120	290	13.962	0	-6.971	-6.965	-6.888	-6.899	-7.220	-6.971	-6.330
120	292	13.632	0	-6.398	-6.392	-6.329	-6.330	-6.638	-6.398	-5.703
120	294	13.257	0	-5.714	-5.708	-5.660	-5.650	-5.943	-5.714	-4.981
120	296	12.752	0	-4.731	-4.725	-4.692	-4.670	-4.946	-4.731	-4.183
120	298	12.190	0	-3.560	-3.554	-3.538	-3.502	-3.760	-3.560	-3.366
120	300	11.659	0	-2.374	-2.368	-2.369	-2.321	-2.559	-2.374	-2.616
120	302	11.152	0	-1.165	-1.159	-1.178	-1.118	-1.333	-1.165	-1.433
120	304	10.672	0	0.061	0.067	0.028	0.100	-0.091	0.061	0.462
120	306	10.219	0	1.294	1.300	1.240	1.323	1.159	1.294	2.510
120	308	9.784	0	2.562	2.568	2.486	2.580	2.444	2.562	4.285
120	310	9.392	0	3.776	3.782	3.677	3.780	3.674	3.776	5.926
120	312	9.073	0	4.818	4.824	4.693	4.803	4.730	4.818	7.293
120	314	8.835	0	5.625	5.631	5.474	5.587	5.549	5.625	8.302
120	316	8.645	0	6.286	6.293	6.108	6.222	6.222	6.286	9.096
120	318	8.456	0	6.966	6.972	6.759	6.873	6.912	6.966	9.885

Table
4.17
(contd.)

Z	A	Q_{ANN}	ℓ	Royer	DK	AKRE	NRB	UDL	WANG	ANN
120	320	8.277	0	7.634	7.640	7.397	7.511	7.591	7.634	10.633
120	322	8.102	0	8.308	8.314	8.041	8.154	8.276	8.308	11.363
120	324	7.921	0	9.029	9.035	8.731	8.844	9.009	9.029	12.117
120	326	7.738	0	9.783	9.789	9.453	9.566	9.775	9.783	12.860
120	328	7.543	0	10.628	10.634	10.266	10.378	10.632	10.628	13.698
120	330	7.346	0	11.509	11.515	11.115	11.227	11.527	11.509	14.585
120	332	7.151	0	12.422	12.428	11.994	12.106	12.453	12.422	15.486
120	334	6.960	0	13.357	13.363	12.896	13.007	13.402	13.357	16.372
120	336	6.760	0	14.374	14.380	13.879	13.991	14.434	14.374	17.294
120	338	6.527	0	15.629	15.635	15.100	15.214	15.706	15.629	18.375
120	340	6.261	0	17.159	17.165	16.597	16.716	17.257	17.159	19.596
120	342	5.974	0	18.922	18.929	18.327	18.454	19.043	18.922	20.909
120	344	5.694	0	20.767	20.774	20.139	20.275	20.912	20.767	22.245
120	346	5.414	0	22.764	22.772	22.103	22.250	22.936	22.764	23.906
120	348	5.146	0	24.823	24.831	24.128	24.286	25.021	24.823	25.825
120	350	4.895	0	26.905	26.913	26.177	26.345	27.130	26.905	27.626
122	290	15.015	0	-8.230	-8.223	-8.117	-8.149	-8.517	-8.230	-7.762
122	292	14.679	0	-7.702	-7.695	-7.598	-7.617	-7.980	-7.702	-7.118
122	294	14.345	0	-7.157	-7.150	-7.065	-7.072	-7.426	-7.157	-6.485
122	296	14.030	0	-6.627	-6.621	-6.548	-6.545	-6.887	-6.627	-5.891
122	298	13.701	0	-6.050	-6.043	-5.985	-5.971	-6.300	-6.050	-5.265
122	300	13.267	0	-5.244	-5.237	-5.194	-5.169	-5.482	-5.244	-4.562
122	302	12.760	0	-4.241	-4.234	-4.207	-4.169	-4.465	-4.241	-3.806
122	304	12.233	0	-3.129	-3.122	-3.112	-3.060	-3.337	-3.129	-3.091
122	306	11.705	0	-1.936	-1.929	-1.936	-1.872	-2.128	-1.936	-2.258
122	308	11.197	0	-0.711	-0.704	-0.730	-0.652	-0.886	-0.710	-0.809
122	310	10.703	0	0.566	0.573	0.527	0.618	0.408	0.566	1.321
122	312	10.242	0	1.842	1.849	1.782	1.884	1.701	1.842	3.422
122	314	9.804	0	3.136	3.143	3.054	3.168	3.012	3.136	5.203
122	316	9.407	0	4.383	4.390	4.278	4.402	4.277	4.383	6.971

Table 4.17 (contd.)

Z	A	Q_{ANN}	ℓ	Royer	DK	AKRE	NRB	UDL	WANG	ANN
122	318	9.045	0	5.590	5.597	5.461	5.594	5.500	5.590	8.526
122	320	8.763	0	6.577	6.585	6.423	6.562	6.502	6.577	9.729
122	322	8.573	0	7.259	7.267	7.077	7.218	7.196	7.259	10.524
122	324	8.385	0	7.960	7.967	7.750	7.892	7.908	7.960	11.312
122	326	8.202	0	8.662	8.669	8.422	8.565	8.621	8.662	12.051
122	328	8.027	0	9.356	9.363	9.086	9.230	9.327	9.356	12.741
122	330	7.850	0	10.085	10.092	9.785	9.928	10.067	10.085	13.442
122	332	7.674	0	10.837	10.844	10.506	10.649	10.832	10.837	14.170
122	334	7.496	0	11.624	11.631	11.261	11.404	11.631	11.624	14.964
122	336	7.309	0	12.482	12.489	12.086	12.229	12.502	12.482	15.811
122	338	7.114	0	13.419	13.426	12.991	13.135	13.454	13.419	16.714
122	340	6.928	0	14.347	14.354	13.886	14.030	14.396	14.347	17.571
122	342	6.738	0	15.341	15.348	14.846	14.992	15.405	15.341	18.404
122	344	6.537	0	16.436	16.443	15.907	16.054	16.515	16.436	19.228
122	346	6.268	0	18.002	18.009	17.441	17.595	18.103	18.002	20.434
122	348	6.014	0	19.577	19.585	18.983	19.144	19.700	19.577	21.593
122	350	5.772	0	21.176	21.184	20.549	20.717	21.321	21.176	22.719
124	296	15.062	0	-7.870	-7.862	-7.758	-7.774	-8.168	-7.870	-7.276
124	298	14.726	0	-7.335	-7.328	-7.234	-7.238	-7.624	-7.335	-6.639
124	300	14.408	0	-6.813	-6.805	-6.724	-6.716	-7.092	-6.813	-6.039
124	302	14.095	0	-6.280	-6.273	-6.205	-6.186	-6.550	-6.280	-5.451
124	304	13.759	0	-5.684	-5.677	-5.623	-5.594	-5.944	-5.684	-4.903
124	306	13.262	0	-4.740	-4.733	-4.694	-4.652	-4.986	-4.740	-4.141
124	308	12.778	0	-3.770	-3.762	-3.740	-3.685	-4.002	-3.769	-3.460
124	310	12.279	0	-2.704	-2.697	-2.692	-2.624	-2.922	-2.704	-2.775
124	312	11.751	0	-1.498	-1.491	-1.504	-1.422	-1.699	-1.498	-1.797
124	314	11.241	0	-0.254	-0.247	-0.278	-0.183	-0.437	-0.254	-0.073
124	316	10.745	0	1.042	1.049	0.997	1.106	0.877	1.042	2.130
124	318	10.269	0	2.374	2.382	2.309	2.432	2.228	2.374	4.294
124	320	9.826	0	3.700	3.707	3.613	3.748	3.572	3.700	6.167

Table
4.17
(contd.)

Z	A	Q_{ANN}	ℓ	Royer	DK	AKRE	NRB	UDL	WANG	ANN
124	322	9.429	0	4.964	4.972	4.855	5.001	4.854	4.964	7.982
124	324	9.052	0	6.245	6.253	6.111	6.268	6.153	6.245	9.608
124	326	8.732	0	7.390	7.398	7.231	7.396	7.314	7.390	10.964
124	328	8.501	0	8.249	8.257	8.064	8.234	8.187	8.249	11.891
124	330	8.313	0	8.970	8.978	8.757	8.929	8.920	8.970	12.637
124	332	8.125	0	9.717	9.725	9.474	9.649	9.679	9.717	13.383
124	334	7.957	0	10.405	10.413	10.132	10.307	10.378	10.405	14.044
124	336	7.782	0	11.145	11.153	10.843	11.019	11.131	11.146	14.746
124	338	7.606	0	11.921	11.929	11.588	11.764	11.919	11.921	15.533
124	340	7.432	0	12.714	12.722	12.349	12.526	12.725	12.714	16.308
124	342	7.258	0	13.535	13.543	13.138	13.316	13.559	13.535	17.106
124	344	7.083	0	14.391	14.399	13.962	14.141	14.429	14.391	17.879
124	346	6.900	0	15.331	15.339	14.870	15.050	15.384	15.331	18.633
124	348	6.711	0	16.338	16.346	15.844	16.025	16.406	16.338	19.409
124	350	6.515	0	17.433	17.441	16.905	17.090	17.518	17.433	20.215
126	302	15.094	0	-7.484	-7.476	-7.374	-7.373	-7.792	-7.484	-6.764
126	304	14.770	0	-6.962	-6.954	-6.863	-6.850	-7.261	-6.962	-6.150
126	306	14.452	0	-6.434	-6.426	-6.348	-6.323	-6.723	-6.434	-5.560
126	308	14.136	0	-5.888	-5.880	-5.816	-5.780	-6.168	-5.888	-5.050
126	310	13.751	0	-5.190	-5.182	-5.133	-5.086	-5.458	-5.190	-4.460
126	312	13.262	0	-4.246	-4.239	-4.205	-4.144	-4.501	-4.246	-3.728
126	314	12.792	0	-3.288	-3.280	-3.263	-3.189	-3.528	-3.288	-3.148
126	316	12.312	0	-2.250	-2.242	-2.243	-2.156	-2.475	-2.250	-2.410
126	318	11.796	0	-1.059	-1.051	-1.069	-0.969	-1.266	-1.059	-1.249
126	320	11.283	0	0.206	0.214	0.176	0.291	0.016	0.206	0.692
126	322	10.788	0	1.515	1.523	1.465	1.595	1.344	1.515	2.948
126	324	10.309	0	2.868	2.876	2.797	2.941	2.715	2.868	5.112
126	326	9.861	0	4.224	4.232	4.132	4.289	4.091	4.224	7.095
126	328	9.452	0	5.546	5.555	5.432	5.601	5.432	5.546	8.972
126	330	9.080	0	6.824	6.832	6.685	6.866	6.728	6.824	10.554

Table 4.17 (contd.)

Z	A	Q_{ANN}	ℓ	Royer	DK	AKRE	NRB	UDL	WANG	ANN
126	332	8.710	0	8.173	8.182	8.011	8.203	8.097	8.173	12.063
126	334	8.461	0	9.125	9.134	8.935	9.134	9.063	9.125	13.067
126	336	8.236	0	10.019	10.028	9.803	10.007	9.972	10.019	13.969
126	338	8.052	0	10.771	10.780	10.526	10.733	10.737	10.771	14.697
126	340	7.889	0	11.458	11.466	11.183	11.391	11.435	11.458	15.336
126	342	7.721	0	12.189	12.197	11.884	12.093	12.178	12.189	16.076
126	344	7.549	0	12.968	12.977	12.633	12.844	12.971	12.968	16.844
126	346	7.375	0	13.781	13.790	13.416	13.628	13.797	13.781	17.582
126	348	7.203	0	14.620	14.629	14.222	14.436	14.649	14.620	18.288
126	350	7.030	0	15.493	15.502	15.064	15.280	15.537	15.493	18.997

4.5.1 Predictions of the α-decay half-lives for unmeasured nuclei using machine learning model.

Also, the machine learning models was used to predict the α-decay half-lives of unmeasured superheavy nuclei as shown in Table 4.18. It is observed that k Nearest neighbours (kNN), Support vector regression (SVR), Decision trees, Random forest and Extra trees could not accurately predict the half-lives of unmeasured superheavy nuclei. The predicted α-decay half-lives by the ANN model correlate with result of the α-decay half-lives calculated by the empirical formulas as shown in Table 4.17. This further established that the ANN model can be reliably used to predict the alpha decay half-lives of measured and unmeasured superheavy nuclei.

Table 4.18: Predicted α-decay half-lives, $\log\left[T_{1/2}(s)\right]$, for $Z = 120 - 126$ using the machine learning models.

				$\log\left[T_{1/2}(s)\right]$					
Z	A	Q_{ANN}	ℓ	ANN	kNN	SVR	DT	RF	ET
120	284	14.966	0	-8.538	-2.935	2.569	-3.948	-3.697	-3.447
120	286	14.630	0	-7.641	-2.938	1.635	-3.948	-3.697	-3.423

Table 4.18 (contd.)

Z	A	Q_{ANN}	ℓ	ANN	kNN	SVR	DT	RF	ET
120	288	14.293	0	-6.956	-2.940	0.657	-3.948	-3.697	-3.399
120	290	13.962	0	-6.330	-2.939	-0.298	-3.948	-3.697	-3.358
120	292	13.632	0	-5.703	-2.935	-1.184	-3.948	-3.697	-3.471
120	294	13.257	0	-4.981	-2.925	-2.047	-3.948	-3.719	-3.486
120	296	12.752	0	-4.183	-2.908	-2.847	-3.948	-3.719	-3.486
120	298	12.190	0	-3.366	-2.897	-3.037	-3.948	-3.719	-3.486
120	300	11.659	0	-2.616	-2.058	-2.398	-3.948	-3.719	-3.348
120	302	11.152	0	-1.433	-1.601	-1.055	-3.948	-3.250	-2.835
120	304	10.672	0	0.462	-1.273	0.764	-2.031	-2.370	-1.590
120	306	10.219	0	2.510	-1.015	2.804	-2.031	-1.685	-0.569
120	308	9.784	0	4.285	-0.869	4.888	1.903	0.776	0.379
120	310	9.392	0	5.926	-0.679	6.747	1.903	0.849	0.367
120	312	9.073	0	7.293	-0.694	8.196	1.903	0.843	0.481
120	314	8.835	0	8.302	-0.718	9.230	1.903	0.907	0.631
120	316	8.645	0	9.096	-0.742	10.004	1.903	0.879	0.615
120	318	8.456	0	9.885	-0.765	10.682	1.903	1.355	0.953
120	320	8.277	0	10.633	-0.785	11.244	2.292	1.577	1.304
120	322	8.102	0	11.363	-0.804	11.704	2.292	2.182	1.910
120	324	7.921	0	12.117	-0.269	12.084	3.178	2.679	2.564
120	326	7.738	0	12.860	-0.277	12.375	3.178	2.807	3.011
120	328	7.543	0	13.698	-0.285	12.590	3.668	4.512	3.634
120	330	7.346	0	14.585	0.191	12.714	3.668	4.696	3.904
120	332	7.151	0	15.486	0.991	12.754	5.087	4.953	4.439
120	334	6.960	0	16.372	1.861	12.717	5.087	5.127	5.034
120	336	6.760	0	17.294	1.897	12.617	6.248	5.916	5.771
120	338	6.527	0	18.375	1.936	12.454	7.511	7.009	6.643
120	340	6.261	0	19.596	1.975	12.225	10.044	8.690	7.646
120	342	5.974	0	20.909	3.120	11.933	9.227	8.857	8.269
120	344	5.694	0	22.245	4.372	11.604	10.554	10.526	10.199
120	346	5.414	0	23.906	4.421	11.247	14.692	11.700	12.060

Table 4.18 (contd.)

Z	A	Q_{ANN}	ℓ	ANN	kNN	SVR	DT	RF	ET
120	348	5.146	0	25.825	6.056	10.883	14.692	13.546	12.799
120	350	4.895	0	27.626	6.078	10.525	14.692	13.741	13.008
122	290	15.015	0	-7.762	-2.911	2.907	-3.948	-3.697	-3.358
122	292	14.679	0	-7.118	-2.911	2.049	-3.948	-3.697	-3.471
122	294	14.345	0	-6.485	-2.908	1.161	-3.948	-3.719	-3.486
122	296	14.030	0	-5.891	-2.903	0.336	-3.948	-3.719	-3.486
122	298	13.701	0	-5.265	-2.895	-0.470	-3.948	-3.719	-3.486
122	300	13.267	0	-4.562	-2.880	-1.378	-3.948	-3.719	-3.486
122	302	12.760	0	-3.806	-2.856	-2.055	-3.948	-3.719	-3.486
122	304	12.233	0	-3.091	-2.814	-2.135	-3.948	-3.719	-3.486
122	306	11.705	0	-2.258	-1.806	-1.465	-3.948	-3.719	-3.397
122	308	11.197	0	-0.809	-1.554	-0.121	-3.948	-3.250	-2.862
122	310	10.703	0	1.321	-1.312	1.719	-2.031	-2.370	-1.659
122	312	10.242	0	3.422	-1.129	3.744	-2.031	-1.685	-0.643
122	314	9.804	0	5.203	-1.017	5.754	1.903	0.802	0.379
122	316	9.407	0	6.971	-0.800	7.517	1.903	0.828	0.364
122	318	9.045	0	8.526	-0.792	8.979	1.903	0.843	0.477
122	320	8.763	0	9.729	-0.801	10.002	1.903	0.907	0.618
122	322	8.573	0	10.524	-0.813	10.633	1.903	0.942	0.639
122	324	8.385	0	11.312	-0.826	11.162	2.292	1.533	1.116
122	326	8.202	0	12.051	-0.838	11.580	2.292	1.728	1.525
122	328	8.027	0	12.741	-0.851	11.892	2.292	2.338	2.110
122	330	7.850	0	13.442	-0.862	12.121	3.178	2.795	2.771
122	332	7.674	0	14.170	-0.873	12.265	3.178	3.033	3.215
122	334	7.496	0	14.964	-0.884	12.331	3.668	4.542	3.643
122	336	7.309	0	15.811	-0.306	12.329	3.668	4.704	3.960
122	338	7.114	0	16.714	0.971	12.260	5.087	4.962	4.508
122	340	6.928	0	17.571	1.831	12.131	5.087	5.484	5.170
122	342	6.738	0	18.404	1.863	11.951	7.511	6.538	5.844
122	344	6.537	0	19.228	1.894	11.726	7.511	7.009	6.562

Table 4.18 (contd.)

Z	A	Q_{ANN}	ℓ	ANN	kNN	SVR	DT	RF	ET
122	346	6.268	0	20.434	1.933	11.433	10.044	8.655	7.665
122	348	6.014	0	21.593	1.965	11.114	9.227	8.856	8.277
122	350	5.772	0	22.719	2.776	10.784	10.554	10.254	9.518
124	296	15.062	0	-7.276	-2.888	3.309	-3.948	-3.719	-3.486
124	298	14.726	0	-6.639	-2.885	2.543	-3.948	-3.719	-3.486
124	300	14.408	0	-6.039	-2.880	1.794	-3.948	-3.719	-3.486
124	302	14.095	0	-5.451	-2.873	1.068	-3.948	-3.719	-3.486
124	304	13.759	0	-4.903	-2.862	0.342	-3.948	-3.719	-3.486
124	306	13.262	0	-4.141	-2.842	-0.579	-3.948	-3.719	-3.486
124	308	12.778	0	-3.460	-2.813	-1.091	-3.948	-3.719	-3.486
124	310	12.279	0	-2.775	-1.958	-1.089	-3.948	-3.719	-3.486
124	312	11.751	0	-1.797	-1.667	-0.419	-3.948	-3.719	-3.456
124	314	11.241	0	-0.073	-1.507	0.880	-3.948	-3.250	-2.864
124	316	10.745	0	2.130	-1.339	2.634	-2.031	-2.423	-1.757
124	318	10.269	0	4.294	-1.201	4.591	-2.031	-1.685	-0.730
124	320	9.826	0	6.167	-1.112	6.473	1.903	0.996	0.403
124	322	9.429	0	7.982	-1.066	8.074	1.903	0.828	0.366
124	324	9.052	0	9.608	-0.865	9.415	1.903	0.843	0.487
124	326	8.732	0	10.964	-0.863	10.371	1.903	0.883	0.603
124	328	8.501	0	11.891	-0.868	10.948	1.903	1.209	0.847
124	330	8.313	0	12.637	-0.875	11.326	2.292	1.543	1.216
124	332	8.125	0	13.383	-0.882	11.609	2.292	2.112	1.869
124	334	7.957	0	14.044	-0.890	11.780	3.178	2.572	2.425
124	336	7.782	0	14.746	-0.897	11.882	3.178	2.779	2.935
124	338	7.606	0	15.533	-0.905	11.912	3.668	4.462	3.543
124	340	7.432	0	16.308	-0.912	11.875	3.668	4.686	3.794
124	342	7.258	0	17.106	-0.919	11.782	3.668	4.708	4.176
124	344	7.083	0	17.879	-0.458	11.639	5.087	4.953	4.593
124	346	6.900	0	18.633	0.975	11.453	5.087	5.518	5.280
124	348	6.711	0	19.409	1.839	11.229	7.511	6.538	5.966

Table 4.18 (contd.)

Z	A	Q_{ANN}	ℓ	ANN	kNN	SVR	DT	RF	ET
124	350	6.515	0	20.215	1.867	10.974	7.511	7.050	6.715
126	302	15.094	0	-6.764	-2.868	3.727	-3.948	-3.719	-3.486
126	304	14.770	0	-6.150	-2.863	3.086	-3.948	-3.719	-3.486
126	306	14.452	0	-5.560	-2.856	2.441	-3.948	-3.719	-3.486
126	308	14.136	0	-5.050	-2.847	1.811	-3.948	-3.719	-3.486
126	310	13.751	0	-4.460	-2.834	1.093	-3.948	-3.719	-3.486
126	312	13.262	0	-3.728	-2.811	0.346	-3.948	-3.719	-3.486
126	314	12.792	0	-3.148	-2.780	-0.028	-3.948	-3.719	-3.486
126	316	12.312	0	-2.410	-1.865	0.044	-3.948	-3.719	-3.486
126	318	11.796	0	-1.249	-1.585	0.685	-3.948	-3.719	-3.473
126	320	11.283	0	0.692	-1.472	1.896	-3.948	-3.275	-2.965
126	322	10.788	0	2.948	-1.353	3.503	-2.031	-2.423	-1.799
126	324	10.309	0	5.112	-1.250	5.286	-2.031	-2.122	-0.906
126	326	9.861	0	7.095	-1.177	6.990	1.903	0.977	0.409
126	328	9.452	0	8.972	-1.133	8.439	1.903	0.828	0.375
126	330	9.080	0	10.554	-1.112	9.574	1.903	0.850	0.489
126	332	8.710	0	12.063	-0.910	10.464	1.903	0.879	0.603
126	334	8.461	0	13.067	-0.911	10.924	1.903	1.355	0.944
126	336	8.236	0	13.969	-0.913	11.220	2.292	1.663	1.367
126	338	8.052	0	14.697	-0.917	11.371	2.292	2.278	2.072
126	340	7.889	0	15.336	-0.922	11.432	3.178	2.795	2.648
126	342	7.721	0	16.076	-0.927	11.434	3.178	2.807	3.052
126	344	7.549	0	16.844	-0.932	11.381	3.668	4.512	3.633
126	346	7.375	0	17.582	-0.937	11.276	3.668	4.700	3.871
126	348	7.203	0	18.288	-0.942	11.128	3.668	4.827	4.383
126	350	7.030	0	18.997	-0.947	10.944	5.087	5.127	4.667

Chapter 5

Conclusion

In this research, the alpha decay half-lives of superheavy nuclei have been studied using empirical formulas and machine learning models. The alpha decay emprical formulas used were AKRE formula, Denisov and Khudenko formula (DK), Royer formula, Viola and Seaborg Semi-empirical formula (VSS), New Ren A (NRA), New Ren B (NRB), Universal Decay Law (UDL), and Wang formula. The machine learning models used were k Nearest neighbour (kNN), Support Vector Regression (SVR), Decision Trees (DT), Random Forest (RF), Extra Trees (ET), and the Artificial neural network (ANN).

The experimental data used in this research were extracted from the NUBASE2020 database. Before the empirical formulas were used to calculate the alpha decay half-lives of the nuclei, new coefficients were obtained for the formulas using a least square fit scheme with input data from the NUBASE2020 database. A total of 549 nuclei were considered from the database containing 189 even-even nuclei, 150 even-odd nuclei, 117 odd-even nuclei, and 93 odd-odd nuclei. The standard deviation was calculated using the experimental alpha decay half-lives and the theoretically calculated alpha decay half-lives. For even-even nuclei, the AKRE formula gave the least standard deviation. New Ren B formula gave the least standard deviation for odd-even nuclei. The Royer formula recorded the lowest standard deviation for even-odd and odd-odd nuclei. Overall, the Royer formula gave the best description of the alpha decay half-lives for all 549 nuclei considered, with a standard deviation value of 0.541078.

The α-decay half-lives were calculated using the improved formulas obtained for nuclei with atomic number in the range $Z = 92 - 118$. This set of nuclei contains both heavy and superheavy nuclei. It was observed that the Royer formula recorded the least deviation with a value of 0.562103 followed by the New Ren B formula with a standard deviation value of 0.612868.

The second stage of the research began with the training of the machine learning algorithms to predict the alpha decay half-lives of nuclei. The dataset of 549 nuclei was split into 80% training set and 20% test set. A 10-fold cross validation scheme was employed. After training, the performance of the machine learning models were determined from their performance in predicting the half-lives in the test set. With a standard deviation value of 0.620593, the ANN gave the best predictions of the half-lives of data in the test set, followed closely by support vector regression, with a standard deviation value of 0.634572.

In order to predict the alpha decay half-lives of unmeasured superheavy nuclei, the Q values are required as inputs. However, the experimental Q values for these unmeasured nuclei are not available. This has prompted us to use artificial neural network again to predict the Q values of nuclei. To achieve this, the experimental Q value of 1080 nuclei were used to train ANN. The data was split again into 80% training set and 20% test set. A 10-fold cross validation scheme was also used. After training, the trained ANN model was then used to predict the Q values of nuclei in the test set. The ANN model was found to give excellent predictions of the Q values of the nuclei in the test set. This served as motivation to use the trained ANN model to predict the Q values of superheavy nuclei with Z = 120–126. The Q values predicted by ANN were termed Q_{ANN}.

The decay energies Q_{ANN} predicted by ANN were then used as inputs to predict the α-decay half-lives of nuclei with atomic number in the range $Z = 120 - 126$ using the improved empirical formulas and machine learning models. The alpha decay half-lives predicted by ANN were found to be of the same order with those predicted by the improved empirical formulas. The other machine learning models did not give good predictions of the alpha decay half-lives of the unmeasured superheavy nuclei.

In conclusion, artificial neural network is a good tool to predict the α-decay half-lives of superheavy nuclei. This is a novel approach in studying the α-decay half-lives of superheavy nuclei and this research will serve as a guideline for future experimental investigations of alpha decay half-lives of the unmeasured superheavy nuclei.

5.1 Recommendation

This thesis studied superheavy nuclei with the aid of machine learning models and empirical formulas. Apart from artificial neural network, the other machine learning models used did not give good predictions of the unmeasured superheavy nuclei. It is recommended that the prediction of the alpha decay half-lives of nuclei be carried out using bayesian neural network. The Bayesian neural network has been recently found to give good descriptions of the Qvalues. The success can be extended to the prediction of the alpha decay half-lives.

In predicting the alpha decay half-lives of the unmeasured superheavy nuclei, we have set the angular momentum carried by the emitted alpha particle to zero. For even-even nuclei, this is correct. However, for other nuclei, the angular momentum may not be zero. It is therefore suggested that future study train classification algorithms to predict the angular momentum carried by the emitted alpha particle.

Bibliography

[1] A. Zdeb, M. Warda, and K. Pomorski. Half-lives for α and cluster radioactivity within a gamow-like model. *Physical Review C*, 87(2), 2013. doi: 10.1103/physrevc.87.024308. URL `https://doi.org/10.1103/physrevc.87.024308`.

[2] E. Rutherford. VIII. uranium radiation and the electrical conduction produced by it. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 47 (284):109–163, 1899. doi: 10.1080/14786449908621245. URL `https://doi.org/10.1080/14786449908621245`.

[3] R. W. Gurney and E. U. Condon. Wave mechanics and radioactive disintegration. *Nature*, 122(3073):439–439, 1928.

[4] W. A. Yahya and B. J. Falaye. Alpha decay study of thorium isotopes using double folding model with NN interactions derived from relativistic mean field theory. *Nuclear Physics A*, 1015:122311, 2021. doi: 10.1016/j.nuclphysa.2021.122311. URL `https://doi.org/10.1016/j.nuclphysa.2021.122311`.

[5] J. Dong, H. Zhang, Y. Wang, W. Zuo, and J. Li. Alpha-decay for heavy nuclei in the ground and isomeric states. *Nuclear Physics A*, 832(3-4):198–208, 2010. doi: 10.1016/j.nuclphysa.2009.10.082. URL `https://doi.org/10.1016/j.nuclphysa.2009.10.082`.

[6] G. Naveya, S. S Kumar, and A. Stephen. A systematic study on α-decay chains of superheavy nuclei, z = 126 & 138. *International Journal of Modern Physics E*, 29(06): 2050034, 2020. doi: 10.1142/s0218301320500342. URL `https://doi.org/10.1142/s0218301320500342`.

[7] Y. Qian and Z. Ren. Half-lives of α decay from natural nuclides and from superheavy elements. *Physics Letters B*, 738:87–91, 2014. doi: 10.1016/j.physletb.2014.09.024. URL `https://doi.org/10.1016/j.physletb.2014.09.024`.

[8] P. de Marcillac, N. Coron, G. Dambier, J. Leblanc, and J. Moalic. Experimental detection of α-particles from the radioactive decay of natural bismuth. 422(6934):876–878, 2003. doi: 10.1038/nature01541. URL `https://doi.org/10.1038/nature01541`.

[9] J. W. Beeman, M. Biassoni, C. Brofferio, C. Bucci, S. Capelli, L. Cardani, M. Carrettoni, M. Clemenza, O. Cremonesi, E. Ferri, A. Giachero, L. Gironi, P. Gorla, C. Gotti, A. Nucciotti, C. Maiano, L. Pattavina, M. Pavan, G. Pessina, S. Pirro, E. Previtali, M. Sisti,

and L. Zanotti. First measurement of the partial widths of Bi 209 decay to the ground and to the first excited states. 108(6), 2012. doi: 10.1103/physrevlett.108.062501. URL https://doi.org/10.1103/physrevlett.108.062501.

[10] J. Bouma. *Elastic scattering and cluster-transfer reactions of 98Rb on 7Li at REX-ISOLDE*. PhD thesis, Leuven U., 2013.

[11] R. A. Rydin. A new approach to finding magic numbers for heavy and superheavy elements. 38(2-3):238–242, 2011. doi: 10.1016/j.anucene.2010.11.004. URL https://doi.org/10.1016/j.anucene.2010.11.004.

[12] J. W. Rohlf and P. J. Collings. Modern physics from α to Z. *Review Of" Modern Physics*, 12:1–1994, 1994.

[13] J. Tracy Jr. A binding energy study of the atomic mass evaluation 2012. 2017.

[14] W. A. Yahya. *A microscopic description of elastic scattering from unstable nuclei within a relativistic framework*. PhD thesis, Stellenbosch: Stellenbosch University, 2018.

[15] H. M. Devaraja, Y. K. Gambhir, M Gupta, and G Münzenberg. Systematics of production cross sections and predictions for the synthesis of new superheavy elements. *Physical Review C*, 93(3):034621, 2016.

[16] E. Olsen and W. Nazarewicz. α-decay energies of superheavy nuclei: Systematic trends. *Phys. Rev. C*, 99:014317, 2019. doi: 10.1103/PhysRevC.99.014317. URL https://link.aps.org/doi/10.1103/PhysRevC.99.014317.

[17] C. Qi. Alpha decay as a probe for the structure of neutron-deficient nuclei. *Reviews in Physics*, 1:77–89, 2016. doi: 10.1016/j.revip.2016.05.001. URL https://doi.org/10.1016/j.revip.2016.05.001.

[18] H. Geiger and J. M. Nuttall. Lvii. the ranges of the α particles from various radioactive substances and a relation between range and period of transformation. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 22(130):613–621, 1911.

[19] V. E. Viola and G. T. Seaborg. Nuclear systematics of the heavy elements—II lifetimes for alpha, beta and spontaneous fission decay. *Journal of Inorganic and Nuclear Chemistry*, 28(3):741–761, 1966. doi: 10.1016/0022-1902(66)80412-8. URL https://doi.org/10.1016/0022-1902(66)80412-8.

[20] B. Alex Brown. Simple relation for alpha decay half-lives. *Physical Review C*, 46(2):811–814, 1992. doi: 10.1103/physrevc.46.811. URL https://doi.org/10.1103/physrevc.46.811.

[21] G. Royer. Alpha emission and spontaneous fission through quasi-molecular shapes. 26(8): 1149–1170, 2000. doi: 10.1088/0954-3899/26/8/305. URL `https://doi.org/10.1088/0954-3899/26/8/305`.

[22] D. N. Poenaru, Ileana-Hania Plonski, and Walter Greiner. α-decay half-lives of superheavy nuclei. *Physical Review C*, 74(1), 2006. doi: 10.1103/physrevc.74.014312. URL `https://doi.org/10.1103/physrevc.74.014312`.

[23] H. C. Manjunatha, N. Sowmya, and A. M. Nagaraja. Semi-empirical formula for alpha and cluster decay half-lives of superheavy nuclei. *Modern Physics Letters A*, 35 (06):2050016, 2019. doi: 10.1142/s0217732320500169. URL `https://doi.org/10.1142/s0217732320500169`.

[24] X. D. Zhang. Machine learning. pages 223–440. Springer Singapore, 2020. doi: 10.1007/978-981-15-2770-8_6. URL `https://doi.org/10.1007/978-981-15-2770-8_6`.

[25] S. Akkoyun, T. Bayram, S. O. Kara, and A. Sinan. An artificial neural network application on nuclear charge radii. *Journal of Physics G: Nuclear and Particle Physics*, 40(5): 055106, 2013. doi: 10.1088/0954-3899/40/5/055106. URL `https://doi.org/10.1088/0954-3899/40/5/055106`.

[26] S. Athanassopoulos, E. Mavrommatis, K.A. Gernoth, and J.W. Clark. Nuclear mass systematics using neural networks. *Nuclear Physics A*, 743(4):222–235, 2004. doi: 10.1016/j.nuclphysa.2004.08.006. URL `https://doi.org/10.1016/j.nuclphysa.2004.08.006`.

[27] S. A. Bass, A. Bischoff, J. A. Maruhn, H. Stöcker, and W. Greiner. Neural networks for impact parameter determination. *Physical Review C*, 53(5):2358–2363, 1996. doi: 10.1103/physrevc.53.2358. URL `https://doi.org/10.1103/physrevc.53.2358`.

[28] N. Costiris, E. Mavrommatis, K. A. Gernoth, and J. W. Clark. A global model of β–decay half-lives using neural networks. *HNPS Proceedings*, 15:210, 2020. doi: 10.12681/hnps.2640. URL `https://doi.org/10.12681/hnps.2640`.

[29] S. S. Schoenholz. Combining machine learning and physics to understand glassy systems. 1036:012021, 2018. doi: 10.1088/1742-6596/1036/1/012021. URL `https://doi.org/10.1088/1742-6596/1036/1/012021`.

[30] A. Decelle, G. Fissore, and C. Furtlehner. Thermodynamics of restricted Boltzmann machines and related learning dynamics. 172(6):1576–1608, 2018. doi: 10.1007/s10955-018-2105-y. URL `https://doi.org/10.1007/s10955-018-2105-y`.

[31] R. Utama, J. Piekarewicz, and H. B. Prosper. Nuclear mass predictions for the crustal composition of neutron stars: A bayesian neural network approach. *Physical Review C*, 93 (1):014311, 2016.

[32] U. Baños Rodríguez, Cristofher Zuñiga Vargas, Marcello Gonçalves, Sergio Barbosa Duarte, and Fernando Guzmán. Alpha half-lives calculation of superheavy nuclei with q α -value predictions based on the bayesian neural network approach. 46(11):115109, 2019. doi: 10.1088/1361-6471/ab2c86. URL https://doi.org/10.1088/1361-6471/ab2c86.

[33] L. M. Raymond and Keith E. H. Radioactivity. pages 33–51. Elsevier, 2020. doi: 10.1016/b978-0-12-812881-7.00003-4. URL https://doi.org/10.1016/b978-0-12-812881-7.00003-4.

[34] M. Schädel. Chemistry of the superheavy elements. 373(2037):20140191, 2015. doi: 10.1098/rsta.2014.0191. URL https://doi.org/10.1098/rsta.2014.0191.

[35] D. Iwanenko. The neutron hypothesis. 129(3265):798–798, 1932. doi: 10.1038/129798d0. URL https://doi.org/10.1038/129798d0.

[36] A. Marinov, I. Rodushkin, D. Kolb, A. Pape, Y. Kashiv, R. Brandt, R. V. Gentry, and H. W. Miller. Evidence for the possible existence of a long-lived superheavy nucleus with atomic mass number A= 292 and atomic number Z≅ 122 in natural *Th*. *International Journal of Modern Physics E*, 19(01):131–140, 2010.

[37] J. P. Adloff. The laboratory notebooks of Pierre and Marie Curie and the discovery of Polonium and Radium. *Czechoslovak Journal of Physics*, 49(1):15–28, 1999.

[38] M. F. L'Annunziata. *Handbook of radioactivity analysis*. Academic press, United Kingdom, 2012.

[39] G. Gamow. The quantum theory of nuclear disintegration. *Nature*, 122(3082):805–806, 1928.

[40] T. Tomoda. Double beta decay. 54(1):53–126, January 1991. doi: 10.1088/0034-4885/54/1/002. URL https://doi.org/10.1088/0034-4885/54/1/002.

[41] J. P. Collman, J. L. Hoard, Nancy Kim, George Lang, and Christopher A Reed. Synthesis, stereochemistry, and structure-related properties of. alpha.,. beta.,. gamma.,. delta.-tetraphenylporphinatoiron (ii). *Journal of the American Chemical Society*, 97(10):2676–2681, 1975.

[42] J. M. Barbosa-Filho, A. A. Alencar, Xirley P Nunes, Anna C Tomaz, José G Sena-Filho, Petrônio F Athayde-Filho, Marcelo S Silva, Maria F Souza, and Emidio V Leitão Da-Cunha. Sources of alpha-, beta-, gamma-, delta-and epsilon-carotenes: A twentieth century review. *Revista Brasileira de Farmacognosia*, 18:135–154, 2008.

[43] A. Kamal. Radioactivity. In *Nuclear Physics*, pages 125–196. Springer, 2014.

[44] G. Audi, O. Bersillon, J. Blachot, and Aaldert Hendrik Wapstra. The nubase evaluation of nuclear and decay properties. *Nuclear physics A*, 729(1):3–128, 2003.

[45] R. H. Stuewer. Gamow's theory of alpha-decay. In *The Kaleidoscope of Science*, pages 147–186. Springer, 1986.

[46] J. H. Cheng, Jiu-Long Chen, Jun-Gang Deng, Xi-Jun Wu, Xiao-Hua Li, and Peng-Cheng Chu. Systematic study of α decay half-lives based on Gamow-like model with a screened electrostatic barrier. *Nuclear Physics A*, 987:350–368, 2019. doi: 10.1016/j.nuclphysa.2019.05.002. URL `https://doi.org/10.1016/j.nuclphysa.2019.05.002`.

[47] H. J. Krappe and K. Pomorski. *Theory of nuclear fission: A textbook*, volume 838. Springer Science & Business Media, 2012.

[48] W. A. Yahya. Alpha decay half-lives of 171-189 Hg isotopes using modified Gamow-like model and temperature dependent proximity potential. *Journal of the Nigerian Society of Physical Sciences*, pages 250–256, 2020. doi: 10.46481/jnsps.2020.139. URL `https://doi.org/10.46481/jnsps.2020.139`.

[49] K.P. Santhosh, Indu Sukumaran, and B. Priyanka. Theoretical studies on the alpha decay of 178–220pb isotopes. *Nuclear Physics A*, 935:28–42, 2015. doi: 10.1016/j.nuclphysa.2014.12.008. URL `https://doi.org/10.1016/j.nuclphysa.2014.12.008`.

[50] D. N. Poenaru, R. A. Gherghescu, and W. Greiner. Single universal curve for cluster radioactivities and α decay. *Physical Review C*, 83(1), 2011. doi: 10.1103/physrevc.83.014601. URL `https://doi.org/10.1103/physrevc.83.014601`.

[51] C. Qi, F. R. Xu, R. J. Liotta, and R. Wyss. Universal decay law in charged-particle emission and exotic cluster radioactivity. *Physical Review Letters*, 103(7), 2009. doi: 10.1103/physrevlett.103.072501. URL `https://doi.org/10.1103/physrevlett.103.072501`.

[52] V. Yu. Denisov and A. A. Khudenko. α-decay half-lives: Empirical relations. *Physical Review C*, 79(5), 2009. doi: 10.1103/physrevc.79.054614. URL `https://doi.org/10.1103/physrevc.79.054614`.

[53] G. Royer. Alpha emission and spontaneous fission through quasi-molecular shapes. *Journal of Physics G: Nuclear and Particle Physics*, 26(8):1149, 2000.

[54] G. Royer. Analytic expressions for alpha-decay half-lives and potential barriers. *Nuclear Physics A*, 848(3-4):279–291, 2010. doi: 10.1016/j.nuclphysa.2010.09.009. URL `https://doi.org/10.1016/j.nuclphysa.2010.09.009`.

[55] D. T. Akrawy and D. N. Poenaru. Alpha decay calculations with a new formula. *Journal of Physics G: Nuclear and Particle Physics*, 44(10):105105, 2017. doi: 10.1088/1361-6471/aa8527. URL `https://doi.org/10.1088/1361-6471/aa8527`.

[56] Z. Ren, C. Xu, and Z. Wang. New perspective on complex cluster radioactivity of heavy nuclei. *Physical Review C*, 70(3), 2004. doi: 10.1103/physrevc.70.034304. URL `https://doi.org/10.1103/physrevc.70.034304`.

[57] D. Ni, Z. Ren, T. Dong, and C. Xu. Unified formula of half-lives for α decay and cluster radioactivity. *Physical Review C*, 78(4), 2008. doi: 10.1103/physrevc.78.044310. URL https://doi.org/10.1103/physrevc.78.044310.

[58] D. T. Akrawy, H. Hassanabadi, Yibin Qian, and K.P. Santhosh. Influence of nuclear isospin and angular momentum on α-decay half-lives. *Nuclear Physics A*, 983:310–320, 2019. doi: 10.1016/j.nuclphysa.2018.10.091. URL https://doi.org/10.1016/j.nuclphysa.2018.10.091.

[59] Z. Y. Wang, Z. M. Niu, Q. Liu, and J. Y. Guo. Systematic calculations of α-decay half-lives with an improved empirical formula. *Journal of Physics G: Nuclear and Particle Physics*, 42(5):055112, 2015. doi: 10.1088/0954-3899/42/5/055112. URL https://doi.org/10.1088/0954-3899/42/5/055112.

[60] S. A. Paulo and John W Clark. Experiments in machine learning of alpha-decay half-lives. *arXiv preprint arXiv:1910.12345*, 2019.

[61] M. Awad and R. Khanna. Support vector regression. In *Efficient Learning Machines*, pages 67–80. Apress, 2015. doi: 10.1007/978-1-4302-5990-9_4. URL https://doi.org/10.1007/978-1-4302-5990-9_4.

[62] S. Shalev-Shwartz and S. Ben-David. *Understanding Machine Learning - From Theory to Algorithms.* Cambridge University Press, England, 2014. ISBN 978-1-10-705713-5.

[63] A. Gron. *Hands-On Machine Learning with Scikit-Learn and TensorFlow: Concepts, Tools, and Techniques to Build Intelligent Systems.* O'Reilly Media, Inc., 1st edition, 2017. ISBN 1491962291.

[64] O. Campesato. *Artificial Intelligence, Machine Learning, and Deep Learning.* Mercury Learning & Information, 2020. ISBN 9781683924661. URL https://books.google.com.ng/books?id=pqnNDwAAQBAJ.

[65] V. K. Vemuri. The hundred-page machine learning book. *Journal of Information Technology Case and Application Research*, 22(2):136–138, 2020. doi: 10.1080/15228053.2020.1766224. URL https://doi.org/10.1080/15228053.2020.1766224.

[66] P. Mehta, M. Bukov, C. H. Wang, Alexandre G.R. Day, Clint Richardson, Charles K. Fisher, and David J. Schwab. A high-bias, low-variance introduction to machine learning for physicists. *Physics Reports*, 810:1–124, 2019. doi: 10.1016/j.physrep.2019.03.001. URL https://doi.org/10.1016/j.physrep.2019.03.001.

[67] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay. Scikit-learn: Machine learning in Python. *Journal of Machine Learning Research*, 12:2825–2830, 2011.

[68] C. J. Stone. Consistent nonparametric regression. *The Annals of Statistics*, 5(4), 1977. doi: 10.1214/aos/1176343886. URL `https://doi.org/10.1214/aos/1176343886`.

[69] S. Haykin and N Network. A comprehensive foundation. *Neural networks*, 2(2004):41, 2004.

[70] D. Arden. Applied deep learning part 1 Artificial neural networks (ANN). https://towardsdatascience.com/applied-deep-learning-part-1-artificial-neural-networks-d7834f67a4f6, 2017.

[71] Arc. The perceptron. https://towardsdatascience.com/the-perceptron-3af34c84838c, July 2019.

[72] W. J. Huang, Meng Wang, F.G. Kondev, G. Audi, and S. Naimi. The AME 2020 atomic mass evaluation (i). evaluation of input data, and adjustment procedures. *Chinese Physics C*, 45(3):030002, 2021. doi: 10.1088/1674-1137/abddb0. URL `https://doi.org/10.1088/1674-1137/abddb0`.

[73] F. G. Kondev, M. Wang, W.J. Huang, S. Naimi, and G. Audi. The NUBASE2020 evaluation of nuclear physics properties. *Chinese Physics C*, 45(3):030001, 2021. doi: 10.1088/1674-1137/abddae. URL `https://doi.org/10.1088/1674-1137/abddae`.

[74] M. Wang, W.J. Huang, F.G. Kondev, G. Audi, and S. Naimi. The AME 2020 atomic mass evaluation (II). tables, graphs and references. *Chinese Physics C*, 45(3):030003, 2021. doi: 10.1088/1674-1137/abddaf. URL `https://doi.org/10.1088/1674-1137/abddaf`.

[75] J. Brownlee. *Data preparation for machine learning: data cleaning, feature selection, and data transforms in Python*. Machine Learning Mastery, 2020.

[76] W. A. Yahya and K. J. Oyewumi. Calculations of the alpha decay half-lives of some polonium isotopes using the double folding model. *arXiv preprint arXiv:2111.05604*, 2021.