APPLICATION OF 2-D ELECTRICAL RESISTIVITY TO GROUND WATER EXPLORATION IN ATBU, YELWA CAMPUS, BAUCHI

BY

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JANUARY; 2018

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A PROJECT SUBMITTED TO PHYSICS DEPARTMENT ABUBAKAR TAFAWA BALEWA UNIVERSITY BAUCHI IN PARTIAL FULFILMENT FOR THE AWARD OF DEGREE OF BACHELOR OF TECHNOLOGY (HONS) IN APPLIED GEOPHYSICS

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JANUARY, 2018

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DECLARATION

I Muazu Suleiman Babadoko hereby declare that this thesis was written by me and it is record of my own research work. It has not been presented before in any previous application for the award of the Degree references made to published literature have duly been acknowledged.

Muazu Suleiman Babadoko

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Signature

Date

CERTIFICATION

This project work has been read, corrected, and approved as meeting to requirement of physics department Abubakar Tafawa Balewa University, Bauchi for the award of B. Tech (Hons) degree in Applied Geophysics.

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DEDICATION

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This project is dedicated to my lovely parents, Alh. Muazu Ibrahim, and other members of the family for their immense support spiritually, mentally and financially throughout the duration of my study.

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My sincere appreciation goes to my project supervisor, Mal. A.D Shehu for the challenges and encouragement he gave to me to ensure that I produce a quality work. I will live to remember for always been there for me for inculcating into me to ensure that this project is done well.

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ABSTRACT

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Three parallel resistivity profiles each of 150 m length were run using the Dipole-Dipole configuration for acquisition of data, while Res2dinV was used for processing the data, and necessary interpretations were carried out. The results of quantitative interpretation of the data show that the study area is dominated by low resistivity values indicating water bearing zone on two profiles. It also shows that all the three profiles indicate three to four layers. The first layer is the top soil having a resistivity value ranging from 40 Ω m to 80 Ω m with Depth value ranging from 0 m to 4.5 m. The second layer is weathered crystalline rock of resistivity value ranging from 90 Ω m to 280 Ω m with Depth value ranging from 5.8 m to 9.5 m. The fourth layer is the fresh basement having resistivity value ranging from 1000 Ω m to 3000 Ω m with Depth value ranging from 12.4 m to ∞ m. The third layer has an indication that this-point is an aquiferous zone, a potential position for sitting borehole which is underlying by partly weathered fractured rock.

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CHAPTER ONE

1.0 Introduction

1.1 Background of the Study

The role of geophysical methods in ground water exploration is vital. The aim is to understand the hidden subsurface hydrogeological conditions adequately and accurately. The basis of any geophysical method is measuring a contrast or anomaly the better the geophysical response and hence the identification. So the efficiency of any geophysical technique lies in its ability to detect and resolve the hidden surface hydrogeological heterogeneities' or variation.

Electrical imaging (2 dimensional) provides detailed view of subsurface structure, the present study is designed to produce appropriate images of the basement with the aim of investigating and delineating appropriate location for underground water exploitation.

A large number of geological studies have been carried out over the past 40 years in the basement. In one such work, Du Preez (1956) reported the presence of welldeveloped, steep-dipping joints in some granitic outcrops in the basement.

Studies have shown that groundwater could be explored using electrical resistivity method (Olufemi and Fasoyi 1993; Olasehinde, 1999; Alile et al, 2008) therefore, the use of such techniques for a ground water exploration has earn an important

place in recent years despite some interpretive limitation (Dogara et al, 1998; Singh et al; 2006). It is therefore expected that the result obtained from this study would indicate detailed groundwater condition.

Ground water accounts for about 90% of the world's fresh water and it is fairly well distributed throughout the world. The exploration and exploitation of groundwater as a major resource to meet the growing population in some urban cities of the basement complex rocks of Nigeria has been a subject of discussion. These works involved a combination of hydrogeological and geo-electrical parameters to delineate aquifer characteristics in the crystalline basement rocks in the Yelwa campus of Abubakar Tafawa Balewa University of Bauchi.

1.1.1 Advantage of groundwater over other sources of water

- i. It is naturally pure and is a reliable source of water
- ii. It is usually hard, which contain in it calcium ions (and some other Di and Trivalent ions which causes hardness of water) hence, groundwater provides us a small amount of beneficial calcium.
- iii. It requires nearly no treatment for usage while surface water requires many processes like sedimentation, filtration, disinfection etc.
- iv. It is free from pathogenic bacteria which causes diseases,
- v. Not easily subjected to contamination externally.



Fig. 1: Map of Bauchi State.

1.2 Relief and Drainage

The study area is situated in a crystalline basement complex rock, generally characterized by low relief with some rock outcrops. The drainage system follows a dendritic pattern.

1.3 Location and accessibility

The study area (International Secondary School Football field, behind Girl's Hostel and ATBU Main football field) lies between longitude $9^{0}46'$ and $9^{0}47'$ East and latitude $10^{0}17'$ North.

(Source: Geology survey of Nigeria surveys sheet Number 149). The area under survey lies by the dual carriage way leading to Dass from Bauchi town.

1.4 Climate and Vegetation

This type of climate consists of two distinct seasons in a year, i.e. wet and dry seasons. The rainy season mostly starts in May and ends in early October. Also, during this season, humidity is generally low and temperatures are high from 13° C to $32 \,^{\circ}$ C (Burnette, 1965).

Dry season usually starts from November to April. This is a long period of dry weather with high temperatures during the day and low temperatures at night. Average temperature during this season is about 24 C (about 70 F) (Burnefte, 1965).

There is a severe harmattan period from December to February. This is the period when the North-East trade winds begin to blow South wards into the country from the Sahara belt. At this period it is generally cooler than normal and less humidity and visibility at certain times is restricted as a result of airborne dust (water survey, 1986). Vegetation in this area is of the savannah type with sporadic thorny bushes, scattered shrubs, and isolated trees. (B.S.A.D.P, 1983).

1.5 AIMS AND OBJECTIVES OF THE STUDY

The aim of this investigation is to delineate potential aquifers for groundwater exploitation, using 2D electrical resistivity method.

Objectives:

- To determine the appropriateness or otherwise of the locations imaged for siting of boreholes.
- ii. To produce appropriate images of the study area with the aim of determining aquifer thickness, depth to the bedrock, and fracture.
- iii. To shed more light on types and extent of the various aquifer in the study area.

CHAPTER TWO: LITERATURE REVIEW

2.1 GEOLOGY OF THE STUDY AREA

The geology Of Nigeria is generally subdivided into three units namely: The basement complex, the volcanic rocks and the sedimentary basins.

Bauchi state is basically underlain by crystalline rocks of Nigerian basement complex mostly Precambrian to early Paleozoic (600-2000ma) in age. These consist of a mixture of granites, gneisses, pegmatite, and some amount of charnokite.

The granites of the basement complex were first investigated by Falconer (1991), and were mapped and examined by Bain (1926) and Oyawoye (1958, 1961, 1962).

The gneisses occur as pendants or xenoliths in the granites and are generally a medium grained granublastic aggregate of quartz, plagioclase and potash feldspar Oyawoye 1958, 1961, 1962) and Oyawoye and Makinjuola (1972).

The granites are coarse grained and are composed essentially of quartz, alkali feldspar, Biotite and some muscovite with accessory hornblend haematite. Many stages of granitization and magmatic activities are displayed and show diverse contact relationships with the metasediments (Oyawoye, 1962). Pegmatite veins within the granites and gneisses are composed largely of potash feldspar and may form very large crystal.

A charnokitic rock occurs around the margin where it forms a small outcrop. The older granite suites are predominantly of porphyritic biotite granite or biotite hornblende granite.

They are usually foliated due to the mafic mineral present in them.

The basement complex had been said to be mostly Precambrian in age (Oyawoye, 1970) and forms part of African shield. However, the basement complex rocks are generally divided into two main groups:

- i. Older basement and
- ii. Younger granite suites



Fig. 2: Geological Map of Bauchi State and Location of Study Area.

2.2 Older Basement Rocks

The older basement rocks are made up of quartz-diorite, granite - gneiss, fayalite quartz monzonite (Bauchite) and migmatite.

2.2.1 Quartz Diorite

Quartz diorite occurs as veins and dykes within the migmatites and granites. The dykes vary in thickness from 10cm to as much as 100cm and also about 1km across. They generally; cut across the structure in the h6st •rock having a sharp

contact with them. The quartz diorite is grained melanocratic rock (Koébe, 1975). 2.2.2 Granite-Gneiss

Granite-gneiss are fine to medium grained, biotite rich granite rocks, found mostly in the southern part of Bauchi town, covering the east and west parts of Bauchi L.G.A and some parts of Ganjuwa and Tafawa Balewa LGA. They are composed of white plagioclase and some microline quartz and fairly abundant biotitehornblende also present in varying amount (Oyawoye, 1958).

2.2.3 Fayalite Quartz Monzonite (Bauchite)

Fayalite bearing charnikitic rocks (Bauchite) and quartzdiorite occur widely in the older granite complexes of Bauchi area. Bauchites are restricted to Bauchi, Dandango, Gwaskoram, and Kubi. (Falconer, 1911). The Bauchite are large bodies up to, 10km across with gradational boundaries whereas the quartz diorite are small discrete intrusion up to about 1km across or dykes close to or intruding into Bauchite. Oyawoye, (1958, 1961).Bauchite was described as coarse grained augitesyenite by Falconer (1911) and later by Bain (1926).

The distinctive features of quartz-diorite and Bauchite were first described by Oyawoye(1958, 1961) who named them Bauchite. Fresh samples of Bauchite are dark green due to the green or brown colour of feldspar (Rahman, 1981). The rocks are massive, homogeneous and foliated with few joints. They have few joints

outcropping as smooth rounded boulders derived from massive unfoliated rocks by weathering.

2.2.4 Migmatite

Migmatites varies in texture from medium to coarse-grained. Generally, the migmatites are foliated with flakes of biotites defining the foliation. They represent a high grade metamorphosed series with excellent branding foliation migmatites are foliated with flakes of biotites defining the foliation. The migmatite is a composite rock of hornblende bearing gneiss and granitic rock. The granite rock is usually biotite granite alternating with the hornblende bearing gneiss. The hornblende is present in varying amount (Eborall, 1989).

2.3 Younger Granite Suite

The younger granite suite of Jurassic age occupies only a small portion of the state. The most prominent occurrence is the zaranda ring complex along Bauchi — Jos road, Kela – Warji and the Ningi — Burra younger granite complex. They are composed of mainly of rhyolites, synites, granite porphyry and volcanic basalts. Others are frachytes and salts (Oyawoye, 1958) Madara and Buskuri at the border of Bauchi — Yobe State (Rayment, 1965). The lacustrine Chad formation overlying the Keri — Keri formation is Pleistocene in age consisting of whitish silts andhands of sand, silt and clays (Rayment, 1965).

2.4 Geology of the Study Area

The crystalline rocks of the area are located in area of high relief with relatively off and low infiltration rate (Offodile, 1983). The Precambrian basement complex comprising mainly the fayalite quartz monzonite (Bauchite), granite, biotitehorblende and the undifferentiated migmatite and gneiss units. These rocks are overlained by a thin Regolith (Likkason and Shemang, 1995). It should be noted that: the crystalline rocks do not normally contain water, but when they are weathered, fractured; jointed and fissured, they are been found to contain appreciable amount of water (D'ike et al., 1994).

Considering the basement complex portion of the state, the weathered zone of this complex forms aquifers with limited resources. The fractured and fissured zones occurring within the fresh basement on the other hand constitute the aquifer with water yield that lend to be constant depending on the rock type. The thickness of the weathered and fractured layers forming the aquifer determined the size of the water bearing capacity.

The maximum groundwater potential in the basement complex of Bauchi state is mostly found along the fractured and fissured zones which constitute the preferential flow paths for groundwater and it depends on the porosity and permeability of the zone.

2.5 Hydrogeology of the Area

The hydrogeology of the study area lies within the basement complex of Bauchi state and comprises generally of low per cent joints and fractures except in few places of a moderate fractures. Most part covered by the Basement, comprise of two main types of aquifers. These include those occurring within the weathered rock overlain by the hard crystalline bed rock and those found in the fractures and the faulted zones of the fresh rock. Characteristically, the weathered Basement is clayey, brown or reddish brown ferruginised laterite.

2.6 Previous work done around the study area

Edok-Eter Mandilas, (1978) in its reports for the Bauchi state water board (BSWB) investigated the various aquifer types in the state with complete borehole logs. Shemang and Umaru (1994) identified areas west of Gudum Hill and Barkumbo valley within Bauchi district to be suitable well field. Adamu (1997) delineated depth to basement and identified the aquifer potentials of Abubakar Tafawa Balewa University Yelwa Campus. The present study aims at providing qualitative and quantitative information on the characteristics of the weathered zone, which relate to the occurrence of an exploitable aquifer around the school premises.

2.7 Ohm's Law

If the current I flow through an object of limited size (fig.3.3), voltage drop (V) is observed. The relation between voltage and current is given by Ohm's law. $\Delta V = IR(ohms \ law) \qquad 3.2$

Where, ΔV is the voltage drop or potential difference from one end of a resistor to the other.Since R is directly proportional to the length of the resistor and inversely proportional to the cross-sectional area of the resistor (fig 3.3) one can define an intrinsic property, electrical resistivity, p as



Fig. 3: flow of current through a conductor of limited size

The basis of the electrical resistivity method is shown in Figure 3.2. We put a known current into the ground and measure the resulting voltage drop to estimate the resistivity of the subsurface. The current can be express as:

$$I = \frac{V}{R} = \sigma A \frac{\Delta V}{I} \approx \sigma A \frac{dV}{dl}$$

$$j = \frac{l}{A} = -\sigma \frac{dv}{dt}$$
 3.4

Where, j is a current density (current divided by area). In three dimensional, $j = \sigma \left\{ \frac{\sigma v}{\sigma x} i + \frac{\sigma v}{\sigma y} j + \frac{\sigma v}{\sigma z} k \right\} = -\sigma \nabla \qquad 3.5$

Where, the minus• sign indicates that current flows in the direction decreasing voltage potential). The vector equivalent of Ohm's law is:

$$j = \frac{E}{\rho} = \sigma E \tag{3.6}$$

Where, E is the electric field. Thus, the gradient of the electrical potential is the electric field, E:

$$E = -\nabla V$$
 3.7

2.8 Electrical Potential Due To a Point Current Source

As it can be seen, in Table 1, the resistivity ranges of different earth materials overlap.

Thus, resistivity measurements cannot be directly related to the type of soil or rock in the subsurface without direct sampling or some other geophysical or geotechnical information.

However, Resistivity method allows determining the structure of the subsurface layers of the rock, the location of ground water table, the location of fault zone etc.

This can be determine from the measurement of potential distribution in the earth's surface and from the geometric characteristics of the probe used to measure this distribution.

Consider a single current electrode on the surface of a medium of uniform resistivity The Current flows radially away from the electrode so that the current distribution is uniform over hemispherical shells centered on the source. Lines of equal voltage (equipotential) intersect the lines of equal current at right angle.



Fig. 4: Current flow for a single surface electrode

The voltage drop between any two points on the surface can be described by the potential gradient. dV/dr is negative because the potential decreases in the direction of current flow.

From Ohm's law,

$$\frac{\partial V}{\partial r} = \frac{\rho l}{2\pi r}$$

3.8

Thus, the potential Vr at distance r is obtained by integration.

$$Vr = \int \partial V = -\int_{2\pi r^2}^{\rho l} \sigma r = \frac{\rho l}{2\pi r}$$

$$3.9$$

In practice; the current is introduced in to the ground by means of two electrodes, i.e. a source and a sink; and to obtain expression for the potential at any point due to this bipolar arrangement (figure 3.3); we need only superimpose two of these solutions.

$$V(p) = \frac{P}{2\pi} \left(\frac{1}{r_1} - \frac{1}{r_2^2} \right)$$
 3.9.1

Where, rl and r2 are the distance of the point p from the source and the sink, respectively.



Fig. 5: Current systems of two electrodes.



Fig.6: (a) The Wenner array. (b) The Schlumberger array constant for some time as L increased during the measurement. However if a. 1/5 L then a increases systematically. The separation of the current electrodes is L, and the separation of the potential electrodes is a (c) dipole-dipole (axial)

2.9 Apparent Resistivity and Electrode Configurations.

In practice, the resistivity method involves placing current in the ground with two (2) electrodes and measuring the voltage drop with two other electrodes.

Consider an arrangement consisting of a pair of current electrodes and a pair of potential electrodes. The current electrodes A and B act as source and sink,

espectively. At the detection electrode M the potential due to the source A is pl/(271AM), while the potential due to the Sink B is -pl/(27trMB).

fhe combined potential at M, is

$$V_{\rm M} = \frac{\rho I}{2\pi} \left(\frac{1}{AM} - \frac{1}{MB} \right)$$
 3.9.2

Similarly, the resultant potential at N is

$$V_N = \frac{\rho I}{2\pi} \left(\frac{1}{AN} - \frac{1}{NB} \right)$$
 3.9.3

The potential difference measured by a Voltmeter connected between M and N

$$V = \frac{1P}{2\pi} \left\{ \left(\frac{1}{AM} - \frac{1}{BM} \right) - \left(\frac{1}{AN} - \frac{1}{BN} \right) \right\}$$
 3.9.4

By rearranging equation 3.9.4, we express resistivity p by

$$\rho = \left\{ \frac{2\pi}{\frac{1}{Am} - \frac{1}{BM} - \frac{1}{AN} + \frac{1}{BN}} \right\} \frac{\Delta V}{I}$$

$$= \rho = K \frac{\Delta V}{I}$$
3.9.5

$$K = \begin{cases} \frac{2\pi}{\frac{1}{Am} - \frac{1}{BM} - \frac{1}{AN} - \frac{1}{BN}} \end{cases}$$

Where, k is called the geometric factors of the electrode arrangement. Where the ground is uniform, the resistivity should be constant and independent of both

lectrode spacing and surface location. When subsurface in homogeneities exist, he resistivity will vary with the relative positions of electrodes.

Wherever these measurements are made over a real heterogeneous earth, as distinguished from the fictitious homogeneous half-space, the symbol p is replaced by pa for apparent resistivity. The resistivity-surveying problem is, reduced to its essence, the use of apparent resistivity values from field observations at various locations and with various electrode configurations to estimate the true resistivity of the several earth materials present at a site and to locate their boundaries spatially below the surface of the site. Hence, equation 16 can be rewritten as:

$$\rho_a = K \frac{\Delta V}{I}$$

P_a, is the resistivity of an equivalent but fictitious half space and depends on electrode geometry and spacing.

In practice, many several different arrays are used. The most widely used configurations are Wenner, Schlumberger and Dipole-Dipole.

2.9.1 Dipole-Dipole Arrangement

Dipole-dipole (Eltran) array: popular in induced polarization (IP) work because the complete separation of current and voltage circuits reduces the vulnerability to inductive noise. A considerable body of interpretational material is available. Information from different depths is obtained by changing n. In principle, the larger the value of n, the deeper the penetration of the current path sampled. Results are usually plotted as pseudo-sections, so the apparent resistivity is given by:



P1, P2: Potential Electrode

Dipole-Dipole Array

The potential electrodes are placed outside the current electrodes. The electrodes may or may not be collinear.



$$\begin{aligned} \rho_{aD} &= \frac{2\pi\Delta V}{\frac{2\pi(n+2) - (n+2)(n+2) - n(n+2)}{n\alpha(n+2)(n+2) - n(n+2)}} \\ \rho_{aD} &= \frac{2\pi\Delta V}{\frac{2\pi^2 + 4n - n^2 - sn - 2}{n\alpha(n+2)(n+2)}} \\ \rho_{aD} &= \frac{2\pi R (n\alpha(n+1)(n+2))}{-2} \\ \rho_{aD} &= -\pi R\alpha (n+1)(n+2)n \\ \rho_{aD} &= -\pi R\alpha (n+1)(n+2)n \end{aligned}$$

$$\rho_a = \pi a n (n+1)(n+2) \frac{V}{r}$$

Depth of investigation = 0.195AN

2.10 Traditional resistivity surveys

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The resistivity method has its origin in the 1920's due to the work of the Schlumberger brothers. For approximately the next 60 years, for quantitative interpretation conventional sounding surveys (Koefoed 1979) were normally used. In this method, the centre point of the electrode array remains fixed, but the spacing between the electrodes is Increased to obtain more information about the deeper sections of the subsurface.



Figure 7: A conventional four electrode array to measure the subsurface resistivity.



Fig.8: Common arrays used in resistivity surveys and their geometric factors.

The measured apparent resistivity values are normally plotted on a log-log graph paper, To interpret the data from such a survey, it is normally assumed that the subsurface consists of horizontal layers. In this case, the subsurface resistivity changes only with depth, but does not change in the horizontal direction. A one-dimensional model of the subsurface is used to interpret the measurements (Figure 3. la). Despite this limitation, this method has given useful results for geological

situations (such the water-table) where the one dimensional model is approximately true. Another classical technique is the profiling method. In this the spacing between the electrodes remains fixed, but the entire array is moved along a straight line. This gives some information about lateral changes in the subsurface resistivity, but it cannot detect vertical changes' resistivity. Interpretation of data from profiling surveys is mainly qualitative.

The most severe limitation of the resistivity sounding method is that horizontal (or lateral) changes in the subsurface resistivity are commonly found, the ideal situation shown in Figure 3a is rarely found in practice. Lateral changes in the subsurface resistivity will cause changes in the apparent resistivity values which might be, and frequently are, misinterpreted as changes with depth in the subsurface resistivity. In many engineering and environmental studies, the subsurface geology is very complex where the resistivity can change rapidly over short distances. The resistivity sounding method might not be sufficiently accurate for such situations.

Despite its obvious limitations, there are two main reasons why I-D resistivity sounding surveys are common. The first reason was the lack of proper field equipment to carry out the more data intensive 2-D and •3-D surveys. The second reason was the lack of practical computer interpretation tools to handle the more complex 2-D and 3-D models.

However, 2-D and even 3-D electrical surveys are now practical commercial techniques with the relatively recent development of multi-electrode resistivity surveying instruments (Griffiths et al. 1990) and fast computer inversion software (Loke 1994).



Figure 3.6. The three different models used in the interpretation of resistivity measurements

2.11 The relationship between geology and resistivity

Resistivity surveys give a picture of the subsurface resistivity distribution. To convert the resistivity picture into a geological picture, some knowledge of typical resistivity values for different types of subsurface materials and the geology of the area surveyed, is important.

Table 1 gives the resistivity values of common rocks, soil materials and chemicals (Keller and Frischknecht 1966, Daniels and Alberty 1966). Igneous and metamorphic rocks typically have high resistivity values. The resistivity of these

rocks is greatly dependent on the degree of fracturing, and the percentage of the fractures filled with ground water. Sedimentary rocks, which usually are more porous and have higher water content, normally have lower resistivity values. Wet soils and fresh ground water have even lower resistivity values. Clayey soil normally has a lower resistivity value than sandy soil. However, note the overlap in the resistivity values of the different classes of rocks and soils. This is because the resistivity of a particular rock or soil sample depends on a number of factors such as the porosity, the degree of water saturation and the concentration of dissolved salts.

The resistivity of ground water varies from 10 to 100 Ohm-m, depending on the concentration of dissolved salts. Note the low resistivity (about 0.2 Ohm-m) of •sea water due to the relatively high salt content. This makes the resistivity method an ideal technique for mapping the saline and fresh water interface in coastal areas.

The resistivity values of several industrial contaminants are also given in Table 1. Metals, such as iron, have extremely low resistivity values. Chemicals which are strong electrolytes, such as potassium chloride and sodium chloride, can greatly reduce the resistivity of ground water to less than I ohm-m even at fairly low concentrations. The effect of weak electrolytes, such as acetic acid, is comparatively smaller. Hydrocarbons, such as xylene, typically have very high

resistivity values.

Resistivity values have a much larger range compared to other physical quantities mapped by other geophysical methods. The resistivity of rocks and soils in a survey area can vary by several orders of magnitude. In comparison, density values used by gravity surveys usually change by less than a factor of 2, and seismic velocities usually do not change by more than a factor of 10. This makes the resistivity and other electrical or electromagnetic based methods very versatile geophysical techniques.

Table 1:Resistivity's of some rocks, Material and chemicals(Keller and Frischknecht 1966, Daniels and Alberty 1966).

Materials	Resistivity (\Omega*m)	Conductivity (Siemen/m)
Igneous and Metamorphic Rocks Granite Basalt Slate Marble Ouartzite	$5 \times 10^{3} - 10^{6}$ $10^{3} - 10^{6}$ $6 \times 10^{2} - 4 \times 10^{7}$ $10^{2} - 2.5 \times 10^{8}$ $10^{2} - 2 \times 10^{8}$	$10^{-6} - 2 \times 10^{-4}$ $10^{-6} - 10^{-3}$ $2:5 \times 10^{-8} - 1.7 \times 10^{-3}$ $4 \times 10^{-9} - 10^{-2}$ $5 \times 10^{-9} - 10^{-2}$
Sedimentary Rocks Sandstone Shale Limestone	$8 - 4 \times 10^{3}$ 20 - 2 \times 10^{3} 50 - 4 \times 10^{2}	$2.5 \times 10^{-4} - 0.125$ $5 \times 10^{-4} - 0.05$ $2.5 \times 10^{-3} - 0.02$
Soils and waters	1 - 100	0.01 - 1

Groundwater (fresh) Sea water Chemicals	10 - 800 10 - 100 0.2	$ \begin{array}{r} 1.25 \times 10^{-3} - 0.1 \\ 0.01 - 0.1 \\ 5 \end{array} $
Iron	9.074 _{x10} -8	1.102x10 ⁷
0.01M Potassium Chloride	0.708	1.413
0.01 M Sodium chloride	0.843	1.185
0.01 M acertic acid	6.13	0.163
Xylene	6.998 _{x10} ¹⁶	1.429x10 ⁻¹⁷

2.12 2-D Electrical Imaging Surveys

Introduction

Limitation of the resistivity sounding method is that it does not take into account horizontal changes in the subsurface resistivity. A more accurate model of the subsurface is a two-dimensional (2-D) model where the resistivity changes in the vertical direction, as well as in the horizontal direction along the survey line. In this case, it is assumed that resistivity does not change in the. Direction that is perpendicular to the survey line. In many situations, particularly for surveys over elongated geological bodies, this is a reasonable assumption. In theory; a 3-D resistivity survey and interpretation model should be even more accurate. However, at the present time, 2-D surveys are the most practical economic compromise between obtaining very accurate results and keeping the survey costs down. Typical I-D resistivity sounding surveys usually involve about 10 to 20 readings, while 2-D imaging surveys involve about 100 to 1000 measurements. In comparison, 3-D surveys usually involve several thousand measurements.

The cost of a typical 2-D survey could be several times the cost of a 1-D sounding survey, and is probably comparable with a seismic survey. In many geological situations, 2-D electrical Imaging surveys can give useful results that are complementary to the information obtained by other geophysical method. For example, seismic' methods can map undulating interfaces well, but will have difficulty (without using advanced data processing techniques)' in mapping discrete bodies such as boulders, cavities and pollution plumes. Ground penetrating radar surveys can provide more detailed pictures but have very limited depth penetration in areas with conductive unconsolidated sediments, such as clayey soils. Twodimensional electrical surveys should be used in conjunction with seismic or GPR surveys as they provide complementary information about the subsurface.

CHAPTER THREE: METHODOLOGY

3.1 Materials

The Materials used are:

- i. Terrameter
- ii. GPS
- iii. Measuring Tape
- iv. Hammer
- v. Electrodes
- vi. Current Source (Car Battery)
- vii. Connecting Cables



Fig.9: The arrangement of electrodes for a 2-D electrical survey.

The same process is repeated for measurements with "3a", "4a" "5a" and "6a" spacing. To get the best results, the measurements in a field survey should be carried out in a systematic manner so that, as far as possible, all the possible measurements are made. This will affect the quality of the interpretation model obtained from the inversion of the apparent resistivity measurements (Dahlin and Loke 1998).

As the electrode spacing increases, the number of measurements decreases. The number of measurements that can be obtained for each electrode Spacing, for a given number of electrodes along the survey line, depends on the type of array used. The Wenner array gives the smallest number of possible measurements compared to the other common arrays that are used in 2-D surveys.

One technique used to extend horizontally the area covered by the survey, particularly for a system with a limited number of electrodes, is the roll-along method. After completing the sequence of measurements, the cable is moved past one end of the line by several unit electrode spacing. All the measurements which involve the electrodes on part of the cable which do not overlap the Original end of the survey line are repeated (Fig.8).

Fig. 10: The use of the roll-along method to extend the area covered by a survey.

3.2 Field Procedure

To obtain a good 2-D picture of the subsurface, the coverage of the measurements must be 2-D as well. As an example, Figure 3.8 shows a possible sequence of measurements for the Dipole-Dipole electrode array for a system with 20 electrodes. In this example, the spacing between adjacent electrodes is "a". The first step is to make all the possible measurements with the Dipole-Dipole array with electrode spacing of "na". For the first measurement, electrodes number 1, 2, 3 and 4 are used. Notice that electrode 1 is used as the first current electrode C1, electrode 2 as the second current electrode C2, electrode 3 as the first potential electrode P1 and electrode 4 as the second potential electrode P 2. For the second measurement, electrodes number 1, 2, 4 and 5 are used for C 1, C2, P1 and p2respectively. This is repeated down the line of electrodes until electrodes 1, 2, 19 and 20 are used for the last measurement with "na" spacing. For a system with 20 electrodes, note that there are 17 (20 - 3) possible measurements with "na" spacing for the Dipole-Dipole array.

After completing the sequence of measurements with "na" spacing, the next sequence of measurements with "na" electrode spacing is made. First electrodes 1, 2, 3 and 4 are used for the first measurement. The electrodes are chosen so that the spacing between adjacent electrodes is "na". For the second measurement, electrodes 1, 2, 4 and 5 are used. This process is repeated down the line until electrodes 1, 2, 18 and 20 are used for the last measurement with spacing "na". For a system with 20 electrodes, note that there are 14 (20 - 2x3) possible measurements with "2a" spacing.

3.3 Pseudosection Data Plotting Method

To plot the data from a 2-D imaging survey, the pseudosection contouring method is normally used. In this case, the horizontal location of the point is placed at the mid-point of the set of electrodes used to make that measurement. The vertical location of the plotting point is placed at a distance which is proportional to the separation between the electrodes. Another method is to place the vertical position of the plotting point at the median depth of investigation (Edwards, 1977), or pseudo-depth, of the electrode array used. The pseudosection plot obtained by contouring the apparent resistivity values is a convenient means to display the data.

The pseudosection gives a very approximate picture of the subsurface resistivity distribution. However the pseudosection gives a distorted picture of the subsurface because the shape of the contours depend on the type of array used as well as the true Subsurface resistivity (Figure 3.9. 1). The pseudosection is useful as a means to present the measured apparent resistivity values in a pictorial form, and as an initial guide for further quantitative interrelation. One common mistake made is to try to use the pseudosection as a final picture of the true subsurface resistivity. As Figure 3.9.1 shows, different arrays used to map the same region can give rise to Very different- contour shapes in 'the pseudo-section plot. Figure 3.9.1 also gives one an idea of the data coverage that can be obtained with different arrays.

One useful practical application of the pseudosection plot is for picking out bad apparent resistivity measurements. Such bad measurements usually stand out as points with unusually high or low values.

3.4 Forward Modeling Program Exercise

The program, RES2DINV, is a 2-D forward modeling program which calculates the apparent resistivity pseudosection for a user defined 2-D subsurface model. With this program, the user can choose the finite-difference (Dey and Monison 1979a) or finite-element (Silvester and Ferrari 1990) method to calculate the apparent resistivity values.

In the program, the subsurface is divided into a large number of small rectangular cells. The program assists the user in choosing the appropriate array for different geological situations or surveys. The arrays supported by this program are the Wenner (Alpha, Beta and the Alpha configuration is normally used for field surveys and Gamma configurations - usually just referred to as the "Wenner" array), Wenner-Schlumberger, pole-pole, inline dipole-dipole, pole-dipole and equatorial.dipole-dipole (Edwards 1977). Each type of array has its advantages and disadvantages. This program will hopefully help in choosing the "best" The arrays most commonly used for resistivity surveys were shown in Figure 2. The choice of the "best" array for a field survey depends on the type of structure to be mapped, the sensitivity of the resistivity meter and the background noise level. In practice, the arrays that are most commonly used for 2-D imaging surveys are the (a)Wenner, (b) dipole-dipole (c) Wenner-Schlumberger (d) pole-pole and (d) poledipole. Among the characteristics of an array that should be considered are (i) the sensitivity of the array to vertical and horizontal changes in the subsurface resistivity, (ii) the depth of investigation, (iii) the horizontal data coverage and (ivy the signal strength. Figures 3.9.2a to e shows the contour pattern for the sensitivity function of the Wenner, Schlumberger and dipole-dipole arrays for a homogeneous earth model. The sensitivity function basically tells us the degree to which a change in the resistivity of a section of the subsurface will influence the potential measured by the array. The higher the value of the sensitivity function, the greater is the influence of the subsurface region on the measurement. For all the three arrays, the highest sensitivity values are found near the electrodes.

At larger distances from the electrodes, the contour patterns are different for the different arrays. The difference in the contour pattern in the sensitivity function plot helps to explain the response of the different arrays to different types of structures. The median depth of investigation gives an idea of the depth to which we can map with a particular array. The median depth values are determined by integrating the sensitivity function with depth. In layman's terms, the upper section of the earth above the "median depth of investigation" has the same influence on the measured potential as the lower section'. This tells us roughly how deep we can see with an array. This depth does not depend on the measured apparent resistivity or the homogeneous earth model. It should be noted that the

depths are strictly only valid for a homogeneous earth model, but they are probably good enough for planning field surveys. If there are large resistivity contrasts near the surface, the actual depth of investigation could be somewhat different. For example, it has been that a large low resistivity body near the surface tends to create a "shadow zone" below where it is more difficult to accurately determine the resistivity values.

CHAPTER FOUR

4.0 Interpretation and Results

The raw field data were processed using RES2DINV (Loke and Barker, 1996, Loke, 1999). This is a computer program that aids the interpretation of a twodimensional (2-D) resistivity for the subsurface for the data obtained from electrical survey.

The results obtained shows measured apparent resistivity pseudosection, the calculated apparent resistivity pseudosection and the inverse model resistivity section of three profiles, but with colour infill instead of line contours. These data have been inverted to produce the image of Figs. 11 to 13. The electrical image revealed three to four layers, ranging from top Soil which is reddish brown, clay, weathered basement to fresh basement. The surface material in the entire area is top soil which is Laterite in nature and has resistivity values ranging from about 40 Ohm-m to 80Ohm-m. Underlying the moderately weathered basement and has resistivity ranging from about 90Ohm-m to 280Ohm-m, is the third layer weathered crystalline rock with resistivity value ranging from 300Ohm-m to 700hm-m, and the fourth layer is the fresh basement which with resistivity value ranging from 1000 Ohm-m to 3,000Ohm-m.

4.1 Profile One

Figure 4.1, shows the resistivity inversion results (iteration 4, 54.4% total average RMS error) for profile 1. Thus, indicating that, good fit between the measured and calculated apparent resistivity data were achieved. The Apparent resistivity (in ohm-metre, Ω m) is plotted against pseudo-depth (in metre). The Profile is 90 m long, and runs in the South to North direction. Low resistivity zones (<45 Ω m) were revealed near the surface with depth between 0m to 4.5m which indicates the topsoil. In this profile water bearing zone about 9.65 m extends from x=48m to 65m along the profile and the depth to the bedrock is shallow, about 6.5m and extends from x= 20m to 45m along the profile. Colour variations in the basement rock are indication of contacts between different rocks which can be interpreted as fractures. The red colour indicate the weathered basement with resistivity value (<800 Ω m). The purple colour with resistivity value (>1000 Ω m) is interpreted as the fresh basement.

profile one: ATBU Football field.



4.2 Profile Two

Figure 4.2 shows the resistivity inversion results (iteration 4, 67.7% total average RMS error) for profile 2. Thus, indicating that, good fit between the measured and ^{calculated} apparent resistivity data were achieved. The Apparent resistivity (in

 $_{\text{phm}}\text{-metre},\ \Omega\text{m})$ is plotted against pseudo-depth (in metre). The Profile is 95 m long, and runs in the South to North direction. Low resistivity zones (<18 Ω m) were revealed near the surface with depth between 0m to 4.5m which indicates the topsoil and at 9.26m to 12.4m which indicate water bearing zone at x=75m to 83m. In this profile the depth to the bedrock is shallow, about 6.5m and extends from x= 20m to 45m along the profile. Colour variations in the basement rock are indication of contacts between different rocks which can be interpreted as fractures. The red colour indicate the weathered basement with resistivity value (<1430 Ω m). The purple colour with resistivity value (>2500 Ω m) is interpreted as the fresh basement.

Profile two: girls Hostel.





4.3 Profile Three

Figure 4.3 shows the resistivity inversion results (iteration 4, 84.4% total average RMS error) for profile 3. Thus, indicating that, good fit between the measured and calculated apparent resistivity data were achieved. The Apparent resistivity (in ohm-metre, Ω m) is plotted against pseudo-depth (in metre). This Profile runs in the South to North direction and it is 60 m long. Low resistivity zones (<10 Ω m) were revealed near the surface with depth between 0m to 1.5m which indicates

the topsoil at x=15m to 20m. In this profile the depth to the bedrock is shallow, about 5.8m and extends from x= 12m to 60m along the profile. Colour variations in the basement rock are indication of contacts between different rocks which can be interpreted as fractures. The red colour indicate the weathered basement with resistivity value (<880 Ω m). The purple colour with resistivity value (>2100 Ω m) is interpreted as the fresh basement.

Profile three: ISS field ATBU.





CHAPTER FIVE

SUMMARY AND CONCLUSION

5.1 Summary

The result of quantitative interpretation of the 2-D electrical resistivity data shows that two of the three profiles from the study areas are dominated with by low resistivity values extending to greater depths.

The interpreted results which are represented by geoelectric sections show the sequence and relationship between the surface lithologies. The weathered layer and fractured zone have been identified as the aquifer in the area.

5.2 High Resistivity Layers

High resistivity values characterize the un-weathered, fresh crystalline bedrock in the study area and are generally represented by values >1000 Ω m. This layer usually underlies low, medium and fairly high resistivity layers. The crystalline Basement rocks normally have no porosity or permeability except where they are fractured, faulted or jointed. Thus they are devoid of underground water which

influences resistivity.

5.3 Fairly High Resistivity Layers

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This unit represents the slightly weathered and fractured zones underlying the weathered mantle. The unit is generally characterized by resistivity values ranging from $300\Omega m$ to $900\Omega m$. The extent of weathering and fracturing is generally limited in crystalline rocks (Offodile, 1991). As a result, the ground water may occur only in small pockets or basins depending on the extent of weathering.

5.4 Medium Resistivity Layers

This is considered to be the top soil which consists of conductive materials varying in composition from clay through laterite to sand, derived from chemical weathering of basement rocks. Variation in resistivity values ranging from 200m to 500m.

5.5 Low Resistivity Layers

This unit is characterized by low resistivity values relative the medium and the high- resistivity units. The unit was interpreted as the weathered crystalline rock of sand, clay, or clayey sands depending on the local variations of the mineralogy of the basement rock. This unit overlies the fractured and /or fresh crystalline This unit may contain water in storage; large enough to drill successful bore holes were

thickly developed.

5.6 Conclusion

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A total number of three profiles were taken along the surface of the ground and the electrical imaging was carried out along the lines. Electrical images provide a more detailed view of the subsurface structure which makes interpretation easier. The inversion results of profile one reveals distinct layers ranging from lateritic clay to fresh basement.

The electrical image of profile two shows intercalation of the laterite clay and ferruginised clay as near surface material with resistivity values ranging from $1\Omega m$ to $100\Omega m$. This profile shows a highly fractured and weathered zone which spreads across almost entirely on whole profile, which makes it suitable for sitting of borehole at any point on the profile.

5.7 Recommendation

The electrical resistivity method should be complemented by the induced Polarization method. The main purpose of recommending the integration of the electrical resistivity method and the induced polarization method is because it will help in delineating clay zones.

The electrical resistivity method can only tell you where moisture is no matter how small in volume, knowing that clay can absorb water in' 'large quantity and cannot

yield' appreciable quantity into a borehole. But induced polarization (IP) will read this clay body as clayey water bearing layer. So with the help of induced polarization (IP) this zones will be delineated, and hence prevent the drilling of water borehole through impermeable clay layers.

Although expensive, the use of multi-core cable which connects large number of electrodes to the system at the same time may be used, such that four electrodes for each measurement are selected automatically by a laptop microcomputer together with an electronic switching unit, should be encouraged for future study.

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