APPLICATION OF QUEUING MODELS TO CUSTOMERS MANAGEMENT IN THE BANKING SYSTEM

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BEING A PROJECT WORK SUBMITTED TO THE DEPARTMENT OF STATISTICS, SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY, AUCHI POLYTECHNIC, AUCHI

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CERTIFICATION

We the undersigned, certify that this project was carried out by **OGHENEOCHUKO MARVELLOUS** with **MATRIC NUMBER**: **ICT/222200091** of the Department of Statistics, school of information and communication technology, Auchi polytechnic Auchi.

We also certify that the work is adequate in scope and quality in partial fulfillment of the requirement for the award of Higher National Diploma (HND) in Statistics.

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DEDICATION

With gratitude, I humbly dedicate this project work to GOD almighty for his grace, provision and guidance throughout my HND program, and to my family for their unrelenting prayers and financial support during this period, their guidance and their time in making this work a success.

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ABSTRACT

Queues are common sight in banks these days especially on Mondays and on Fridays. Hence queuing theory which is the mathematical study of waiting lines or queue is suitable to be applied in the banking sector since it is associated with queue and waiting line where customers who cannot be served immediately have to queue(wait) for service. The aim of this project is to determine the average time customers spend on queue and the actual time of service delivery, thereby examining the impact of time *wasting and cost associated with it. The primary data were collected from the UBA branch, the data were collected based on the arrival pattern and service pattern of customers. The methodology employed followed the birth and death Markovian process. We further used the chi-square test to test the arrival pattern to determine if it follows a Poisson distribution and also tested the service pattern to determine if it follows an exponential distribution. The results obtained from the chi-square test showed that the arrival pattern follows a Poisson distribution and that the service pattern follows an exponential distribution, hence it can be analyzed using Markovian birth and death process. We were now forced to recommend based on the analysis, that the Bank management should increase the number of servers, so as to help reduce the time customers spend on queue and also reduce cost incurred from waiting. We now concluded that the objective of this project was achieved.*

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND OF STUDY

Queue is a common sight in banks especially on Mondays and Fridays. The word queue comes via French and the Latin “Cauda Customers” meaning, waiting in line to receive services in any service system is inevitable and that is why queue management has been where the manager faces huge challenge. Hence, queuing theory is suitable to be applied in the banking system. Since it is associated with queue or waiting line where customers who cannot be served immediately have to queue (wait) for service for a long time and time being a resource ought to be managed effectively and efficiently because time is money.

Queuing theory is the mathematical study of waiting lines or queues. It is generally considered a branch of operations research because the results are often used when making business decisions about the resources needed to provide service (Wikipedia, 2008). Queuing theory is also known as the theory of overcrowding; it is the branch of operational research that explores the relationship between the demand on a service system and the delays suffered by the users of that system (Sharma, 2013).

Queuing theory is a major topic for applied mathematics that deals with phenomenon of waiting and arises from the use of powerful mathematical analysis to describe production processes (Masurdi, 2011). The study of queues deals with quantifying the phenomenon of waiting in lines using representative measures of performance, such as average queue length, average waiting time in queue and average facility utilization (Taha 2002).

Queuing theory can also be applied to a variety of operational situations where it is not possible to accurately predict the arrival rate (or time) of customers and service rate (or time)) of service facility of facilities. Queuing theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service and the probability of encountering the system in certain states, such as empty, full or having an available server or having to wait a certain time to be served (M.K, 2011), queuing models provide the analyst with a powerful tool for evaluating the performance of queuing systems (Nicol, 2001).

1.2 STATEMENT OF PROBLEM

This study is required to investigate the expected waiting time of customers and the actual waiting time in banks, where the gap between the actual and expected waiting time can be analyzed to know how to improve on the efficiency and effectiveness of their bank. Such problems are;

- How poor service facilities has affected the overall bank performance.
- How poor service pattern affects queue discipline.
- How service facilities has affected the time of customers

- How poor service delivery impacts on time.
- How poor service's delivery has affected queuing in banks.

1.3 AIMS AND OBJECTIVES OF THE STUDY

The aim of this study is to determine the amount of average time customers spend on a queue and actual time of service delivery. Therefore the objectives of this study are as follows:-

- To examine the impact of time wasting or weak performance of service delivery.
- To improve on the efficiency and effectiveness of their operations by assigning more bank clerks to solve customers' bank related issues.
- To help bank mangers improve customer's management by addressing the bank clerks before the service opening of the bank.
- To improve on time management which is an important resource?

1.4 SIGNIFICANCE OF STUDY

This study when completed will be significant to many people and organizations especially banks in Nigeria. First of all, it will add to the literature on queuing theory and management which will be accessed by lecturers and scholars.

Most importantly, Bank Managers will benefit a lot from this study as they will apply this theory in their various banks, thereby reducing the amount of time spent on queues which might and improve lead on to their overall efficiency and effectiveness.

1.5. RESEARCH QUESTIONS

In terms of the analysis of queuing situations, the types of questions in which we are interested in are typically concerned with measures of system performance which includes;

- To what extent does the service time differ from the actual time that customers have to wait before being served?
- To what extent does poor service pattern affect queue discipline?
- To what extent do service facilities affects queue discipline?
- To what extent does the average service time affect the overall performance of the bank?

1.6 SCOPE AND LIMITATIONS OF STUDY

We will be studying one bank and our concern is the single queue multiple service point because of lack of time and resources to embark on large scale study on majority of banks. One bank to our mind is fair judgment. The queue discipline is first in first out (FIFO) and the arrival time is strictly random.

We also consider a Poisson distributed arrival times and exponentially distributed service times.

1.6.1 PROBLEM DEFINITION

Single-channel queuing system with multi where S>1) are operated by banks with the hope of giving customers maximum satisfaction and also make their profit, the problem arises when these objectives are not reached.

1.7 DEFINITIONS OF TERMS ASSOCIATED WITH QUEUING MODELS

We now look at the definition of terms associated with queuing theory;

Queue – A queue can be defined as an aggregation of items waiting for a service function.

Queuing theory –This is the construction of mathematical model of varying forms of queuing systems.

Arrival –This element is concerned with the rate of entry by customers into the system.

Queuing discipline –This element is concerned with what goes on between the arrival times of a customer and when service is rendered to him/her.

Server – An operation fed by a queue.

Phase – A queue and its connected servers, or router to a server.

Arrival pattern –This is the manner in which customers arrive in the system for service.

Service pattern –This is the rate in which the service channel renders service to a customer.

Balking – This is the refusal of a customer to join the queue if the queue is long.

Reneging – This is the withdrawal of a customer from the queue because of the length of the waiting line.

Jockeying –When a customer withdraws from a queue to join another one because the new queue is shorter.

Arrival means rate number of (λ): arrival per unit time.

Service rate (μ): Mean number of customers that can be served at 100% utilization by each individual server per unit time (usually per hour a day).

1.8 QUEUING MODEL NOMENCLATURE

In queuing theory, a standard terminology is employed. It is often called the Kendall's notation.

The Kendall's notations for specifying queuing are;

V – This indicates the arrival pattern

W – This denotes the service pattern

X – This signifies the number of available servers

Y – This represents the system's capacity

Z – This designates the queue discipline

But we majorly restrict ourselves to the first three notations on the assumption that customers leave the system immediately after service.

CHAPTER TWO

INTRODUCTION TO THE STUDY

2.1. LITERATURE REVIEW

Queuing theory is the mathematical study of waiting lines or queue. The theory enables mathematical analysis of several related processes, including the arrival at the (back of the) queue, waiting in the queue (essentially a storage process) and being served at the front of the queue. The queue permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service and the probability of encountering the system in certain states such as empty, full, having an available server or having to wait a certain time to be served.

Queuing theory has become one of the most important, valuable and arguable one of the most universally used tool by an operational researcher. It has applications in diverse fields including telecommunications, traffic engineering, computing and design of factories, shops, offices, banks and hospitals.

A queuing model of a system is an abstract representation whose purpose is to isolate those factors ability to meet service related demands to those whose occurrences and durations are random. (Sztrik, 2010).

The study of queue deals with quantifying the phenomenon of waiting in lines using representative measures of performance, such as average queue length, average waiting time in queue and average facility utilization (Taha, 2002).

Some of the analysis that can be derived using queuing theory include the expected waiting time in the queue, the average waiting time in the system, the expected queue length, the expected number of customers served at one time, the probability of balking customers, as well as the probability of the system to be in certain states, such as empty or full (Makwana, 2012).Queuing models are used to represent the various types of queuing systems that arise in practice, the models enable in finding an appropriate balance between the cost of service and the amount of waiting (Nafees, 2007).

Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems (Nicol, 2001).

Any system in which arrivals place demands upon a finite capacity resource maybe termed as queuing systems, if the arrival times of these demands are unpredictable, or if the size of these demands is unpredictable, then conflicts for the use of the resource will arise and queues of waiting customers will form and the length of these queue depend on two aspects of the flow pattern: First, they depend on the average rate. Secondly, they depend on the statistical fluctuations of this rate (Klenrock, 1975).

The first study of queuing theory was done by a Danish mathematician, which resulted into the worldwide acclaimed Erlang telephone model (Erlang, 1909). He examined the telephone network system and tried to determine the effect of fluctuating service demands on calls on utilization of automatic dial equipment.

The original problem Erlang treated was the calculation of this delay for one telephone operator and in 1917; the results were extended to the activities of several telephone operators.

Development is largely along the lines initiated by Erlang and the main publications were those of (Molina, 1927 and Fry, 1928).

2.2 QUEUE CHARACTERISTICS

Queuing systems are characterized by five components:-

- The arrival pattern of customers.
- The service pattern of customers.
- The number of servers.
- The capacity of the facility to hold customers.
- The order in which the customers are served.

2.2.1 ARRIVAL PATTERN

This is the manner in which customers arrive in the system for service. The arrival pattern forms a stochastic process. It may be described in two ways;

- Arrival may be measured by the instrument of the time between two successive arrival i.e. inter-arrival times.
- Arrivals may be measured in a particular time interval e.g. arrival in five minutes.
- The arrival time may be constant or random, the random case resulting in a Poisson distribution. The inter-arrival time gives a more accurate description of arrival pattern in cases of related successive arrival.

2.2.2 SERVICE PATTERN

This is the rate in which the service channel renders service to a customer. The service pattern is usually specified by the service time, the time required by one server to serve one customer. The service time may be deterministic or it may be a random variable whose probability distribution is presumed known. It may depend on the number of customers already in the facility or it may be state independent.

2.2.3 QUEUE DISCIPLINE

This is the order in which customers in a queue are served. Some queue behaviors are related to queue discipline, which are as follows:

- Balking
- Reneging
- jockeying

Types of queue discipline

I. First in First out (FIFO): In this type of queue discipline, customers are served in order of arrival also known as first come first served queue discipline.

II. Last in First out (LIFO): In this type of queue discipline, the last customer to arrive is served first.

III. Service in Random Order (SIRO): In this of queue discipline customers are served at random.

IV. Processor sharing (PS): The customers are served at the same time so that

V. the average waiting for all customers is about the same.

VI. Priority Ordering (PRI): in this type of queue discipline, customers with the highest priority are served first.

2.2.4 VARIETIES OF QUEUING SYSTEM

There are four basic varieties of queuing situations

1. Single channel, single phase: The simplest form of queue is a simple line of a customer waiting to be served by a single service point called single channel/single phase. It is applicable in buttery, banks, post offices etc.
2. Multi-channel, single phase: This can also be described as queuing situation with single queue and several points. It is applicable in banks especially.
3. Single channel, multiple phases: This is a system which is often seen in some supermarkets and elsewhere, it is where single queue have to pass through multiple phases. It is applicable in clinics, shopping malls etc
4. Multi-channel, multi-phase
5. The multiple channel, multiple phase case might be represented by two or more parallel production lines as in.

2.3 BRIEF HISTORY OF UNITED BANK FOR AFRICA

The United Bank for Africa (UBA) was incorporated in Nigeria as a public company on the 23rd February 1961 by a consortium of five major international financial institutions to take over the banking business carried on in Nigeria since 1949 by the British and French Bank Limited (a subsidiary of French Bank now known as Banque Nationale De Paris). This bank received its banking license on 17th May 1961 and commenced business on the 1st of October 1961.

By 1961, the bank had grown from its one office structure to 10 branches located in Lagos, Kano, Kaduna, and Port-Harcourt. The branches that pioneered its early development of the bank included Kano branch established inn1954, Ebute-metta branch (May 1955), Apapa branch (March 1956), Port-Harcourt branch (Aggrey road, October 1956) and Lagos East (December 1958), Kaduna branch (July 1960), Ikeja branch (November 1960), and Port-Harcourt main branch (1961). At incorporation, UBA has staff strength of approximately 650 staffs nationwide. Its deposit was valued at 1.898 million naira, its loans and advances amounted to 8.677 million and its total assets were worth 11.903 million naira. By 1990, 9.6 billion naira, loans and advances exceeding 2.5 billion naira, while a total asset hits over 11 billion naira. That year, the branch network had reached 190 while staff strength was over 8,000 persons. The bank as a pace setter, was the first among international banks in Nigeria to be registered under the Nigeria law and to sell its shares to the Nigeria public. It was the first bank to introduce mobile banking service in the country and the first to establish a branch at a University campus (UBA Annual Report 1992). The University of Lagos, Akoka branch, was opened on October 3rd, 1963. The bank was also the first to open a branch at the Kainji Dam site (New Bussa), the centre piece of Nigeria's first five Nigeria's longest banks with assets o all over the country. The bank has had an impressive growth rate internationally rated as one of the top 500 banks in the world, as at 1987. UBA is active in all aspects of commercial banking, including the operation of current deposit and savings account, staff and medium term finance to industry, commerce, agriculture, housing and government bodies (UBA Annual Report 1992). In addition, the bank provides international banking services, corporate finance, share registration, computer services through specialized division and active particularly in the cleaning house of the central bank of Nigeria. As a good corporate citizen, UBA is strongly committed to the social responsibilities and identities with the communities in which it is represented.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 RESEARCH DESIGN

The main aim of this research is to show how the management of United Bank of Africa will go about the reduction of the waiting time of customers. This piece of work will also check if increasing the number of server will reduce the waiting time as well as putting the profit in consideration.

Hence, the objectives of this project will be achieved by analyzing the real life observed data, then constructing a new model of system and using statistical analytical tools like Poisson, exponential and chi-square distribution to study pattern and reaction to change in the system.

3.2 AREA OF STUDY

This research centers on the waiting area of the banking hall of United Bank of Africa i.e. the withdrawal points of the bank.

3.3 FORMULATION OF MODEL FOR THE SYSTEM

To formulate a model for this system, we put the following assumptions into consideration:

- The arrival of customers into the system is discrete form Poisson distribution with arrival rate λ.
- The queuing discipline is first come first serve.
- There is only a two-service channel i.e. (M│M│2)
- The service channel can only render service of finite rate exponentially distribution with service rate, µ.
- The calling population (i.e. the number of customers calling for service is finite.
- The number of customer in the system at the time of initiating the observation is assumed to have arrived in the first unit time.
- The waiting area for the customers in the system is N, which either limited or unlimited. Hence, the model can be formulated appropriately, by using a system for the investigation introduced; (V│W│X│Y│Z) from chapter 1.

V- Which is the arrival distribution or pattern is Poisson as indicated earlier.
W- The service time distribution is distribution is exponential as indicated earlier
X- The number of available server in the system is two from the assumption above.
Z- This represents the queue discipline which is first come first served
(FIFO).
Hence, with the above assumptions and approach the formulated model is

(M│M│2│N│FCFS) by Kendall’s notation.

3.4 SOLUTION OF THE DEFINED PROBLEM

There is no unique model for single channel; the particular approach one adopts depends on some factors. These factors include the purpose of the solution exercise, the information available about the system and the tools for the work on the system. For this investigation, analytical method of solution is adopted.

3.4.1 ALGORITHM FOR SOLVING THE PROBLEM

1. Considering the system characteristics.
2. Collection of data based on.
   - Arrival distribution.
   - Service time distribution.
   - The time arrival of each customer.
3. The time service commerce for each customer leaves the system using the M│M│S model since the system and a single queue Estimating the parameter λ and μ fr.
4. Testing of the data distributions for statistical conformity to the assumed theoretical probability distribution using the chi-square tests for goodness for fit.
5. Analytical solution to the system model using the values of the parameters λ, μ and N.

3.5 METHOD OF DATA ANALYSIS

First we use the chi-square distribution to study the pattern and reactions to change of the system i.e. identification of customers, server and queue characteristics that are apparent in the system. After which we analyze the data collected from the bank based on

- The time arrival of each customer.

- The time in which the service commences before each customer leaves the system using the M│M│S model since the system and a single queue.

3.5.1 CHI SQUARE DISTRIBUTION

This is a unique statistical test designed to investigate the agreement of a set of observed frequency and expected frequency on the assumption of a theoretical model for the phenomenon being studied. The test is used for investigating dependency or independency of two attributes of classification.

3.5.2 M/M/S MODEL

In this model, it shows that If $_{\mu}{}^{1}$is the mean service time for one server to handle one customer, then the mean rate of service completion when there are customers in the system is the probability of zero customers in the system (P) and the probability of n Customer in the system (P) are given by:

1. The time service commences for each customer in the system.
2. The time the customer leaves the system.

3.6. METHOD OF DATA COLLECTION

The data was collected primarily by direct observation at the bank. Thus, the researcher recorded the following events as it happened in the system using a wrist watch

1. The time of arrival of each customer.
2. The time service commences for each customer in the system.
3. The time the customer leaves the system.

These events were observed at the withdrawal station of the banking hall. A form was designed for this exercise and the above required information was recorded in the form.

One week of three working days was spent to collect relevant data.

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CHAPTER FOUR

DATA ANALYSIS AND RESULTS

4.1 MATHEMATICAL INSTRUMENTS EMPLOYED IN QUEUING THEORY

4.1.1 POISSON DISTRIBUTION

In a Poisson process, the probability of occurrence of an event is independent of what has happened immediately preceding the present observation. We may be interested in what happens over a continuous interval. A random variable X is said to be Poisson distributed if its probability density functions, pdf can be written as

k –λ P(x=k) = $^{\lambda e}{}_{k!}$k = 0, 1, 2…

Where λ is the parameter of the dist variance) is a constant and K is any non-negative integer. The essence of this distribution is that the arrival and services rate are non-negative integer though with infinite point but has a discrete distribution hence we employed the use of Poisson distribution if we define x (s, s+t), as the number of occurrence in the interval (s, s+t), then the following is true,

k –λt

P {x (s, s+t)(λt)e

=k} = k! , k = 0, 1..,

regardless of the location of this interval.

4.1.2 EXPONENETIAL DISTRIBUTION

We have discussed the Poisson probability distribution which gives the probability of the number of occurrence of an event, gives an in certain time period T. for the same Poisson process, we could ask about the waiting time between successive event words, what is the probability distribution of time, t, between events? This probability distribution is called exponential distribution.

The variable T is said to have an exponential distribution with parameter if the probability density function (pdf) of T is given by

fT (t) ={ $\lambda e^{-\lambda t}$, t>0

0 & elsewhere

The cumulative density function, cdf, is given by

∞ ()dt=1$^{-\lambda}$

P (T< t) = ∫ ft -e 0 t

∞ ()dt=e$^{-\lambda}$ P (T > t) =∫ ft 0 t

Thus, the expected value of T, i.e. the expectation of T and its variance are respectively given:

$E(T) = \frac{1}{\lambda}$

$Var(T) = \frac{1}{\lambda^2}$

4.1.3 STOCHASTIC PROCESS

If we represent the number of customers in a system up to time t by X. Then X is a random variable assuming the values of 0,1,2,3.. . A stochastic process is an Indexed collection of random variable X for tET, all having the same T associated probability space. If the parameter set T is countable, then the stochastic process is said to be discrete time process. If T is an open or closed interval of the real time, then the process is called continuous time stochastic process can also be defined as system that change in accordance with probability laws.

4.1.4 MARKOV PROCESS

A Markov process is a stochastic process with the property that given the value of X, the probability of X , where is independent of t t+s X ,μ<t. That is, the conditional distribution of the future event X given the μ t+s present X and the

past X , μ< t is independent of the past. More formally, the t μ process if {X, μ<t } tET is said to be a Markov process.

4.1.5 MARKOVIAN BIRTH AND DEATH PROCESS

Birth and death process is a special class of Markov process. It is a time process in which the defining condition is that state transition which takes place between neighboring states only. A birth and death process is one that is appropriate for modeling changes in the size of a population. A queuing system is a birth and death process because a birth occurs when a customer arrives for service and the death occurs when a customer leaves after service. Thus, most queuing models, especially the single ones can be analyzed as birth and death process. In the context of queuing theory;

- Birth is the arrival or entrance of new customer into the queuing system.
- Death is the departure of the served customer.
- The state of the system at time t, (t > 0) is given by N (t).

Thus, the birth and death process describes probabilistically how N (t) changes as t changes.

Generally speaking, the birth and death process states that individual birth and death occurs randomly, where their mean occurrence rates depend only upon the current state of the system.

- λ, the expected number of arrivals into the system where N-customers n, are already in the system.
- μ, the expected number of customers completing service in a unit time n, when N customers are in the system (service rate).

4.1.6 QUEUING MODELS

We will discuss only the single queue models which are:

- One server (Queue with M|M|I model)
- Multiplier servers (Queue with M|M|).

4.1.7 ONE SERVER (QUEUE WITH M|M|I MODEL)

An M|M|I system is a queuing system arrival time, with parameter λ; exponential parameter μ, one server, no limits on the system capacity and a queue discipline of first come first served. The constant while the constant, μ is the average service rate of customers. Both are in units of customers per time. The expected inter-arrival time and the expected time to

serve one customer are $\frac{1}{\Lambda}$ and $\frac{1}{\mu}$ respectively. M|M|I s and referred to as system with single-server, infinite capacity, queuing systems having Poisson input and exponential service times.

4.1.8 MULTIPLE SERVERS (QUEUE WITH M|M|S)

An M|M|S system is a queuing process independent, identically, distributed, exponential service times (which does not depend on the state of the system) with S servers; infinite capacity and first come first serve (FCFS) queue discipline. The arrival pattern being stated independent, λ= λ for all n. the service server are timed and also independent, but since the number of servers that actually attend to customers (i.e. are not idle) does not depend on the number of customers in the system, the effective time it takes the system to process customers through the service facility is state dependent.

M|M|S system are often referred to as queue, infinite capacity queuing systems having Poisson input and exponential service times.

4.1.9 DATA ANALYSIS AND RESULTS

The data under study was collected from United Bank for Africa. Appendix II is the table showing the number of arrivals taken every ten minutes.

We now use the χ (chi^2 square) test of goodness of fit to test the hypothesis that

H_0 = arrival distribution is Poisson

H_1 = arrival distribution is not Poisson

Let n = number of arrivals in 10 mins

F_n= frequency of number of arrivals

P_n= probability of number of arrivals.

TABLE 4.1.9

N	F	Nfn	Pn	en
0	6	0	0.1178	8.4816
1	14	14	0.2519	18.1368
2	26	52	0.2694	19.3968
3	18	54	0.1921	13.8312
4	6	24	0.1027	7.3944
5	2	10	0.0439	3.1608
TOTAL	**72**	**154**		

Source: arrival pattern in UBA Bank

Since $\chi 2$ test is used, we experience when an approximate theoretical investigations indicate that the approximation is usually satisfactory, provided that e ≥ 5. If the expected frequency of a cell does not exceed five, this cell should be combined with one or more other cells until the above condition is satisfied.

TABLE 4.2

N	0	1	2	3	4 or5
f_n	6	14	26	18	8
e_n	8.4816	18.1368	19.3968	13.8312	10.5552

2 $^{\Sigma}$(fe) χ^2 cal = $^{n-\ n}$ = 5.7927 e n

To find^2 we check the distribution χ table

Degree of freedom = number of observed value – number of estimated parameter - 1 = n-1-1.

Number of observed from table 4.2 is 5 and the number of estimated parameter is 1. Hence the degree of freedom = 5-1-1 =3

Taking =5%, from ^{2}distributionheχ table,^{2} for 3df is 5cri

$\chi^2_{tab} = \chi^2_{3,0.95}$=7.8147.

Clearly$^2_{cal}$>^{2} $_{tab}\chi\chi$ Hence, the null hypothesis is accepted and thus there is no reason on the basis of this test for doubting that queuing model can be applied to this data. This also implies that the arrival pattern follows a Poisson distribution.

Appendix 1is the table showing the time of arrival, the time service begins, the time service ends and the service time.

We use^2 (chi-square)χ test of goodness of fit to test the hypothesis that:

H_0 = service time distribution is exponential

H_1= service time distribution is not exponential

Let n = service time in minutes

F_n= Frequency of the service times

P_n = probability of service time

TABLE 4.3

T	f_n	P_n	e_n
$0 \leq T<6$	95	0.5985	92.169
$6\leq T<12$	47	0.2403	37.0062
$12\leq T<18$	7	0.0965	14.861
$18\leq T<24$	5	0.0387	5.9598

$e_n = {}_n\sum f(P_n) = 154P_n \; P_n = \mu. \;{}^{\mu-}$

system capacity

Where $\mu =$ $\overline{\text{time taken to be served}}$ $= \frac{154}{1012} = 0.1521$

^{b} μ-T $P(a<b)\leq T = \int \mu e.$ dT a

$= e^{-\mu b} + e^{-\mu a}$

${}_{-\mu a}$ $-\mu b = e - e$

$-$ .1521(0) -0.1521(6)

$P(0\leq T<6)e \quad = -e \quad = 0.5985$

$P(6\leq T<12)e^{-0.1521(6)} - e = {}^{-0.1521(12)} = 0.2384$

$P(12 \leq T<18)=e^{-0.1521(12)}-e^{-0.1521(18)}=0.0965$

$P(18 \leq T<24)=e^{-0.1521(18)}-e^{-0.1521(24)}=0.0387$

As stated, x^2 is earlier used only when χ the approximation is good. Approximation is usually satisfactory provided that the e≥5.

i

Hence, table 4.3 above now yields

TABLE 4.4

T	0≤T<6	6≤T<12	12≤T<1	18≤T<3
f_n	95	47	7	5
e_n	92.169	37.0062	14.861	5.9598

χ^2_{cal} = i- i = 5.8983ei

to find $x^2_{tab,}$we check the chi χ-square distribution table.

Degree of freedom = 4-1-1=2

Taken α = 5%,distributionfromtable, 5%thecritical χvalue of χ for 2df is

2 2 $_{2,}\chi_{0.95}$ = 5.99

χtab =

Clearly, $^2_{cal} < ^2_{tab}\chi$ Thereforeχ the null hypothesis is accepted and this implies that the service Pattern follows an exponential distribution.

4.5 PARAMETERS ESTIMATION

From Appendix 1 (i.e. the data collected at the bank) The system capacity, N =154 customers Inter-arrival time for 154 customers, T =714 minutes

Time taken by 154 customers to be served, S=1012 minutes Now the

$$\text{Arrival rate, } = \frac{N}{T} = \frac{154}{714} = 0.2157$$

$$\text{Service rate, } \mu = \frac{N}{S} = \frac{154}{1012} = 0.1521$$

λ0.2157Traffic in=tensity=1.4181(ℓ) = μ 0.1521

This implies that μ = 0.1521

= 0.2157 C = 2

$\frac{\lambda}{\mu}$ = 1.4181

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATION

5.1 SUMMARY

Considering the analytical solution, the capacity of the system under study is 154 customers and the arrival rate is 0.2157 while the service rate is 0.1521. This shows that the arrival rate of the system is greater than the service rate, this imply that customers have to queue up, though the queue will not be long. Probability that the servers are idle is 0.29 which shows that the servers will be 29% idle and 71% busy.

The expected number in the waiting line is 2.3465. The expected number in the system is 3.7646. The expected waiting time in the queue is 10.8785 and the expected total time lost waiting in one day is 18.7719 hours.

From the foregoing Queue measures, the average cost per day for waiting is ₦938.597 and from the calculation of the comparing solutions, the average cost per day from waiting is ₦74.72. There had been a saving in the expected cost of ₦938.597 - ₦74.72 = ₦863.877

5.2 CONCLUSION

We now conclude that adding one more server will help reduce the time customers spend on queue and as well help to reduce the cost incurred from waiting. Hence the objective of this project is achieved.

The advantage of using this single system with multiple servers is that a slow server does not affect the movement of the queue i.e if a server is slow it does not affect the movement of the queue because next customer can go to the next available server instead of waiting for the slow server.

5.3 RECOMMENDATION

We recommend that the management of United Bank of Africa should increase the number of the servers at the withdrawal point so as to reduce the time customers have to wait to receive services, thereby minimizing the cost incurred from waiting.

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APPENDIX 1

DAY 1

	Arrival time	Service begin	Service ends	Service time
1.	10:00am	10:00 am	10:07 am	7:00
2.	10:02am	10:02 am	10:7 am	5:00
3.	10:03am	10:07 am	10:11 am	3:00
4.	10:08am	10:08 am	10:13 am	5:00
5.	10:13am	10:13 am	10:25 am	12:00
6.	10:20am	10:20 am	10:25 am	5:00
7.	10:23am	10:25 am	10:30 am	5:00
8.	10:26am	10:26 am	10:31 am	5:00
9.	10:28am	10:30 am	10:38 am	8:00
10.	10:32am	10:32 am	10:40 am	8:00
11.	10:37am	10:38 am	10:43 am	5:00
12.	10:46am	10:46 am	10:51 am	5:00
13.	10:52am	10:52 am	11:05 am	13:00
14.	10:58am	10:58 am	11:07 am	9:00
15.	11:05am	11:05 am	11:10 am	5:00
16.	11:07am	11:07 am	11:16 am	9:00
17.	11:12am	11:13 am	11:17 am	4:00
18.	11:15am	11:16 am	11:21 am	5:00
19.	11:32am	11:32 am	11:37 am	5:00
20.	11:40am	11:40 am	11:45 am	5:00
21.	11:41am	11:41 am	11:59 am	18:00
22.	11:50am	11:50 am	11:55 am	5:00
23.	11:51am	11:55 am	12:00 pm	5:00

24.	12:00pm	12:00 pm	12:12 pm	12:00
25.	12:07pm	12:07 pm	12:12 pm	5:00
26.	12:12pm	12:12 pm	12:22 pm	10:00
27.	12:13pm	12:13 pm	12:25 pm	12:00
28.	12:14pm	12:22 pm	12:27 pm	5:00
29.	12:30pm	12:30 pm	12:35 pm	5:00
30.	12:38pm	12:38 pm	12:43 pm	5:00
31.	12:42pm	12:42 pm	12:47 pm	5:00
32.	12:43pm	12:43 pm	12:50 pm	7:00
33.	12:45pm	12:48 pm	12:53 pm	5:00
34.	12:47pm	12:50 pm	12:57 pm	7:00
35.	12:50pm	12:53 pm	1:00 pm	7:00
36.	12:52pm	12:57 pm	1:02 pm	5:00
37.	12:59pm	1:00 pm	1:05 pm	4:00
38.	1:04pm	1:04 pm	1:08 pm	3:00
39.	1:05pm	1: 07 pm	1:10 pm	5:00
40.	1:06pm	1:08 pm	1:13 pm	8:00
41.	1:07pm	1:10 pm	1:18 pm	5:00
42.	1:10pm	1:13 pm	1:18 pm	14:00
43.	1:21pm	1:21 pm	1:35 pm	5:00
44.	1:27pm	1:28 pm	1:33 pm	5:00

DAY 2

	Arrival time	Service time	Service end	Service time(mins)
1.	10:05am	10:05am	10:11am	6:00
2.	10:07am	10:07am	10:12am	5:00
3.	10:10am	10:11am	10:16am	5:00
4.	10:20am	10:20am	10:29am	9:00
5.	10:23am	10:23am	10:30am	7:00
6.	10:27am	10:29am	10:34am	5:00
7.	10:33am	10:33am	10:37am	4:00
8.	10:36am	10:36am	10:41am	5:00
9.	10:42am	10:42am	10:47am	5:00
10.	10:50am	10:50am	11:01am	11:00
11.	10:57am	10:57am	11:02am	5:00
12.	11:01am	11:01am	11:06am	5:00
13.	11:03am	11:05am	11:10am	5:00
14.	11:05am	11:06am	11:14am	8:00
15.	11:24am	11:24am	11:28am	4:00
16.	11:26am	11:28am	11:40am	12:00
17.	11:40am	11:40am	11:45am	5:00
18.	11:47am	11:47am	11:52am	5:00
19.	11:52am	11:52am	11:56am	4:00
20.	12:02am	12:02am	12:10pm	8:00
21.	12:07am	12:07am	12:12am	5:00
22.	12:08am	12:10am	12:15am	5:00
23.	12:08am	12:12am	12:17am	5:00
24.	12:22am	12:22am	12:27am	5:00

25.	12:27am	12:27am	12:33pm	6:00
26.	12:35pm	12:35pm	12:40pm	5:00
27.	12:43pm	12:43pm	12:47pm	4:00
28.	12:50pm	2:50pm	12:55pm	5:00
29.	12:54pm	12:54pm	12:59pm	5:00
30.	12:56pm	12:56pm	1:04pm	9:00
31.	12:59pm	12:59pm	1:04pm	5:00
32.	1:02pm	1:04pm	1:11pm	9:00
33.	1:03pm	1:04pm	1:09pm	5:00
34.	1:10pm	1:110m	1:16pm	5:00
35.	1:13pm	1:13pm	1:23pm	9:00
36.	1:16pm	1:16pm	1:20pm	4:00
37.	1:20pm	1:23pm	1:30pm	7:00
38.	1:25pm	1:25pm	1:43pm	18:00
39.	1:27pm	1:30pm	1:35pm	5:00
40.	1:34pm	1:40pm	1:45pm	5:00
41.	1:41pm	1:41pm	1:50pm	9:00
42.	1:49pm	1:49pm	1:54pm	5:00
43.	1:54pm	1:54pm	1:59pm	5:00

DAY 3

Arrival time	Service begin	Service ends	Service time
1. 10:00am	10:00 am	10:05 am	5:00
2. 10:02am	10:02 am	10:11 am	9:00
3. 10:03am	10:05 am	10:10 am	5:00
4. 10:10am	10:11 am	10:16 am	5:00
5. 10:14am	10:14 am	10:20 am	6:00
6. 10:17am	10:17 am	10:25 am	8:00
7. 10:18am	10:20 am	10:25 am	5:00
8. 10:20am	10:25 am	10:33 am	8:00
9. 10:26am	10:26 am	10:33 am	7:00
10. 10:29am	10:33 am	10:38 am	5:00
11. 10:32am	10:35 am	10:40 am	5:00
12. 10:36am	10:38 am	10:46 am	8:00
13. 10:38am	10:40 am	10:45 am	5:00
14. 10:42am	10:46 am	10:51 am	5:00
15. 10:49am	10:49 am	10:54 am	5:00
16. 10:57am	10:57 am	11:02 am	5:00
17. 11:00am	11:00 am	11:07 am	7:00
18. 11:00am	11:02 am	11:07 am	5:00
19. 11:04am	11:07 am	11:12 am	5:00
20. 11:10am	11:10 am	11:18 am	8:00
21. 11:22am	11:22 am	11:28 am	6:00
22. 11:25am	11:25 am	11:30 am	5:00
23. 11:26am	11:28 am	11:32 am	4:00
24. 11:33am	11:33 am	11:38 am	5:00
25. 11:35am	11:35 am	11:47 am	12:00

26.11:37am	11:41 am	11:46 am	5:00
27.11:40am	11:43 am	11:50 am	7:00
28.11:44am	11:47 am	11:52 am	5:00
29.11:45am	11:50 am	11:54 am	4:00
30.11:46am	11:55 am	12:00 pm	5:00
33.11:58am	12:03 pm	12:09 pm	6:00
34.12:01pm	12:08 pm	12:15 pm	7:00
35.12:05pm	12:09 pm	12:14 pm	5:00
36.12:10pm	12:15 pm	12:21 pm	6:00
37.12:14pm	12:20 pm	12:25 pm	5:00
38.12:16pm	12:21 pm	12:30 pm	9:00
39.12:20pm	12:25 pm	12:30 pm	5:00
40.12:31pm	12:31 pm	12:49 pm	18:00
41.12:36pm	12:40 pm	12:45 pm	5:00
42.12:40pm	12:45 pm	12:55 pm	10:00
43.12:44pm	12:49 pm	12:54 pm	5:00
44.12:49pm	12:54 pm	1:04 pm	5:00
45.12:54pm	12:55 pm	1:12 pm	18:00
46.1:02pm	1:04 pm	1:12 pm	8:00
31.11:52am	11:56 am	12:01 pm	5:00
32.11:56am	12:00 pm	12:08 pm	8:00