THE THERMAL AND ELECTRICAL CONDUCTIVITY OF SELECTED METALS

BY

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DEDICATION

To the memory of my late father,

Mr. Yohanna Pam Kim.

ACKNOWLEDGEMENT

My unfeigned gratitude goes to God almighty for his abundant grace and mercy throughout my academic pursuit in Abubakar Tafawa Balewa University, Bauchi.

Grateful acknowledgement must be made to my dear mother Mrs. Martha Pam, an ideal mother whose so much love, care and understanding in all that I do, has been a great source of inspiration for me. It is also a pleasure to thank my brothers not forgeting my queen sister for assisting me appropriately.

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ABSTRACT

In the experimental determination of the conductivity of metals, a number of different methods of measurement are required for various classes of materials having different ranges of conductivity values. A particular method may thus be preferable over the others for a given material, and no one method is suitable for all the conditions of measurment. The appropriateness of a method is further determined by such considerations as the physical nature of the materials. The geometry of samples available, the required accuracy of results depending on the nature of equipment and funds entailed.

A model each of insulated copper and aluminium rods was used as test specimen. The steady state method in which the test specimen is subjected to a temperature profile which is time invariant was used in determining the thermal conductivity directly by measuring the rate of heat flow per unit area and temperature gradient after equilibrium has been reached. The four wire method in which the voltage is measured with change in Ammeter current was used to determine the electrical conductivities.

The method employed in each case showed results which are in reasonable agreement with each other.

CHAPTER ONE

PRELIMINARY SURVEY

INTRODUCTION

1.1 General

Conduction is, basically, the transmission of energy by molecular motion. Conductivity is, then, the physical property denoting the case with which a particular substance can accomplish this transmission. The conductivity of a material is found to depend on the chemical composition of the substance, or substances, of which it is composed, the phase (i.e gas, liquid, or solid) in which it exists, its crystalline structure if a solid, the temperature and pressure to which it is subjected, and whether or not it is a homogeneous material.

The factors with greatest influence are chemical composition, phase changes, and temperature. Usually the first two of these do not enter a case in which one is interested in a particular material, and, hence, only the temperature effect has to be accounted for.

Generally speaking, a liquid is a better conductor than a gas and that a solid is a better conductor than a liquid. These facts are best illustrated by considering the three phases of single substance; such as mercury.

TABLE 1.1 Thermal Conductivity of Various Materials

Substance	Thermal Conductivity W/M-°C
Gases	
Freon - 12(0°C, 1atm)	0.0083
Air (0°C, 1 atm)	0.0241
Liquids	
CO_2 (Sat. liquid, 0°C)	0.105
Glycerine, Pure 10°C)	0.282
Water (Sat. Liquid, 0°C)	0.562
Solids	
Glass, plate (20°C)	0.76
Ice (0°C)	2.22
Magnesite brick (204°C)	3.81
Quartz (20°C)	7.6
Stainless steel (18% Cr, 8% Ni) (0°C)	16.3
Iron, pure (0°C)	73
Zinc, Pure (0°C)	112
Aluminium, Pure (0°C)	202
Copper, Pure (0°C)	386
Silver, Pure 10°C	417

Consider table 1.1 above, as a solid at - 193°C, mercury has a thermal conductivity of 48w/m-°C, as a liquid at 200°C its thermal conductivity is reported to be 0.0341 w/m-°C. These differences can be explained partially by the fact that while in a gaseous state, the molecules of a substance are spaced relatively far apart and their motion is random. This means that energy transfer by molecular impact is much more slower than in the case of a liquid in which the motion is still random but in which the molecules are more closely packed. The same is true concerning the difference between the thermal conductivity of the liquid and solid phases; however, other factors become important when the solid state is formed.

Refering to table 1.1 one sees that a solid having a crystalline structure, such as quartz, has a higher thermal conductivity than a substance in an amorphous solid state, such as glass.

Also, metals, crystalline in structure, are seen to have greater thermal conductivities than do nonmetals. In the case of the amorphores solids, the irregular arrangment of the molecules inhibits the effectiveness of the transfer of the energy by molecular impact, and hence, the thermal conductivity is of the same order of magnitude as that observed for liquids. On the other hand, in a solid having a crystalline structure, there is an additional transfer of heat energy as a result of a vibratory motion of the crystal lattice as a whole, in the direction of decreasing temperature. Imperfections in the lattice structure tend to distort and scatter these "thermoelastic" vibrations and, hence, tend to decrease their intensity.

In the case of metallic conduction, still a third mechanism of energy transfer, in addition to the molecular communication and lattice vibration mentioned above, comes to play. When the crystal of a nonmetallic substance is is formed, the valence electrons (i.e the outermost electrons) are shared among atoms to form the chemical bond which holds the atoms together as a molecule. In a metal crystal hovever, these valence electrons become detached and are free to move within the lattice formed by the remaining positive ions of the metal atoms. When a difference exists between the temperatures of different parts of the metal, a general drift of these free electrons which makes the metals so much better as conductors than other solids. These electrons account for the observed proportionality between thermal and electrical condictivities of pure metals.

It is known that the thermal conductivity of metals is directly proportional to the absolute temperature and the mean free path of the molecules. The mean free path tends to decrease with increasing temperature so that the net variation is the result of opposing influences. Pure metals generally have thermal conductivities which decrease with temperature, but the presence of impurities or alloying elements, even in minute amounts, may reverse this trend.

1.2 Definition of the Thermal and Electrical Conductivities

1.21 Thermal Conductivity

The thermal conductivity of a given metallice rod is defined by the relation $q_E = - \lambda A \frac{\partial t}{\partial x}$

where A is the cross-sectional area, q_E the heat flux and $\partial t/\partial x$ is the temperature gradient. The proportionality constant λ is called the thermal conductivity with units in $W/M-{}^{o}C$. In this process the quantity being transported is the molecular energy. Energy is transported from a high temperature region to a low tempterature region by the molecular motion. The minus sign is due to the convention that the heat flow is taken to be positive if ∂t is negative in the direction of increasing X, the normal displacement.

1.22 Electrical Conductivity

The electrical conductivity of a metallic rod is defined by

$$\delta = \left(\frac{\ell}{m}\tau\right)(Nee)$$

$$= \mu e(Nee)$$

with units in $(\Omega m)^{-1}$. μe is known as the mobility and Nee the charge density. Thus we may have high conductivities because there are lots of electrons around or because they can require high drift velocities (by having high mobilities). The electrical conductivity, like the thermal conductivity, is directly proportional to temperature.

1.3 Experimentation

The various methods for the measurement of thermal conductivity fall into two categories: the steady-state and the non-steady state methods. In the steady state method of measurement, the test specimen is subjected to a temperature profile which is time invariant, and the thermal conductivity is determined directly by measuring the rate of heat flow per unit area and temperature gradient after equilibrium has been reached. In the nonsteady-state method, the temperature distribution in the specimen varies with time, and measurement of the rate of temperature change, which normally determines the thermal diffusivity with a further knowledge of the density and specific heat of the test material.

The primary concern in most methods of measurement is to obtain a controlled heat flow in a prescribed direction such that the actual boundary conditions in the experiment agree with those assumed in the theory.

Theoritically, the simplest method to obtain a controlled heat flow is to use a specimen in the form of a hollow sphere with a heater in the center. The heat supplied by the internal heater passes through the specimen in a radial direction without loss. However, in reality it is very difficult to fabricate a spherical heater which produces uniform heat flux in all radial directions. It is also difficult to fabricate spherical specimens and to measure the heat input and the temperature gradient in this experiment arrangement.

A more commonly used method of controlling heat flow in the prescribed direction is the use of guard heaters (combined with thermal insulation in most cases). So adjusted that the temperature gradient is zero in all directions except in the direction of heat flow. In most methods of measuring thermal conductivity, a cyclindrical specimen with geometry ranging from long rod to short disk is utilized, and the heat flow is controlled to be in either the longitudinal (axial) or radial direction as the case may be.

1.3.1 Equipment

The apparatus used for the different experiments are listed below.

Tripod base, Bench Clamp, Support rod (630 mm) Support rod (1,000 mm), Universal Clamp, Right angle clamp, Supporting block, Glass beaker, Heat conductive paste, distilled water, Rheostat, heater, Digitat thermometer, temperature probe (immersion and surface type, Heat conductive rods (copper and Aluminium), Aquarium pump, Power controller, Calorimeter, Calorimeter vessel with heat conductor connection, stepping transformer with rectifier, voltmeter, Ammeter, Measuring amplifier, stop watch (w interruption type), connecting cord (red and blue).

1.3.2 Aim and Scope

The primary aim of this experimental research is to determine the thermal and electrical conductivities of selected metals (of which copper and Aluminium were used) due to their availability), and compare their conductivities.

To do this, the experiement was done in three phases;

(a) The heat capacity of the calorimeter was determined in a preliminary experiment.

(b) A constant temperature gradient was created in the metal rod using two heat reservoirs (boiling water and iced water) and the thermal conductivity determined.

c) The electrical conductivity of each one of the metals was then determined by recording a current - voltage characterstic and to check the Wiedemann-franz law.

1.3.3 Set-up and Performance

The experiment was set-up as shown in figure 2.1 and the performance was conducted in three phases (a,b,C) as stated earlier in secion 1.31.

(a) To determine the heat capacity of the lower calorimenter, the calorimeter was first weighed at room temperature (27°C). The calcorimeter was then filled with water and the equilibruim temperature of water determinal by the method of mixtures. The mass of the water was then obtained so that the heat capacity of the calorimeter can be calculated. The essence of this preliminary experiment is to determine the heat capacity of the calorimeter which is then used in calculating the other factors leading to determination of the thermal condicutivity.

The effect of the ambient temperature and the circulating pump on the rise in the the temperature of the water (2°C, no pieces of ice) in the calorimeter was also determined by measuring the temperature over a period of approximately half an hour at two minutes interval.

(b) To determine the thermal conductivity, the experiment was set-up as shown in figure 1.2 with the upper end of the metal rod smeared with a generous amount of heat conductive paste before putting the top calorimeter in place.

The water in the top calorimeter was heated with an immersion heater and maintained at the boiling point with the aid of the power controller. There are grooves in the rod for the surface temperature probe, and these were also smeared with the heat conductive paste.

The lower end of the rod sits in a calorimeter full of water, the temperature being maintained at the lowest degree possible (about 2°C) with ice cubes.

When a constant temperature gradient had been established in the metal rod (i.e. when the values on the surface temperature probes no longer changed as shown on the electronic/ digital thermometer), the ice from the bottom calorimeter was removed and the rise of temperature of the water, initially at 2°C measured and the difference in the temperature between the two probes on the rod also measured for approximately five minutes.

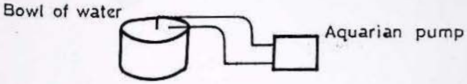


Fig. 1.1 Determination of the effect of ambient temperature

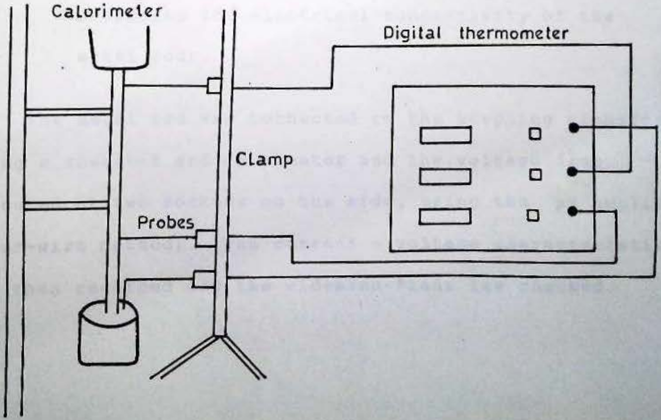


Fig. 1.2 Experiment set-up for determining thermal conductivity of metal rods.

(c) The experimental set-up for measuring the electrical conductivity was set up as shown in fig. 1.3 below.

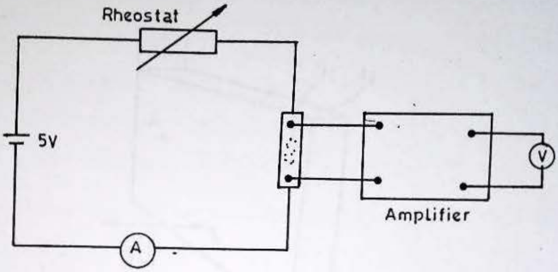


Fig. 1.3 circuit diagram showing the equipment used in measuring the electrical conductivity of the metal rods.

The metal rod was connected to the stepping transformer using a rheostat and an ammeter and the voltage drop measured at two sockets on the side, using the μV amplifier (four-wire method). The current - voltage characteristic was then recorded and the widemann-Franz law checked.

CHAPTER TWO

THEORITICAL BACKGROUND

2.1 Theory of Thermal Conductivity of Metals

The basic law governing heat conduction may best be illustrated by considering a simple, idealized situation in fig. 2.1.

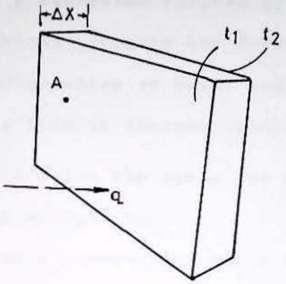


Fig. 2.1 A Plate of Material

Consider a plate of material having a surface area A and a thickness ΔX. Let one side be maintained at a temperature t_1, uniformly over the surface, and the other side at temperature t_1. Let q denote the rate of heat flow (i.e, energy per unit time) through the plate, nelgecting any edge effects. Experiment has shown that the rate of heat flow is directly proportional to A and (t_1 - t_2) but inversely proportional to ΔX, i.e.

$$q = \frac{\lambda A t_1 - t_2}{\Delta X} \quad . \ldots \quad 2$$

The constant of proportionality, λ, is called the thermal conductivity of the material of which it is composed. It is a property dependent only on the composition of the material, not the geometrical configuration.

Sometimes a gross quantity, unit thermal conductance, is used to express the heat conducting capacity of a given physical system, so that if $C = \lambda/DX$ denote the unit thermal conductance,

$$q = CA\ (t1 - t2) \quad \ldots\ldots \quad 2.15$$

Thus, it is seen that the thermal conductance is the conductivity of a substance divided by its thickness. It is no longer a physical property but depends, as well, on the geometrical configuration at hand, and thus, is a less general quantity than is thermal condictivity.

Equation 2.1 forms the basis for the fundamental relation of heat conduction.

Consider now a homogeneous solid as depicted in fig. 2.2 below

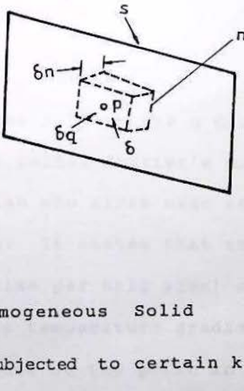


Fig 2.2 A homogeneous Solid

If the solid is subjected to certain known boundary temperatures; selecting a point P on the surface S, one can select a wafer of material having an area δA, which is part of the surface S containing P, and having a thickness δn in the direction of the normal drawn to the surface at P. If the difference between the temperature of the back face of the wafer and its front face is δt, and if δA is chosen small enough so that δt is essentially uniform over it, the rate of heat heat flow across the wafer, δq, is, by equation 2.4

$$q = -\lambda \delta A \frac{\delta t}{\delta n} \quad .... \quad 2.2$$

The minus sign is due to the convention that the heat flow taken to be positive if δt is negative in the direction of increasing n, the normal displacement.

Forming the ratio δq/δA and allowing the earea $\delta A \rightarrow 0$, one obtains what is termed the flux of heat conducted through the thickness δn at the point P,

$$fn = \frac{dq}{dA} = -\lambda \frac{\delta t}{\delta n}$$

Further, allowing $\delta n \rightarrow o$, one arrives at the flux of heat accross S at P in terms of the temperature gradient in the n direction, δt/δn:

$$fn = \frac{-\lambda \delta t}{\delta n} \quad ...... \quad 2.3$$

Where fn is the flux in the n direction. The statement in equation 2.3 is called fourier's condition law after the french mathematician who first made an extensive analysis of heat conduction. It states that the flux of heat conducted (energy per unit time per unit area) across a surface is proportional to the temperature gradient taken in a direction normal to the surface at the point in question.

From equation 2.3 , total rate of heat transferred across the surface S would be

$$q = -\int_s \frac{\delta \lambda}{\delta n} dA \quad .... \quad 2.4$$

Generally speaking, $\delta t/\delta n$ may vary over S, but in many instances it is possible to select the surface as one on which the gradient is everywhere the same. This is the situation in the case depicted in fig. 2.1, in which every plane normal to ΔX in such a surface. In the case of a hollow cylinder with uniform outside and inside surface temperatures, every concentric interior cyclindrical surface is isothermal with a uniform temperature gradient normal to it. In such cases

$$q = -\lambda A \frac{\delta t}{\delta n} \qquad \ldots\ldots\ldots \qquad 2.5$$

Where A is the total area of the finite surface.

2.2 The Theory of Electrical Conductivity

Suppose a potential difference U is applied between the two ends of a solid of length L. Then an electric field

$$\varepsilon = \frac{U}{L} \qquad . \qquad . \qquad . \qquad 2.6$$

is present at every point in the solid, causing an acceleration

$$a = \frac{\ell\varepsilon}{m} \qquad \ldots\ldots \qquad 2.7$$

Thus the electrons, in addition to their random velocities, will acquire a velocity in the direction of the electric field. We may assume that this directed velocity is completely lost after each collision, because an electron is much lighter than a lattice atom. Thus only the part of this velocity that is picked up in between collisions counts. If we write τ for the average time between two collisions, the final velocity of the electron will be $a\tau$ and the average velocity

$$\text{Vave} = \frac{1}{2}\, d\tau \qquad \ldots\ldots \qquad 2.8$$

V_{ave}.

This is simple enough but not quite correct. We shouldn't use the average time between collisions to calculate the average velocity but the actual times, and then average.

The correct derivation is fairly lengthy but all it gives is a factor of 2. Numerical factors like 2 or 3 or π are generally not worth worrying about in simple models but just to agree with the formulae generally quoted in the literature we shall incorporate that factor 2, and use

$$V_{ave} = a\tau \quad \ldots\ldots\ldots \quad 2.9$$

The average time between collisions (τ) is also referred to as mean free time, relaxation time, and collision time. Similarly, the average velocity is also referred to as the mean volocity or drift velocity. We shall call them 'collision time' and 'drift velocity', (denoting the latter by V_D).

The relationship between drift velocity and electric field may be obtained from eqns. 2.7 and 2.8 as follows:

Since $V_D = a\tau$

and $a = \frac{\ell}{m}\varepsilon$

then $V_D = \left(\frac{\ell}{m}\tau\right)\varepsilon, \quad \ldots\ldots \quad 2.10$

where the proportionality constant in parentheses is called the 'mobility.' The higher the mobility, the more mobile the electrons.

Assuming now that all electrons drift with their drift velocity, the total number of electrons crossing a plane of unit area per second is

$$J = Ne \; eV_D \quad \ldots\ldots\ldots \quad 2.11$$

Notice that it is only the drift velocity, created by the electric field, that comes into the expression. The random velocities do not contribute to the electric current because they average out to zero. They give rise however to 'electrical noise' in a conductor. Its value is usually much smaller than the signals we are concerned with.

From equations 21.10 and 2.11 the current density is found by substituting V_D from 21.10 into 2.10 i.e,

$$V_D = \left(\frac{\ell}{m} \tau \right) \varepsilon$$

$$\text{and } J = Ne\ C\ V_D$$

$$\text{hence } J = \frac{Ne\ C^2 \tau}{m} \varepsilon \qquad \ldots\ldots \qquad 2.12$$

This is a linear relationship which may be recognized as Ohm's Law

$$J = \sigma \varepsilon \qquad \ldots\ldots \qquad 2.13$$

where σ is the electrical conductivity. We can write it in the form

$$\delta = \left(\frac{\ell}{m}\tau\right)(NeC)$$

$$= \mu e\ (NeC) \qquad \ldots\ldots\ldots \qquad 2.14$$

That is, we may regard conductivity as a product of two factors, charge density (NeC) and mobility (Me). Thus we may have high conductivities because there are lots of electrons around or because they can acquire high drift velocities (by having high mobilities). In metals, incidentally the mobilities are quite low (about two orders of magnitude below those of semiconductors); so their high conductivity is due to the high density of electrons.

Ohm's law further implies that σ is a constant, which means that τ must be independent of electric field. It seems reasonable at this stage to assume that the charge and mass of the electron and the number of electrons present will be independent of the electric field.

From our model so far it is more reasonsble to assume that ℓ, the distance between collisions (usually called the mean free path) in the regularly spaced lattice, rather than τ, is independent of electric field. But ℓ must be related to τ by the relationship

$$\ell = \tau \ (V_{th} + V_D) \quad \ldots\ldots\ldots \quad 2.15$$

Since V_D varies with electric field, τ must also vary with the field unless

$$V_{th} >> V_D$$

As Ohm's law is accurately true for most metals this inequality should hold. In a typical metal $\mu e = 5X10^{-3} M^2 V^{-1} S^{-1}$, which gives a V_D of $5X10^{-3}$M/S for an electric field of 1 V/M. The thermal velocity at room temperature according to $\frac{1}{2} MV_{th}^2 = \frac{3}{2} KT$ 2.16

which actually gives too low a value for metals is

$$V_{th} = \left(\frac{3KT}{M}\right)^{\frac{1}{2}} \cong 1V^5 \ M/S \quad \ldots\ldots\ldots \quad 2.17$$

Thus there will be a custant relationship between current and electric field accurate to about 1 part in 108. This is less true for semiconductors as they violate Ohm's law at high electric fields.

This important consideration can be emphasized in another way. Let us draw the graph (fig 2.1) of the distribution of particles in velocity space, i.e. with rectilinear ones representing velocities in three dimensions Vn, Vy, Vz.

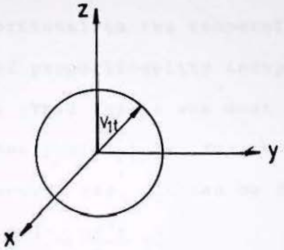


Fig. 2.1 Distribution of electrons in velocity space.

With no electric field present the distribution is spherically symmetric about the origin. The surface of of a sphere of radius V_{th} represents all electrons moving in all possible directions with that r.m.s. speed. When a field is applied along the X-axis (ray), the distribution is minutely perturbed (the electrons acquire some additional velocity in the direction of the X-axis) so that its centre shifts from (O, O, O) to about (V_{th}/IV^8,O, O,).

Taking copper, a field of 1V/M causes a current density of $10^8 A/M^2$. It is remarkable that a current density of this magnitude can be achieved with an almost negligible pertubation of the electron velocity distribution.

2.3 Wiedemann-Franz Law and the Lorente Number

The Wiedemann-Franz law states that for metals at not too low temperature the ratio of the thermal conductivity to the electrical condictivity to the electrical conductivity is directly proportional to the temperature, with the value of the constant of proportionality independent of the particular metal. This result was most important in the history of the theory of metals, for it supported the picture of an electron gas. It can be explained by using

$$\delta = \frac{n\ell^2\tau}{m}$$

and $$K = \frac{\Pi^2 n k^2 T\tau}{3m}$$

$$\Rightarrow \underline{K} = \frac{\Pi^2 K_B{}^2 T n\tau/3m}{n\ell^2\tau/m} = \frac{\Pi^2}{3}\left(\frac{KB}{\ell}\right)^2 T \quad \ldots\ldots \quad 2.18$$

Where the electron concentration is n, and τ is the collision time

The loventz number L is defined as

$$L \equiv \frac{K}{\delta T}$$

and according te 2.18

$$L = \frac{\Pi^2}{3}\left(\frac{K_B}{\ell}\right)^2 = 2.72\text{X}10^{-13}5u/deg^2 \quad \ldots\ldots \quad 2.19$$

$$= 2.45\text{X}10^{-8} \ \underline{\text{Watt-ohm}} \ deg^2$$

This remarkable result involves neither n nor m. It does not involve τ if the relaxation times are identical for electrical and thermal processes.

Experimental values of L at $0^{o}C$ and dt $100^{o}C$ as given in table 5 below are in good agreement with 3.19.

Table 2.1 Experimental Lorentz numbers:

L X 10^{8} Watt $^{-}$ Ohm/deg^{2}

Metal	$0^{o}C$	$100^{o}C$
Ag	2.31	2.37
Au	2.35	2.40
Cu	2.23	2.33
Pb	2.47	2.56
5n	2.52	2.49
Zn	2.31	2.33

On purely classical theory with a maxwellian distribution of velocities, the result is $L = 3\ (UB/_{e})^{2}$, very close to 2.19 and also in fair agreement with experiment.

At low temperatures, $T \ll \theta$ the Lorentz number tends to decrease; for pure copper near 15k the value is an order of magnitude smaller than 2.19. The reason is attributed to a difference in the collision averages involved in the thermal and electrical conductivities; the thermal and electrical relaxation times are not identical.

2.4 DIFFUSION

When there is a concentration gradient of impurity atoms or vacancies in a solid, there will be a flux of these through the solid. In equilibrium the vacancies will be distributed uniformly. The net flux j_N of atoms of one species in a solid is related to the gradient of the concentration N of this species by a phenomenological relation called fick's law:

$$j_N = -D \text{ grad } N \qquad ----- \qquad 3.21$$

Here j_N is the number of atoms crossing unit area in unit time; the constant D is the diffusion constant or diffusivity and has units c m^3/s. The minus sign means that diffusion occurs away from regions of high concentration. In order to diffuse, an atom must surmount the potential energy barrier presented by its neighbours.

Electric conduction in gases arises as a result of motion of the free electrons present in the gas. These electrons are liberated as a result of collisions between the few high-energy molecules in the system. Most gases at room temperature do not have many such high-energy molecules and, thus, have very few free electrons. If here are no external electric fields impressed on the as, the electrons will be distributed uniformly throughout he gas volume. In this instance there is no net electron ransport at a particular location in the gas, which of urse, is what we could expect. As an electric field is impressed on the gas, the charged particles are accelerated according to

$$Fe = q_e\ E = Mede = Me\ \frac{dNe}{dt}\quad ve,\ Ne = 0 \text{ at } t = 0$$

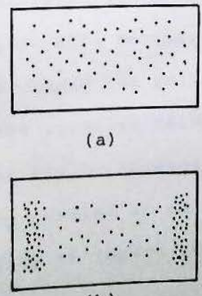
where qe is the charge on the electron and E is the electric field strength. Integrating the above

$$ve = \frac{dNe}{dt} = \frac{qe}{Me}\ Et \qquad ---- \qquad 2.20$$

The electron velocity ve is randomly oriented in the case of a zero electric field, so that there is no preferred direction of motion. The motion described by above equation then expresses the directed velocity the electron would attain t seconds after the electric field is turned on, assuming it had no collisions.

To take the effects of collisions into account, we assume that the electrons are brought to rest after each collision, reaccelerated by the electric field, and then experience another collision.

Consider the system shown in Fig. 2.4 a. The container is filled with a mixture of molecules having different masses and sizes.



(a)

(b)

Fig. 2.1 Thermal diffusion of unlike molecules.

(a) uniform distribution with no temperature gradients:

(b) concentration gradients resulting from temperature gradients.

As shown, the two types of molecules are distributed uniformly throughout the volume when the temperature is uniform. Now suppose a temperature gradient is imposed on the system, as shown in Fig. b. As a result of this temperature gradient, it is observed experimentally that the lighter molecules tend to concentrate in the higher temperature region and the heavier molecules tend to concentrate in the lower temperature region. This is known as thermal diffusion because the diffusion results from a temperature gradient instead of a concentration gradient.

Consider the typical collision shown below

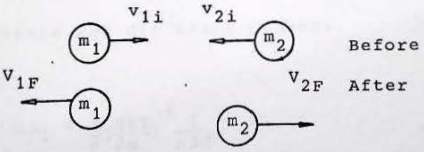


Fig 2.5 typical collision for analysis of thermal diffusion.

The rate of transport will be governed by the speeds at which the molecules leave the collision, which, in turn, are dependent on the relative velocity of the molecules before impact. For Fig 2.5 molecule 1 gives up momentum to molecule 2, thereby pushing it to the right. The higher the relative velocity at impact the faster molecule 2 is pushed to the right.

The foregoing collision process may be related to the thermal diffusion in the following way: At any one z location in the gas mixture all molecules have the same average kinetic energy (since T is uniform at any z location), but the lighter molecules have a higher average velocity than the heavier molecules. The greatest momentum exchange occurs in a collision between unlike molecules having greatest relative velocity. This situation occurs when a light molecule from a warm region collides with a heavy molecule from a cool region. After the collision the light molecule is pushed back toward the cool region. The basic thermal diffusion effect, is thus, a result of the fact that the unlike molecule collisions having the greatest momentum exchange will be those tending to force the light molecules into the hot region.

Hence the molecular size as well as molecular mass can influence the diffusion process.

Thus

$$D_{12} = \frac{3}{8}\left(\frac{\pi k T}{2m}\right)^{\frac{1}{2}} \frac{1}{n\pi d^2} \qquad \text{-----} \qquad 2.21$$

where

$$m = \frac{m_1 m_2}{m_1 + m_2}$$

$$d_{12} = \frac{d_1 + d_2}{2}$$

d_1 and d_2 being diameters of the two molecules which are assumed to behave as perfectly elastic spheres. The quantity D_{12} designates the diffusion coefficient for type one molecules diffusing through type two molecules.

2.5 SPECIFIC HEAT

Rapid advances in the frontiers of science and technology have brought about a general realization of the fact that the present limitations in many technical developments are a direct result of inadequate knowledge of the thermophysical properties of materials. In the high-temperature range ($T > 1000$ K), interest in the determination of specific heats of materials has been hastened because of the requirements in space programs as well as industrial applications.

The measurement of specific heat at cryogenic temperatures ($Cp \cong C_v$ for $T \leqslant 4$ k) provides us with a direct means to test theoretical models of a system. For instance, precise specific heat measurements were needed to test the validity of Debye's and Einstein's theory for specific heat of solids at low temperatures.

When a quantity of heat Q is added to a system so that there is a change in temperature, $T_2 - T_1$, then the mean heat capacity of the mass m of the substance is defined by

$$\bar{c} = \frac{Q}{T_2 - T_1} \qquad \text{------} \qquad 2.22$$

The limiting value of the above ratio as the temperature changes by dT is defined as the true heat capacity, i.e., where dQ is used instead of dQ to indicate that it is not exact differential.

In order to obtain a quantity that is independent of the mass, m, of a substance, equation 2.23 is divided by m i.e.

$$C = \frac{c}{m} = \frac{dQ}{mdT} \qquad \text{-----} \qquad 2.24$$

where q is the amount of heat per unit mass, so that equation 3.33 may also be written as

$$C = \frac{dq}{dT} \qquad \text{-----} \qquad 2.25$$

Raising the temperature of a unit mass of a substance by an amount dT, however, does not define the process in a thermodynamic sense; for instance, it will take a different amount of heat dq if the process is at constant pressure than when the process is at constant volume. As a matter of fact there are an infinite number of different processes for a system at temperature T to change to a temperature T + dT. It is clear, therefore, that an infinite number of specific heats could also be defined for a substance. The two processes that are most commonly used in thermodynamics are those at constant volume and constant pressure. For these two processes equation 2.24 may be written

$$C_p = \left(\frac{dq}{dT}\right)_p \qquad \text{-----} \qquad 2.25$$

and

$$C_v = \left(\frac{dq}{dT}\right)_v \qquad \text{-----} \qquad 2.26$$

Experimentally, the values of the specific heat measured are either at constant pressure Cp, or at constant volume C_v. The units most commonly used for specific heat are cal/g/k, joules/Kg/K.

CHAPTER THREE

3.0 RESULTS AND EVALUATION

3.1 DETERMINATION OF THE HEAT CAPACITY OF THE CALORIMETER

If different parts of a body differ in temperature, heat is conducted between them. In this experiment the temperature gradient is unidimentional along the rod. The amount of heat energy dQ flowing in time dt depends on the cross-sectional area A of the rod and the temperature gradient $\partial T/\partial x$ perpendicular to this area.

$$\frac{dQ}{dt} = -\lambda . \; A . \frac{\partial T}{\partial x} \qquad \text{-----} \qquad 3.1$$

where λ is the thermal conductivity of the material which we have copper and Aluminium in this case.

The temperature distribution in the body depends generally on position and time and varies the conduction equation

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho . c} \cdot \frac{\partial^2 T}{\partial x^2} , \qquad \text{-----} \qquad 3.2$$

where ρ is the density and c the specific heat capacity of each one of the metal rods.

If the two ends of the metal rod of length l are kept at constant temperatures T_1 and T_2 by two heat reservoirs (see fig. below)

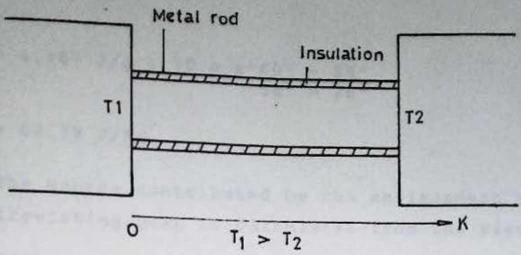


Fig3 1a Two ends of metal rod kept at constant temperature T_1 and T_2. A stationary state is reached where,

$$\frac{\partial T}{\partial t} = 0 \qquad ------ \qquad 3.3$$

after some time.

We obtain the following from (3.2) and (3.3)

$$T(x) = \frac{T_2 - T_1}{l} \cdot x + T_1 \qquad ------ \qquad 3.4$$

The heat capacity C of the calorimeter being used in first determined is the mixing experiment. Using the relation

$$C = C_W \cdot M_W \cdot \frac{\theta_W - \bar{\theta}}{\bar{\theta} - \theta_K} \qquad ------ \qquad 3.5$$

where

C_W = 4.187 J/g K, the specific heat capacity of water
θ_K = 26 °C, is the temperature of the calorimeter (room temperature).
θ
θ_W = 60 °C, is the temperature of the hot water
$\bar{\theta}$ = 54 °C, is the temperature of the mixture,

and

M_W = 70 g is the mass of the water

$$\Rightarrow C = 4.187\ \text{J/g K}\ 70\ \text{g} \times \frac{60^\circ - 54^\circ}{54^\circ - 26^\circ}$$

$$= 62.79\ \text{J/Kg.}$$

The energy contributed by the environment and by the circulating pump is calculated from the rise in temperature $\Delta\theta$ of the cold water in the calorimeter vessel (see figure 3.1)

i.e.

$$\Delta\theta \;=\; (C_w \,.\, M_w + C) \;-\; \Delta\theta$$

where $\Delta\theta = \theta - \theta_o$ and θ_o = temperature at time t = 0 hence, since $C_w = 4.187$ J/g K, $M_w = 70$ g

$$\Rightarrow \Delta\theta \;=\; (4.187\ \text{J/g K} \times 70\ \text{g} + 62.79\ \text{J/K}) \;.\; \Delta\theta$$

$$= \;355.88 \;.\; \Delta\theta$$

$\Delta\theta$ is calculated for each value of $\Delta\theta$ and the result is as tabulated as shown in table 3.1

TABLE 3.1 Quantity of heat (KJ) contributed by environment as a function of time

time (min)	temp rise (°C)	Δθ (OC)	Qenv (kJ)
0	2.0	0	0
2	2.5	0.5	0.18
4	3.5	1.5	0.53
6	4.5	2.5	0.88
8	5.0	3.0	1.07
10	5.9	3.9	1.39
12	6.5	4.5	1.60
14	7.0	5.0	1.78
16	7.5	5.5	1.96
18	8.0	6.0	2.14
20	8.9	6.9	2.46
22	9.3	7.3	2.60
24	10.0	8.0	2.85
26	10.5	8.5	3.02
28	11.2	9.2	3.27
30	11.9	9.9	3.52

Hence to find the thermal conductivities of each one of the metals, a constant temperature gradient was created in each of the metals using two heat reservoirs (boiling water and iced water). When the pieces of ice were removed, the rise in the temperature of the cold water was measured as a function of time and the heat conductivity of the metal rod calculated.

For copper (cu) (see table 3.2)

Length ΔL = 0.42 m

Diameter = 0.24 m

3.2 DETERMINATION OF THE THERMAL CONDUCTIVITY COPPER

TABLE 3.2 Temperature difference as a function of time

T_2 (°C)	T_1 (°C)	$\Delta T = T_2 - T_1$ (°C)	time (min)	Q (kJ)	temp rise (K)
10.40	1.11	9.29	0	0	-
11.10	1.47	9.63	0.50	0.16	0.50
12.20	1.22	10.98	1.00	0.25	0.80
12.40	1.81	10.59	1.50	0.36	1.12
12.70	1.88	10.82	2.00	0.58	1.83
13.00	2.50	10.50	2.50	0.64	2.01
13.40	2.82	10.58	3.00	0.95	3.00
13.70	3.40	10.30	3.50	1.08	3.41
13.90	3.36	10.54	4.00	1.23	3.88
14.30	4.1	10.20	4.50	1.41	4.43
14.60	4.6	10.00	5.00	1.56	4.91

From equation 3.2 above,

$$\frac{\lambda}{\rho . c} \frac{\partial^2 T}{\partial x^2} = 0 \quad \text{since} \quad \frac{\partial T}{\partial t} = 0 \text{ in equation (3.3)}$$

$$\Rightarrow \quad \frac{\partial^2 T}{\partial x^2} = 0 \quad \text{since } \lambda , \rho , \ c \text{ are constants}$$

$\Rightarrow \frac{\partial T}{\partial x} = c$ and $T = Cx + D$ where C and D = constants.

$$\Rightarrow T(x) = \frac{T_2 - T_1}{\phi} x + T_1$$

where $\frac{\partial T}{\partial x} = \frac{T_2 - T_1}{\phi}$

also $\frac{dQ_{rod}}{dt} = \frac{dQ_{tot}}{dt} - \frac{dQ_{env}}{dt}$ ------ 3.8

where $\frac{dQ_{rod}}{dt} = \frac{dQ}{dt}$ in equation (3.1)

Now,

$\frac{\partial T}{\partial x} = \frac{T_2 - T_1}{l}$ where l = length of the rod(cu) = 42×10^{-2} m

$\Rightarrow$ at 0.5 min, $\frac{\partial T}{\partial x} = \frac{9.63}{42 \times 10^{-2}\ \text{m}} = 22.92$ ^{o}C/M

at 1.0 min, $\frac{\partial T}{\partial x} = \frac{10.98}{42 \times 10^{-2}\ \text{m}} = 26.14$ "

at 1.5 min, $\frac{\partial T}{\partial x} = \frac{10.59}{42 \times 10^{-2}\ \text{m}} = 25.21$ "

at 2.0 min, $\frac{\partial T}{\partial x} = \frac{10.82}{42 \times 10^{-2}\ \text{m}} = 25.76$ "

at 2.5 min, $\frac{\partial T}{\partial x} = \frac{10.50}{42 \times 10^{-2}\ \text{m}} = 25.00$ "

at 3.0 min, $\frac{\partial T}{\partial x} = \frac{10.58}{42 \times 10^{-2}\ \text{m}} = 25.19$ "

at 3.5 min, $\frac{\partial T}{\partial x} = \frac{10.54}{42 \times 10^{-2}\ \text{m}} = 24.52$ "

at 4.0 min, $\frac{\partial T}{\partial x} = \frac{10.20}{42 \times 10^{-2}\ \text{m}} = 24.29$ "

at 4.5 min, $\frac{\partial T}{\partial x} = \frac{10.20}{42x10^{-2}\ m} = 24.29\ {}^{o}C/M$

at 5.0 min, $\frac{\partial T}{\partial x} = \frac{10.00}{42x10^{-2}\ m} = 23.80\ {}^{o}C/M$

Also,

at 0.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.16}{0.5x60\ sec} = 2.67x10^{-3}$, and $\frac{\partial Q_{env}}{\partial t}$

$= \frac{0.18}{2x60} = 1.5\ x\ 10^{-3}$ KJ/sec

at 1.0 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.09}{0.5x60sec} = 1.5x10^{-3}$, $\frac{\partial Q_{env}}{\partial t} = \frac{0.35}{2x60} = 2.92\ x\ 10^{-3}$

at 1.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.11}{0.5x60sec} = 3.67x10^{-3}$, $\frac{\partial Q_{env}}{\partial t} = \frac{0.35}{2x60} = 2.92\ x\ 10^{-3}$

at 2.0 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.22}{0.5x60sec} = 7.34x10^{-3}$, $\frac{\partial Q_{env}}{\partial t} = \frac{0.19}{2x60} = 1.59\ x\ 10^{-3}$

at 2.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.06}{0.5x60sec} = 2.0x10^{-3}$, $\frac{\partial Q_{env}}{\partial t} = \frac{0.32}{2x60} = 2.67\ x\ 10^{-3}$

at 3.0 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.31}{0.5x60sec} = 1.03x10^{-2}$, $\frac{\partial Q_{env}}{\partial t} = \frac{0.21}{2x60} = 1.75\ x\ 10^{-3}$

at 3.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.13}{0.5x60sec} = 4.34x10^{-3}$, $\frac{\partial Q_{env}}{\partial t} = \frac{0.18}{2x60} = 1.5\ x\ 10^{-3}$

at 4.0 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.15}{0.5x60sec} = 5.0x10^{-3}$, $\frac{\partial Q_{env}}{\partial t} = \frac{0.18}{2x60} = 1.5\ x\ 10^{-3}$

at 4.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.18}{0.5x60sec} = 6.0x10^{-3}$, $\frac{\partial Q_{env}}{\partial t} = \frac{0.18}{2x60} = 1.5\ x\ 10^{-3}$

at 5.0 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.15}{0.5x60sec} = 5.0x10^{-3}$, $\frac{\partial Q_{env}}{\partial t} = \frac{0.32}{2x60} = 2.67\ x\ 10^{-3}$

hence using equation 3.8 i.e. $\frac{dQ_{rod}}{dt} = \frac{dQ_{tot}}{dt} - \frac{dQ_{env}}{dt}$

we obtain thefollowing results for change in quantity of heat of the copper rod with time, (in KJ/sec)

at 0.5 min, $\frac{dQ_{rod}}{dt} = 2.67 \times 10^{-3} - 1.5 \times 10^{-3} = 1.17 \times 10^{-3}$

at 1.0 min, $\frac{dQ_{rod}}{dt} = 1.5 \times 10^{-3} - 2.92 \times 10^{-3} = -1.42 \times 10^{-3}$

at 1.5 min, $\frac{dQ_{rod}}{dt} = 3.67 \times 10^{-3} - 2.92 \times 10^{-3} = 75.0 \times 10^{-3}$

at 2.0 min, $\frac{dQ_{rod}}{dt} = 7.34 \times 10^{-3} - 1.59 \times 10^{-3} = 5.75 \times 10^{-3}$

at 2.5 min, $\frac{dQ_{rod}}{dt} = 2.0 \times 10^{-3} - 2.67 \times 10^{-3} = 67.0 \times 10^{-3}$

at 3.0 min, $\frac{dQ_{rod}}{dt} = 1.03 \times 10^{-2} - 1.75 \times 10^{-3} = 8.45 \times 10^{-3}$

at 3.5 min, $\frac{dQ_{rod}}{dt} = 4.34 \times 10^{-3} - 1.5 \times 10^{-3} = 2.84 \times 10^{-3}$

at 4.0 min, $\frac{dQ_{rod}}{dt} = 5.0 \times 10^{-3} - 1.5 \times 10^{-3} = 3.5 \times 10^{-3}$

at 4.5 min, $\frac{dQ_{rod}}{dt} = 6.0 \times 10^{-3} - 1.5 \times 10^{-3} = 4.5 \times 10^{-3}$

at 5.0 min, $\frac{dQ_{rod}}{dt} = 5.0 \times 10^{-3} - 2.67 \times 10^{-3} = 2.33 \times 10^{-3}$

The values for $\frac{\partial T}{\partial x}$ and $\frac{dQ_{rod}}{dt}$ are then tabulated as shown below and the graph of $\frac{dQ_{rod}}{dt}$ versus $\frac{\partial T}{\partial x}$ plotted where the slope is $\frac{dQ}{dt} / \frac{\partial T}{\partial x}$ as in equation (3.1)

Hence the value of λ (thermal conductivity) is determined from equation 3.1

TABLE 3.3 Rate of heat energy versus temperature gradient

$\partial T/\partial x$ ($^\circ C/M$)	$\frac{dQ_{rod}}{\partial t}$ ($\frac{KJ}{sec}$)
	$(\times 10^{-3})$
22.92	1.17×10^{-3}
26.14	-1.42×10^{-3}
25.21	7.5×10^{-3}
25.76	5.75×10^{-3}
25.00	-6.7×10^{-3}
25.19	8.45×10^{-3}
24.52	2.84×10^{-3}
25.10	3.5×10^{-3}
24.29	4.5×10^{-3}
23.80	2.33×10^{-3}

Now,

$$\frac{dQ_{rod}}{dt} = -\lambda . A . \frac{\partial T}{\partial x}$$

$$\frac{dQ_{rod}}{dt} / \frac{\partial T}{\partial x} = -\lambda . A \qquad ---- \qquad 3.1a$$

where $\frac{dQ_{rod}}{dt} / \frac{\partial T}{\partial x}$ = slope and A = cross-sectional area of the copper rod.

from fig. 3.2

$$\frac{dQ_{rod}}{dt} = 3.6 \times 10^{-3} \text{ kJ/sec}, \quad \frac{\partial T}{\partial x} = 26 \ {}^\circ\text{C/m}$$

$$\Rightarrow \quad \frac{dQ_{rod}}{dt} / \frac{\partial T}{\partial x} = \frac{3.6 \times 10^{-3} \text{ kJ sec}^{-}}{26 \ {}^\circ\text{C m}^{-1}} = 1.38 \times 10^{-4} \text{KJ M/Sec}^\circ\text{C}$$

Hence from 3.1a

$$\frac{dQ_{rod}}{dt} / \frac{\partial T}{\partial x} = - \lambda . A$$

$$\Rightarrow 1.38 \times 10^{-4} = - \lambda . A , \; - \lambda = \frac{1.3.8 \times 10^{-4}}{A} \text{ kJ sec}^{-1}{}^{o}C^{-1} \text{ m}$$

$$\text{but } A = \Pi r^2 = \Pi \times 1.2 \times 10^{-2} \text{ m} = 3.77 \times 10^{-2} M^2$$

Therefore

$$(-\lambda)^2_{cm} = \frac{1.38 \times 10^{-4} \text{ kJ}^{-1}}{3.77 \times 10^{-6} \text{ m}} \text{sec}^{-1} \; \lambda = \frac{1.38 \times 10^{-4}}{3.77 \times 10^{-6}} = 366.00 \text{ w/m° c}$$

$$= 366 \text{ W/m-°c}$$

3.3 DETERMINATION FOR THE ELECTRICAL CONDUCTIVITY OF COPPER

The experiment for the electrical conductivity of copper was conducted as described in section 1-3-2 c and the result as stated below:

TABLE 3.4 Voltage versus current through the conductor.

Volt	Ampere
0.83	0.12
0.84	0.30
0.85	0.40
0.86	0.50
0.88	0.60
0.89	0.70
0.90	0.80
0.91	0.90

At room temperature the conduction electrons in the metal have a much longer average path than the positive nuclei, and so it is chiefly electrons that conduct heat in metals.

The electrical conductivity σ is determined from the resistance of the rod R and it's dimensions (i.e. $l = 0.42$ no and A = area = $1.58\ m^2$

Now,

$$\sigma = \frac{1}{A.R} \qquad \text{-----} \qquad (4.9)$$

but,

$$v = IR \Rightarrow R = \frac{V}{I}$$

where v = voltage and I = current

$$\Rightarrow R = \frac{\Delta V}{\Delta I}$$ from the graph of V versus I (i.e. fig 3.6) for copper where

$$\frac{\Delta V}{\Delta I} = \text{Slope of the graph} = R$$

hence from (figure 4.6, $\Delta V = 0.88$ v, $\Delta I = 0.78$ A

$$\Rightarrow \quad R = \frac{0.08}{0.78} = 0.10\ \Omega$$

Therefore,

$$\sigma_{CU} = \frac{0.42\ m}{1.58m \times 0.10m} = \frac{0.42}{0.16}$$

$$= 2.63\ (\Omega m)^{-1}$$

3.4 LORENTZ NUMBER AS DETERMINED FROM THE THERMAL AND ELECTRICAL CONDUCTIVITY OF COPPER:

The resulting relationship between the thermal conductivity λ and the electrical conductivity σ is expressed by the wiedemann-franz law:

$$\frac{\lambda}{\sigma} = <L.\ T \qquad ----- \qquad 3.10$$

The Lorentz number L, which is determined experimentally according to equation 4.10, is obtained from the theory of the electron gas (for temperatures above the Debye temperature) and is represented by

$$L = \frac{\Pi^2}{3} \cdot \frac{K^2}{e^2}$$

where K = Boltzmann's constant =

e = electron change = 1.6×10^{-19} C

Π = constant = 3.14

$$=> \quad L = \frac{\Pi^2}{3} \frac{K}{(1.6 \times 10^{-19}\ c)^2}$$

$$= 2.4 \times 10^{-8}\ w\Omega\ /K^2$$

From experiment, using equation (3.10) where

λ_m = 366.00 w/m °C

σ = 2.63 $(\Omega m)^{-1}$

T = constant = 335 K

$$=> \quad L_{cu} = \frac{\lambda}{\sigma}\frac{1}{T} = \frac{366\ w/m\ °C}{\frac{2.63(\Omega m)^{-1}}{1000}} \times \frac{1}{300\ K}$$

$$= \frac{366}{242355.75}$$

$$= 1.52 \times 10^{-3}\ w\Omega/k^2$$

3.5 RESULTS AND EVALUATIONS FOR ALUMINIUM

3.5 DETERMINATION OF THE THERMAL CONDUCTIVITY OF ALUMINIUM

The table below shows the experimental result for thermal determination of thermal conductivity.

TABLE 3.5 Temperature difference as a function of time for Aluminium

T_2 °C	T_1 °C	$\Delta T (T_2 - T_1)$ °C	time (x60) sec	Q kJ	temp rise °C
23.90	8.10	15.80	0	0	-
24.30	8.57	15.73	0.50	0.25	0.48
24.60	8.97	15.63	1.00	0.33	0.63
24.80	9.40	15.40	1.50	0.50	0.94
25.00	9.68	15.32	2.00	0.53	1.02
25.35	10.00	15.35	2.50	0.77	1.48
25.65	10.40	15.30	3.00	0.90	1.72
27.70	10.50	15.25	3.50	1.45	2.78
25.80	10.60	15.20	4.00	1.62	3.10
26.10	11.00	15.10	4.50	2.01	3.85

Now, we again find $\frac{\partial T}{\partial x} = \frac{\Delta T}{l}$ just as in the case of copper where $l = 42 \times 10^{-2}$ m = 0.42 m at 0.5 min, $\frac{\partial T}{\partial x} = \frac{15.73}{0.42\mathrm{m}} = 37.45$ ^{o}C/M

at 1.0 min, $\frac{\partial T}{\partial x} = \frac{15.63}{0.42\mathrm{m}} = 37.21$ "

at 1.5 min, $\frac{\partial T}{\partial x} = \frac{15.40}{0.42\mathrm{m}} = 36.67$ "

at 2.0 min, $\frac{\partial T}{\partial x} = \frac{15.32}{0.42\mathrm{m}} = 36.48$ "

at 2.5 min, $\frac{\partial T}{\partial x} = \frac{15.35}{0.42\mathrm{m}} = 36.55$ "

at 3.0 min, $\frac{\partial T}{\partial x} = \frac{15.30}{0.42\mathrm{m}} = 36.43$ "

at 3.5 min, $\frac{\partial T}{\partial x} = \frac{15.25}{0.42\mathrm{m}} = 36.31$ "

at 4.0 min, $\frac{\partial T}{\partial x} = \frac{15.20}{0.42\mathrm{m}} = 36.19$ "

at 4.5 min, $\frac{\partial T}{\partial x} = \frac{15.10}{0.42\mathrm{m}} = 35.95$ "

Hence,

as in the case of copper (using equation 3.8)

at 0.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.25}{30\mathrm{secs}} = 8.34\mathrm{x}10^{-3}$ and $\frac{\partial Q_{env}}{\partial t} = 1.5\mathrm{x}10^{-3}$

at 1.0 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.08}{30\mathrm{secs}} = 2.67\mathrm{x}10^{-3}$ and $\frac{\partial Q_{env}}{\partial t} = 2.92\mathrm{x}10^{-3}$

at 1.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.17}{30\mathrm{secs}} = 5.67\mathrm{x}10^{-3}$ and $\frac{\partial Q_{env}}{\partial t} = 2.92\mathrm{x}10^{-3}$

at 2.0 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.03}{30\mathrm{secs}} = 1.00\mathrm{x}10^{-3}$ and $\frac{\partial Q_{env}}{\partial t} = 1.59\mathrm{x}10^{-3}$

at 2.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.24}{30\mathrm{secs}} = 8.00\mathrm{x}10^{-3}$ and $\frac{\partial Q_{env}}{\partial t} = 2.67\mathrm{x}10^{-3}$

at 3.0 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.13}{30\,\text{secs}} = 4.34\text{x}10^{-3}$ and $\frac{\partial Q_{env}}{\partial t} = 1.75\text{x}10^{-3}$

at 3.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.55}{30\,\text{secs}} = 0.84\text{x}10^{-3}$ and $\frac{\partial Q_{env}}{\partial t} = 1.5\text{x}10^{-3}$

at 4.0 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.17}{30\,\text{secs}} = 5.67\text{x}10^{-3}$ and $\frac{\partial Q_{env}}{\partial t} = 1.5\text{x}10^{-3}$

at 4.5 min, $\frac{\partial Q_{tot}}{\partial t} = \frac{0.39}{30\,\text{secs}} = 1.3\text{x}10^{-2}$ and $\frac{\partial Q_{env}}{\partial t} = 1.5\text{x}10^{-3}$

Therefore, using equation 3.8 as before:

at 0.5 min, $\frac{dQ_{rod}}{dt} = (8.34 - 1.5) \text{ x } 10^{-3} = 6.84 \text{ x } 10^{-3}$ KJ/sec

at 1.0 min, $\frac{dQ_{rod}}{dt} = (2.67 \text{ x } 2.92) \text{ x } 10^{-3} = -0.25 \text{ x } 10^{-3}$ "

at 1.5 min, $\frac{dQ_{rod}}{dt} = (5.67 - 2.92) \text{ x } 10^{-3} = 2.75 \text{ x } 10^{-3}$ "

at 2.0 min, $\frac{dQ_{rod}}{dt} = (1.00 - 1.59) \text{ x } 10^{-3} = -0.59 \text{ x } 10^{-3}$ "

at 2.5 min, $\frac{dQ_{rod}}{dt} = (8.00 - 2.67) \text{ x } 10^{-3} = 5.33 \text{ x } 10^{-3}$ "

at 3.0 min, $\frac{dQ_{rod}}{dt} = (4.34 - 1.75) \text{ x } 10^{-3} = 2.59 \text{ x } 10^{-3}$ "

at 3.5 min, $\frac{dQ_{rod}}{dt} = (1.84 - 1.5) \text{ x } 10^{-3} = 0.34 \text{ x } 10^{-3}$ "

at 4.0 min, $\frac{dQ_{rod}}{dt} = (5.67 - 1.5) \text{ x } 10^{-3} = 4.17 \text{ x } 10^{-3}$ "

at 4.5 min, $\underline{dQ_{rod}} = (1.3 \text{ x } 10^{-2} - 1.5) \text{x} 10^{-3} = 11.5 \text{ x } 10^{-3}$ "

TABLE 3.6 TheTable below shows results of $\frac{dQ_{rod}}{dt}$ and $\partial T/\partial x$

$\frac{dQ_{rod}}{dt}$ (x10$^{KJ/}$)sec	$\frac{\partial T}{\partial x}$ $\frac{^{o}C}{M}$
6.84	37.45
-0.25	37.21
2.75	36.67
-0.59	36.48
5.33	36.55
2.59	36.43
0.34	36.31
4.17	36.19
11.5	35.95

The graph of $\frac{dQ_{rod}}{dt}$ versus $\frac{\partial T}{\partial x}$ is plotted as shown in figure 4.9 and $\frac{DdQ_{rod}}{dt} / \frac{\partial T}{\partial x}$ is called the slope and this is then substituted in equation 3.1 to find λ_{Al}

Now,

$$\frac{dQ_{rod}}{dt} = - \lambda \ . \ A \ . \ \frac{\partial T}{\partial x}$$

$$\frac{dQ_{rod}}{dt} / \frac{\partial T}{\partial x} = \lambda \ . \ A$$

where $\frac{dQ_{rod}}{dt} / \frac{\partial T}{\partial x}$ = slope and A = area of cross-sectional of the Aluminium rod.

from fig. 3.9

$$\frac{dQ_{rod}}{dt} = 4.2 \text{ kJ/sec}, \quad \frac{\partial T}{\partial x} = 38 \text{ °C/m}$$

$$\text{Slope} = \frac{dQ_{rod}}{dt} \bigg/ \frac{\partial T}{\partial x} = \frac{4.2 \text{ kJ/s}}{38 \text{ °C/m}} = 0.110 \text{ kJsec}^{-1} \text{ °C m}^{-1}$$

$$\Rightarrow ?\text{Slope} = -\lambda \cdot A \Rightarrow 0.110 = -\lambda \times A$$

$$\text{where } A = \Pi r^2 = (\Pi \times 1.2 \times 10^{-2} \text{ m})^2 = 3.77 \times 10^{-2} \text{ m}$$

$$\Rightarrow \lambda_{Al} = \frac{0.11 \text{ kJ sec °C}^{-1}}{3.77 \times 10^{-2} \text{ m}} = 170.8 \text{ W/m - °C}$$

3.6 ELECTRICAL CONDUCTIVITY OF ALUMINIUM

The experiment for the electrical conductivity of copper was conducted according to section 1.32 and the result as stated below:

TABLE 3.7 Voltage versus current through the conductor

Voltage (volts)	Current (Amp)
0.85	0.12
0.87	0.20
0.89	0.30
0.92	0.40
0.95	0.52
0.97	0.60
0.99	0.70

The electrical conductivity σ_{Al} is determined as in the case of copper from the resistance of the rod and it's dimensions.

Now, from equation 3.9

$$\sigma = \frac{1}{A.R}$$

where $A = 1.58\ m^2$, $1 = 0.42\ m$

and R from fig. 3.10 i.e. graph of ΔV against ΔL

$$R = \frac{\Delta V}{\Delta I} = \text{slope of the graph}$$

$$=> \quad R = \frac{0.99}{0.70} = 1.28\ \Omega$$

Therefore,

$$\sigma = \frac{0.42\ m}{1.58 m^4\ x 1.28} = = \frac{0.42}{2.02}$$

$$\sigma_{AL} = 0.21\ (\Omega m)^{-1}\ .$$

3.7 THE LORENTZ NUMBER AS DETERMINED FROM THE THERMAL AND ELECTRICAL CONDUCTIVITY OF ALUMINIUM

From the experiment using Aluminium, and applying equation(3.10) where

$\lambda_{AL} = 170.8\ W/m\ °C$

$\sigma_{AL} = 0.21\ (\Omega m)^{-1}$

$T = \text{constant} = 419\ K$

$$=> L_{AL} = \frac{\lambda_{AL}}{\sigma_{AL}} \frac{1}{T} = \frac{170.8\ W/m\ °C}{0.21 (\Omega m)^{-}} \times \frac{1}{419\ K}$$

$$= \frac{170.8}{24021.27}$$

$$= L_{AL} = 7.1 \times 10^{-3}\ W\ \Omega\ /\ K^2$$

3.8 Summary of results as arrived at from the various experiments carried out.

Table 3.8: Summary of experimental results

	λ W/M^{-o}C	R (Ω)	σ (ΩM)$^{-1}$	L (WΩ/K^2)
CU	366.00	0.10	2.63	1.52×10^{-3}
AL	170.80	1.28	0.21	7.10×10^{-3}

The table below shows standard results

Table 3.9: Table of standard values as recorded elsewhere.

	λ W/M^{-o}C	R. (Ω)	σ. (ΩM)$^{-1}$	L (WΩ/K^2)
CU	386.00	1.0	5.8	2.2×10^{-8}
AL	202.00	2.18	3.0	2.1×10^{-8}

CHAPTER FOUR

4.1 CONCLUSION AND OUTLOOK

Although disprepancies could still be found between our results and more standard values, table 3.8 shows that the results were to an appreciable extent comparable to results found elsewhere (Table 3.9), in which the experiment was carried out under even more favourable conditions.

The results also show that thermal conductivity is proportional to electrical conductivity for each metal, since electrons are carriers both of heat and of electricity in metals. And the ratio of the thermal to electrical conductivity known as the Lorentz number is found to be directly proportional to the temperature, and the Lorentz number is independent of the particular metal hence there is very small difference between the Lorentz numbers for copper and Aluminium as seen in table 3.8 for our case and also table 2.1 for standard values.

Tables 3.2 and 3.5 also depict the general trend in which the quantity of heat flowing through the insulated metal rods increase with increasing temperature, hence the rate of heat flow is proportional to the temperature difference.

The scattering of electrons intensifies with rising temperature, owing to the increase of thermal non-uniformity and in pure metals this leads to a decline in thermal and electrical conductivity.

It can be seen that most results didn't turn out as expected but still depicted general patterns as seen in standard measurements. Hence the choice of method as it affects the make of material is always very important in experimental work like this. The nature of equipment has to be checked also.

Methods choosen should always agree with the nature of environment. The method used here was choosen so as to suit the test material (insulated metal rods). Insulated metal rods were used in order to minimise heat contribution by environment or heat loss by the rod.

The points to be noted in the event of furthering this kind of research work are:

1. To find the most appropriate method for a given test material.
2. To minimise errors due to choice of material
3. To see if it is possible to find the best way of determining the conductivity of metal rods of this nature and for this environment.
4. To check whether there are other less tedious and more precise ways of calculating the various values in the course of finding these conductivities.

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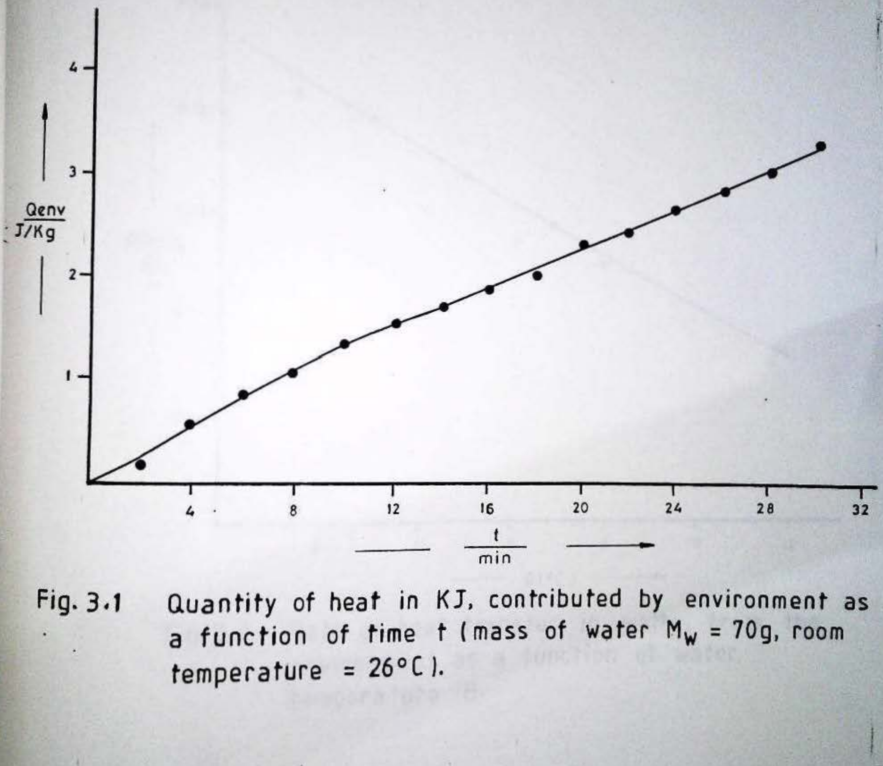

Fig. 3.1 Quantity of heat in KJ, contributed by environment as a function of time t (mass of water M_w = 70g, room temperature = 26°C).

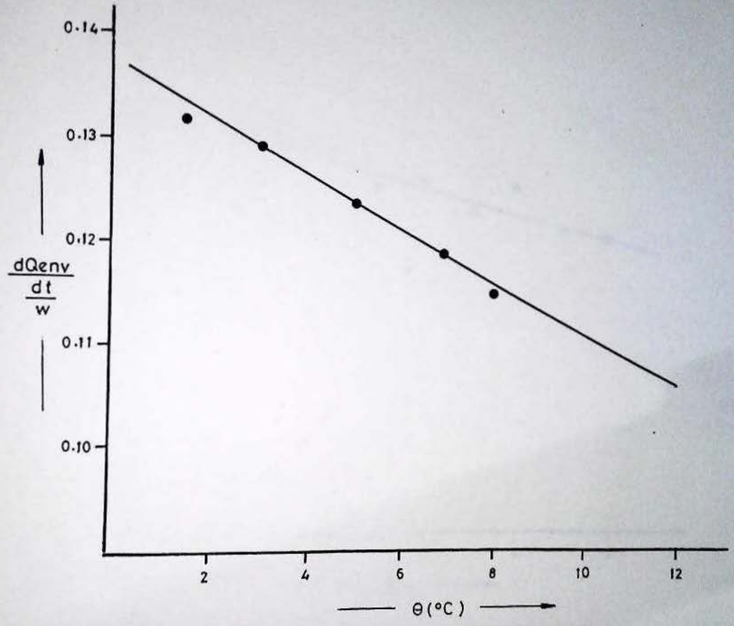

Fig. 3.2 Rate of heat transfer, in watts, from the environment as a function of water temperature θ.

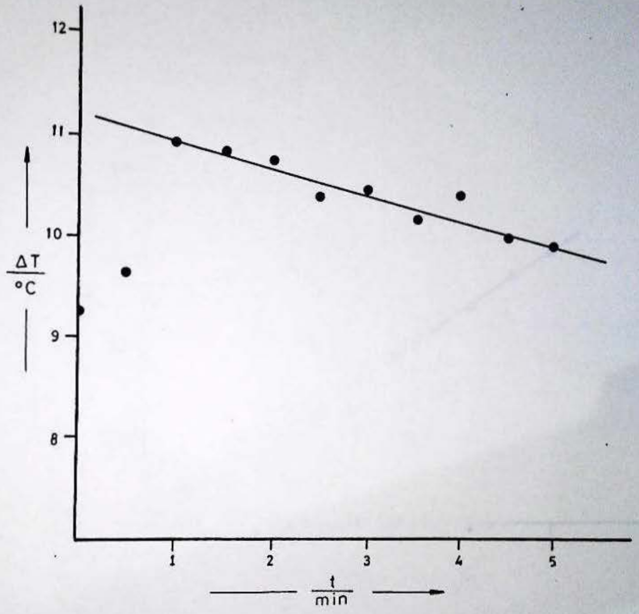

Fig. 3.3 Calorimetrically determined temprature difference ΔT (°C) as a function of time for Copper.

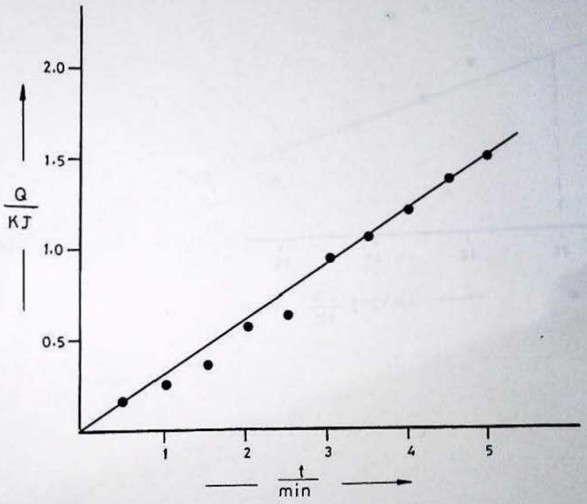

Fig. 3.4 Calorimetrically determined quantity of heat Q along the rod against time for Copper.

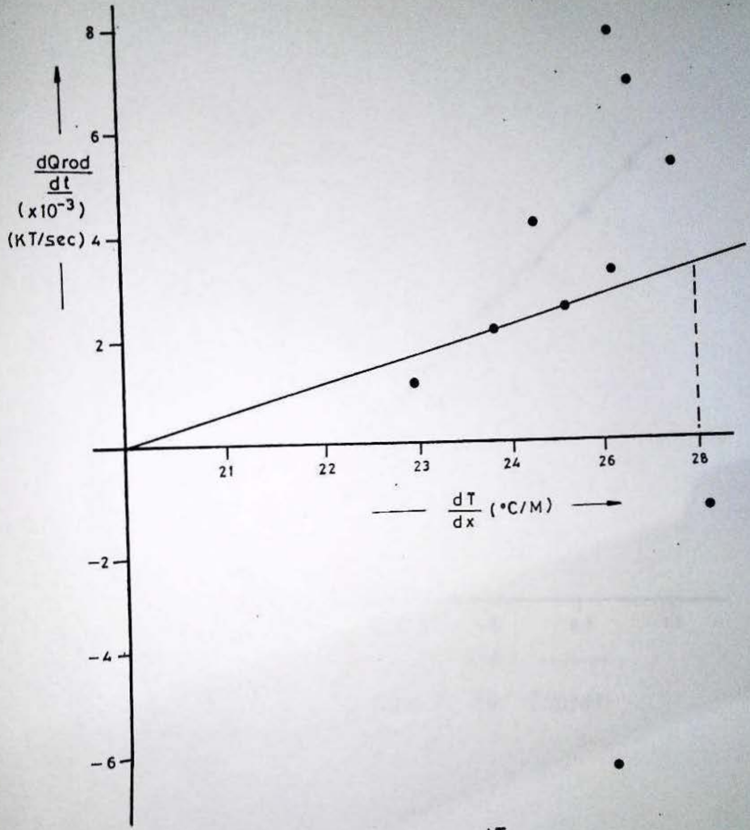

Fig. 3.5 Graph of $\frac{dQ}{dt}$ rod Vs $\frac{dT}{dx}$ for copper.

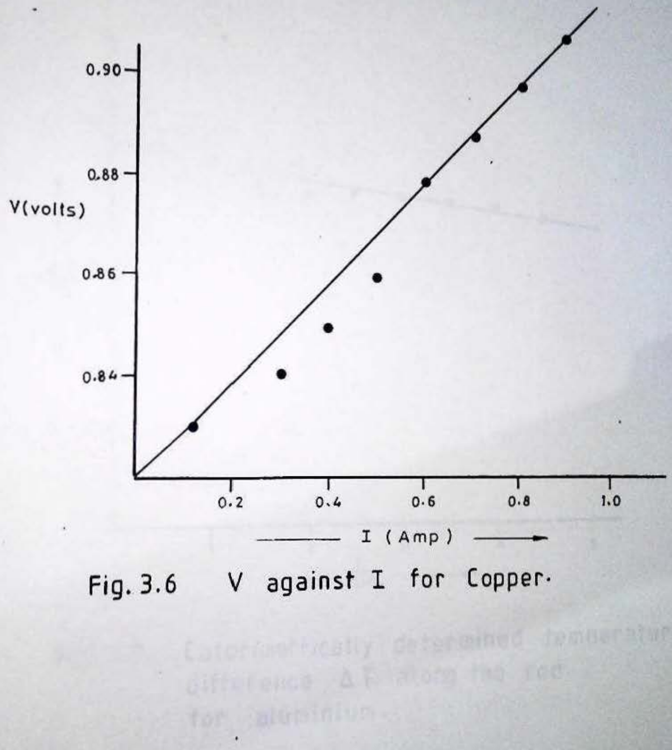

Fig. 3.6 V against I for Copper.

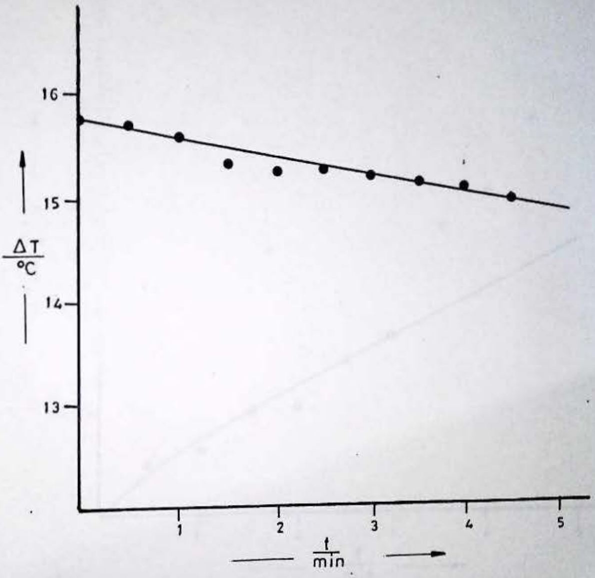

Fig. 3.7 Calorimetrically determined temperature difference ΔT along the rod for aluminium.

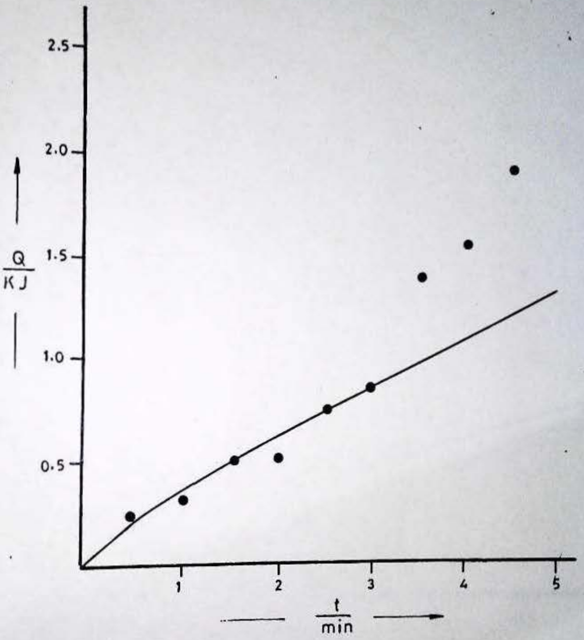

Fig. 3.8 Calorimetrically determined quantity of heat as a function (Q/KJ) as a function of time ($\frac{t}{min}$) for AL.

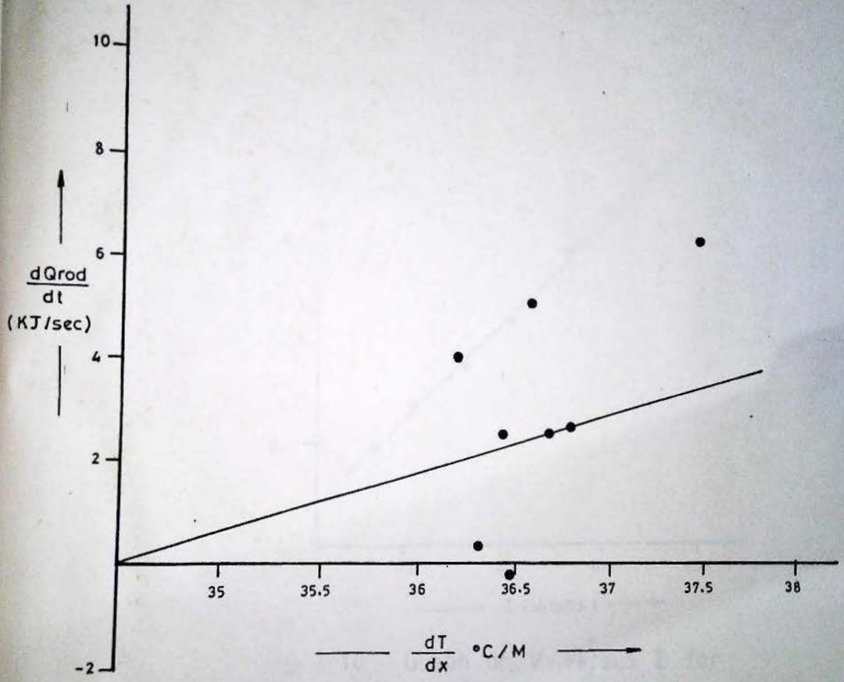

Fig. 3.9 Graph of $\frac{d\,Qrod}{dt}$ Vs $\frac{dT}{dx}$ you aluminium.

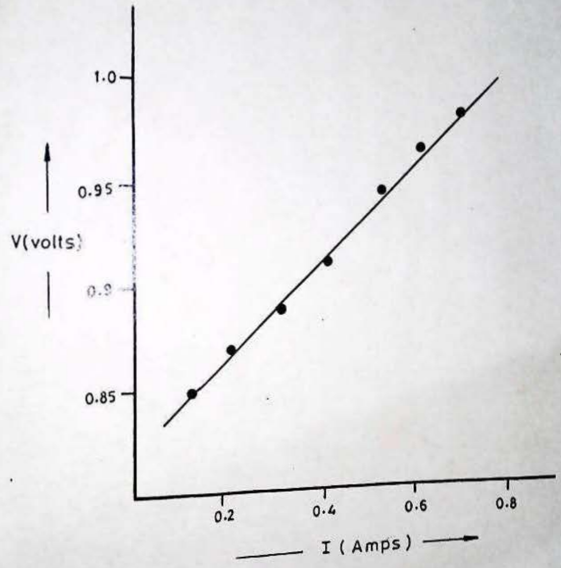

Fig. 3.10 Graph of V versus I for aluminium.