A SIMULATION FOR ENERGY DISSIPATION IN PHYSICAL SYSTEMS

BY

YAKUBU HUNIRA NGADDA

SEPTEMBER, 1988

A SIMULATION FOR ENERGY DISSIPATION IN PHYSICAL SYSTEMS

BY

YAKUBU HUNIRA NGADDA

Department of Physics,
Ahmadu Bello University,
Zaria.

A THESIS SUBMITTED TO THE POST-GRADUATE SCHOOL
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR
THE AWARD OF M.Sc. (THEORETICAL PHYSICS).

September, 1988

DECLARATION

I hereby declare that this thesis has been written by myself and that it is a record of my own research work at Ahmadu Bello University, Zaria. It has not been accepted in any previous application for a higher diploma or degree.

All sources of information have been specifically acknowledged in the reference section.

Y.H. Ngadda

CERTIFICATION

This project thesis entitled *A SIMULATION FOR ENERGY DISSIPATION IN PHYSICAL SYSTEMS* by Yakubu Hunira Ngadda meets the regulations governing the award of the degree of M.Sc. (Theoretical Physics) of Ahmadu Bello University and is approved for its contribution to knowledge and literary presentation.

Prof. E.D. Mshelia
Chairman, Supervisory Committee

24/1/89
Date

Dr. I.V.S. Rathore
Member, Supervisory Committee

Date

Dr. J. Adetunji
Head of Department of Physics

Date

Dean, Postgraduate School

Date

ABSTRACT

A formalism for energy dissipation in physical systems using coupled oscillators is hereby presented. The model demonstrates energy transfer from a free translational motion of a centre of mass into intrinsic modes represented by four oscillators of the same force constant. First, from the classical energy of the system, the kinetic and potential energy matrices are deduced. These matrices are used to transform the natural modes of vibration into normal coordinates which decouple the vibration into the various modes with particular frequencies. This transformation has been done through the application of Lagrangian dynamics. A classically decoupled total Hamiltonian obtained is written in terms of its quantum mechanical equivalence. The time-independent Schrödinger's equation applied to the decoupled Hamiltonian of the system quantizes it, giving the usual quantum mechanical eigenvalues and eigenfunctions for both intrinsic and translational motions.

Collective amplitudes of the motion, or the dissipation functions of the internal excitation energy, obtained as a multiple integral in terms of the translational and intrinsic wave functions is solved analytically exactly.

Numerical values of the normalized collective amplitudes have been computed and examples of probability distributions for the intrinsic excitation are presented graphically.

DEDICATION

To my wife
Mrs. Zharakahyel Y. Ngadda
for her forbearance, impressionable
loyalty, incredible capability for hard work
and a good heart.

ACKNOWLEDGEMENTS

I am indebted to Professor E.D. Mshelia of the Ahmadu Bello University, Bauchi Campus, for the motivation and supervision of this dissertation. The profundity of his knowledge and experience in the fields of Theoretical Nuclear Physics and Mathematics led this work through to a success.

My special gratitude goes to Dr. S.B. Elegba of the Centre for Energy Research and Training, Ahmadu Bello University, Zaria, for having spared his precious time to read through the manuscript. My regards extend to my course supervisor, Dr. I.V.S. Rathore of the Physics Department, Ahmadu Bello University, Zaria, for his willingness and concern in acting as a link between the two campuses of this university, in connection with this project.

I seize this opportunity to acknowledge Dr. J. Adetunji, the Head of Physics, Ahmadu Bello University, Zaria, for his useful advice and outstanding co-operation in connection with this undertaking.

It is a necessity to acknowledge the entire staff of the Computer Centre, A.B.U. Bauchi Campus, for their splendid co-operation and assistance in the computing analysis. I am particularly indebted to Mr. R.O. Anumba for the time-sharing effort in perfecting the computer program.

Lastly, but not the least, I will not forget Mr. D.Y. Kutchin for his advisory criticisms.

TABLE OF CONTENTS

Page

CHAPTER ONE

CHAPTER TWO

CHAPTER THREE

CHAPTER FOUR

CHAPTER FIVE

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

A nuclear phenomenon, not yet well understood, which plays a very important role in the field of fusion and fission dynamics is the dissipation of energy or friction. By dissipation we mean the conversion of the kinetic energy of relative motion into other degrees of freedom, for example intrinsic excitations (Mshelia et al, 1975, 1981). During the historical development of quantum mechanics there has been little interest in a formalism for systems with dissipations. This is because most quantum mechanical analyses deal with microscopic phenomena, while dissipation is a macroscopic concept. However, there has been some discussion of dissipation in quantum mechanics in general, and with reference to the harmonic oscillator in particular. One method (Kanai, 1948; Kerner, 1958; Stevens, 1958) starts with the classical equations of motion for a system with dissipation due to velocity-dependent force, finding a Lagrangian which leads to these equations of motion, and then quantizing it by conventional formal methods. This method has been shown to be unrealistic (Senitzky, 1960) and cannot describe a true physical system with quantum mechanical properties because it leads to the violation of Heisenberg's uncertainty principles. The other method consists of considering the dissipation as being due to the coupling of two systems, the undamped harmonic oscillator and the system which produces the damping. Similar systems considered in classical physics are the following: when dealing with dissipation in an electrical system specified by a value for the resistance, or dissipation in a mechanical system specified by a value of the coefficient of friction, one is really concerned with the interaction of two types of systems, one is the harmonic oscillator which one wants to describe completely, and the other a complicated system - the loss mehcanism - which one can describe approximately

Some attempts on the treatment of dissipation for nuclear systems have shown that the energy of collective degrees of freedom is converted into intrinsic excitation energy (Swiatecki et al, 1972). For example, in deep inelastic reactions, the heavy ions lose their total kinetic energy and are repelled only by their Coulomb interaction, indicating that the kinetic energy of relative motion has been dissipated into intrinsic degrees of freedom. Many authors have used frictional forces in order to describe the kinetic energy loss theoretically (Bass, 1980). Beck and Gross, 1973 have, in a semi-classical treatment given a relationship between the frictional force, acting between two ions moving on classical orbits, and the imaginary optical-model potential. More recently Glas and Mosel, 1974 have numerically demonstrated the effects of friction in heavy-ion reactions. By using the Landau-Zener approximation, these authors calculated the probabilities for inelastic processes and fusion for a head-on collision of the $^{16}O + {}^{16}O$ system. This work is promising because it has the following features: the inelastic excitations occur at the rearrangement point of nuclear shells, which varies with the mass of the nuclei. This radius varies as $R_o = r_o A^{1/3}$ with $r_o = 0.9$ fm according to the two-centre shell model (Glas and Mosel, 1974). Also various studies have been made with respect to a microscopic and collective description of nuclear-energy dissipation (Hasse, 1978; Gross and Kalinowski, 1978). The various methods of approach include time-dependent perturbation expansions (Beck and Gross, 1973; Hasse, 1979), linear response theory (Hofmann and Siemens, 1976, 1977), quantal master equations, Fokker-Planck equations random-matrix theory for complex nuclear reactions and time-dependent reaction theories (Mshelia et al, 1984).

A number of complicated oscillatory systems can be analysed in terms of simple ones by converting to normal modes of vibration whenever the interparticle forces are linear functions of the relative displacements. These normal modes are equivalent to independent harmonic oscillators. The one-dimensional harmonic oscillator is important for the quantum mechanical treatment of the vibrations of atoms in molecules and crystals or nuclear vibrations. It is also important in the study of inelastic reactions between heavy ions where the energy of the collective degree of freedom is converted into intrinsic excitation energy (Mshelia et al, 1975).

In this work we present a model for energy dissipation based on quantum mechanics, with some classical background theory. Dissipation is considered as due to the coupling of collective degrees of freedom to intrinsic excitations. A simplified model of four oscillators coupled to a degree of freedom of free motion, which can be solved exactly by analytical methods, is hereby described. This is an extension of a model developed by Mshelia et al, 1981. This simple model can be extended to a more realistic but complex case of heavy ion collision. The four oscillators represent intrinsic degrees of freedom relative to a fifth particle whose motion is translational only The model is simple enough and has been solved exactly by analytical methods.

In Chapter Two a description of the model of four oscillators coupled to a free motion is made using classical mechanics. Normal coordinates are obtained for a linear vibration of five particles. These coordinates decouple the vibrations into different modes of motion. A decoupled Hamiltonian is thus obtained. In Chapter Three we obtain the quantum mechanical equivalence of the total Hamiltonian being split into two parts: collective and intrinsic Hamiltonians. The time-independent Schrödinger's equation is applied to these Hamiltonians and the quantum mechanical eigenvalues and eigen-functions obtained. These turn out to be the usual harmonic oscillator eigenvalues and eigenfunctions. The dissipation function of the internal excitation energy is calculated by expanding the total wave function in the orthonormal set of intrinsic wave functions. This dissipation function or the collective amplitude is solved analytically exactly.

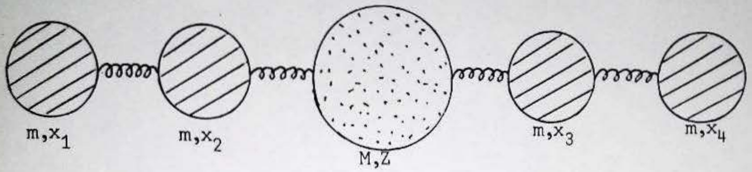
In Chapter Four the result obtained in Chapter Three is appl to the model. We computed some numerical values of the collective amplitudes and presented graphical examples of probability distributions for the intrinsic excitation. Finally in Chapter Five conclusions and further work on the model are proposed.

CHAPTER TWO

2. THE MODEL

2.1 THE CLASSICAL PROBLEM

2.1.1 Linear Vibration of Five Particles



The model consists of five particles on a straight line and only vibrations along the straight line joining the particles are considered. The potential energies of the particles are approximated by assuming four springs of the same force constant c joining the four particles of the same mass, m, to the central particle of mass **M.** The coordinate Z denotes the position of the particle of mass M relative to which the potential energy of the system is determined.

The kinetic and potential energies are thus given by:

$$T = \tfrac{1}{2}m\dot{x}_1^2 + \tfrac{1}{2}m\dot{x}_2^2 + \tfrac{1}{2}m\dot{x}_3^2 + \tfrac{1}{2}m\dot{x}_4^2 + \tfrac{1}{2}M\dot{Z}^2 \qquad \text{-----} \quad (1)$$

where $\dot{x}_1$, i = 1, 2, 3, 4, and $\dot{Z}$ are velocities.

$$V = \tfrac{1}{2}c[(x_1-Z)^2 + (x_2-Z)^2 + (x_3-Z)^2 + (x_4-Z)^2]$$

$$= \tfrac{1}{2}c[x_1^2+x_2^2+x_3^2+x_4^2+4Z^2 - 2(x_1+x_2+x_3+x_4)Z] \qquad \text{-----} \quad (2)$$

Expressed in matrix form,

$$T = \tfrac{1}{2} \left\{ \begin{matrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & M \end{matrix} \right\} \left\{ \begin{matrix} \dot{x}_1^2 \\ \dot{x}_2^2 \\ \dot{x}_3^2 \\ \dot{x}_4^2 \\ \dot{Z}^2 \end{matrix} \right\} \qquad \text{-----} \quad (3)$$

$$V = \tfrac{1}{2}c \begin{vmatrix} x_1^2 & 0 & 0 & 0 & -x_1Z \\ 0 & x_2^2 & 0 & 0 & -x_2Z \\ 0 & 0 & x_3^2 & 0 & -x_3Z \\ 0 & 0 & 0 & x_4^2 & -x_4Z \\ -x_1Z & -x_2Z & -x_3Z & -x_4Z & 4Z^2 \end{vmatrix}$$

$$= \tfrac{1}{2}(x_1x_2x_3x_4Z) \begin{vmatrix} c & 0 & 0 & 0 & -c \\ 0 & c & 0 & 0 & -c \\ 0 & 0 & c & 0 & -c \\ 0 & 0 & 0 & c & -c \\ -c & -c & -c & -c & 4c \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ Z \end{vmatrix} \qquad \text{-----} \quad (4)$$

We denote the kinetic and potential energy matrices by

$$[T_{ij}] = \begin{vmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & M \end{vmatrix} \qquad \text{-----} \quad (5)$$

and $$[V_{ij}] = \begin{vmatrix} c & 0 & 0 & 0 & -c \\ 0 & c & 0 & 0 & -c \\ 0 & 0 & c & 0 & -c \\ 0 & 0 & 0 & c & -c \\ -c & -c & -c & -c & 4c \end{vmatrix} \qquad \text{------} \quad (6)$$

These matrices are obtained directly by inspection of equations (1) and (2) or on application of the Lagrangian equation of classical mechanics,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0 \qquad \text{-----} \quad (7)$$

where $L = T - V$ and $\dot{q}$ is the first derivative with respect to time of the generalized coordinate, q (Goldstein, 1978).

We see that the kinetic energy matrix is diagonal while the potential energy matrix is nondiagonal due to the products x_1Z, x_2Z, x_3Z, and x_4Z. These off-diagonal terms give rise to the coupling of the collective and intrinsic motions. In the theory of vibrations it is possible to find a transformation to normal coordinates. These coordinates are uncoupled, thereby making the problem completely separable into independent motions, each with a particular normal frequency. We present this in the foregoing sections.

2.1.2 The Eigenfrequencies

By considering the case of small oscillations about positions of stable equilibrium and applying the Lagrangian formalism one obtains the eigenvalue equations from which the eigenfrequencies are calculated. These eigenvalue equations are given by

$$\sum_j (V_{ij} - \omega^2 T_{ij}) A_j = 0 \qquad \text{-----} \quad (8)$$

where V_{ij} and T_{ij} are the potential and kinetic energy matrix elements respectively. The direction of the line along which the oscillations take place is the direction of the vector $\vec{A}$ whose scalar components are A_j. Assuming the non-trivial case, where $A_j \neq 0$, the $f \times f$ matrix

$$[V_{ij}] - \omega^2 [T_{ij}]$$

is singular so that the secular determinant

$$\det([V_{ij}] - \omega^2 [T_{ij}]) = 0 \qquad \text{-----} \quad (9)$$

gives f values of ω^2. Thus the roots of the determinant provide the modal frequencies ω_i of the vibrations. We label a modal vector associated with the frequency ω_i as $\vec{A}_i$.

With $[T_{ij}]$ and $[V_{ij}]$ given by equations (5) and (6), the modal frequencies ω_i are written as:

$$\left.\begin{aligned} \omega_1^2 &= \frac{c}{mM}(4m + M) = c/\mu \\ \omega_2^2 &= c/m \\ \omega_3^2 &= c/m \\ \omega_4^2 &= c/m \\ \omega_5^2 &= 0 \end{aligned}\right\} \qquad \text{-----} \qquad (10)$$

where $\mu = mM/(4m + M)$.

Thus the eigenmode corresponding to the eigenfrequency ω_1 describes the motion in which all the five particles vibrate about their common equilibrium configuration. The eigenmodes corresponding to the eigenfrequencies ω_2, ω_3 and ω_4 are degenerate. The eigenfrequency ω_5 vanishes. This implies that its corresponding eigenmode describes a uniform translational motion of the system as a whole. This corresponds to the Z-coordinate.

2.1.3 Natural Modes of Motion and Normal Coordinates

We proceed to find a transformation to normal coordinates bearing in mind that we are dealing with a system for which the energy is small enough to ensure that the system does not depart appreciably from the equilibrium configuration.

A Lagrangian system is said to be harmonic with respect to the set of coordinates q_j if it is of the form

$$L = \tfrac{1}{2} \sum_{ij} T_{ij}\, \dot{q}_i\, \dot{q}_j - \tfrac{1}{2} \sum_{ij} V_{ij}\, q_i\, q_j \qquad \text{-----} \qquad (11)$$

where the matrices $[T_{ij}]$ and $[V_{ij}]$ are real, constant, **symmetric and positive definite** (Pars, 1965 and Desloge, 1982). The results of motion of harmonic systems can be summarized as follows:

Consider a Lagrangian system of f degrees of freedom that is harmonic with respect to the set of variables $\vec{q}_i$. The system whose Lagrangian is given by equation (11) has the following properties:

(a) There is a set of possible modes of motion in which the point in configuration space $\vec{q}$ representing the instantaneous configuration of the system oscillates in simple harmonic motion about the origin along a straight line passing through the origin. These modes of motion are called natural modes of motion. The directions in $\vec{q}$-space of the straight lines along which such motion is possible are called modal directions. A vector $\vec{A}$ whose direction is a modal direction is called a modal vector. In a natural mode of motion all the particles in the system are oscillating with the same frequency and are either in phase or 180° out of phase with one another.

(b) Associated with each modal direction there is a single angular frequency of oscillation ω_i, called a natural or modal frequency. The value of a particular modal frequency must be one of the f positive roots of equation (9).

(c) If any of the roots of equation (9) is substituted into equation (8) then any vector $\vec{A}$ whose components, A_j, satisfy the resulting set of equations will be a modal vector, $\vec{A}_i$, with which the given frequency ω_i is associated. If the frequency ω_i is unique, equation (8) will determine a unique one dimensional subspace within which $\vec{A}_i$ must lie; that is, the vectors will all lie along a single direction. If the frequency ω_i is one of a set of l identical frequencies, equation (8) will determine a unique l dimensional subspace within which $\vec{A}_i$ must lie.

(d) From the set of all modal directions it is always possible to choose a set of f linearly independent modal directions. This follows from the fact that a set of modal directions is linearly independent if the corresponding modal vectors are linearly independent.

We can now define normal coordinates as follows: The set of coordinates corresponding to the set of orthonormalized modal vectors $\vec{A}_1, \vec{A}_2, ---, \vec{A}_f$, designated by $g_1, g_2, ---, g_f$ are called normal coorinates (Desloge, 1982). The transformation is given by the equation

$$g_r = \sum_{ij} A_{ri} T_{ij} q_j \qquad ----- \qquad (12)$$

On application of this equation, with proper choice of the modal vectors, the normal coordinates are obtained as:

$$\left.\begin{aligned} g_1 &= \frac{1}{4}(-x_1 - x_2 - x_3 - x_4 + 4Z) \\ g_2 &= x_1 - x_2 \\ g_3 &= x_2 - x_3 \\ g_4 &= x_3 - x_4 \\ g_5 &= \frac{1}{(4m + M)}\{mx_1 + mx_2 + mx_3 + mx_4 + MZ\} \end{aligned}\right\} \qquad ---- \qquad (13)$$

The physical meaning of these coordinates is self-explanatory from equation (13): g_1 is the relative coordinate between Z and the centre of the intrinsic coordinates at $\frac{1}{4}(x_1 + x_2 + x_3 + x_4)$; g_2, g_3 and g_4 are the relative coordinates between the intrinsic coordinates x_1, x_2, x_3 and x_4 ; and g_5 is the centre-of-mass coordinate.

2.1.4 The Decoupled Classical Hamiltonian

By solving simultaneously equations (13) we obtain, in terms of the normal coordinates, x_1, x_2, x_3, x_4 and Z, and when substituted into equations (1) and (2) give the decoupled kinetic and potential energies as:

$$T = \tfrac{1}{2}(\mu_1\dot{g}_1^2 + \mu_2\dot{g}_2^2 + \mu_3\dot{g}_3^2 + \mu_4\dot{g}_4^2 + \mu_5\dot{g}_5^2) \quad \text{-----} \quad (14)$$

where the corresponding masses are given by

$$\left.\begin{aligned} \mu_1 &= \frac{4mM}{4m + M} \\ \mu_2 &= 3/4\ m \\ \mu_3 &= m \\ \mu_4 &= 3/4\ m \\ \mu_5 &= 4m + M \end{aligned}\right\} \quad \text{-----} \quad (15)$$

and

$$V = \tfrac{1}{2}c(\ \mu_1\omega_1^2g_1^2 + \mu_2\omega_2^2g_2^2 + \mu_3\omega_3^2g_3^2 + \mu_4\omega_4^2g_4^2) \quad \text{----} \quad (16)$$

Having obtained the decoupled energies we can now write down the total classical Hamiltonian as

$$\begin{aligned} H &= T + V \\ &= \tfrac{1}{2}(\mu_1\dot{g}_1^2 + \mu_2\dot{g}_2^2 + \mu_3\dot{g}_3^2 + \mu_4\dot{g}_4^2 + \mu_5\dot{g}_5^2) \\ &+ \tfrac{1}{2}(\mu_1\omega_1^2g_1^2 + \mu_2\omega_2^2g_2^2 + \mu_3\omega_3^2g_3^2 + \mu_4\omega_4^2g_4^2) \quad \text{-----} \quad (17) \end{aligned}$$

Thus the complicated motions of the particles have been transformed into an equivalence of uncoupled oscillators, with one free particle, since its motion about the Z-coordinate is translational only with kinetic energy of the centre-of-mass as $\tfrac{1}{2}\mu_5\dot{g}_5^2$ and frequency $\omega_5 = 0$.

The solution of Schrodinger's equation with the quantized Hamiltonian (see next section) is equivalent to that of a collection of independent harmonic oscillators.

CHAPTER THREE

3 THE QUANTUM MECHANICAL APPROACH

In this section, a quantum mechanical description of our model is presented. Here we obtain solutions of Schrodinger's time-indepdent wave equation by incorporating the classical formulae obtained in Chapter Two. A collective wave function obtained as a multiple integral has been solved analytically. This collective wavefunction provides the probability for intrinsic excitation.

3.1 Eigenvalues and Eigenfunctions of the total Hamiltonian

The oscillators described by the coordinates x_1, x_2, x_3 and x_4 represent the intrinsic degrees of freedom and the translational motion described by Z represents the collective degree of freedom. Thus the Hamiltonian is split into collective and intrinsic parts as follows:

$$\hat{H} = \hat{H}_{coll}(Z) + \hat{H}_{int}(x_1, x_2, x_3, x_4, Z) \qquad \text{-----} \quad (18)$$

where the collective and intrinsic Hamilton operators are respectively defined thus:

$$\hat{H}_{coll}(Z) = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial Z^2} \qquad \text{-----} \quad (19)$$

and

$$\hat{H}_{int}(x_1, x_2, x_3, x_4, Z) = -\frac{\hbar}{2m}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} + \frac{\partial^2}{\partial x_4^2}\right) + \frac{1}{2}c[(x_1-Z)^2 + (x_2-Z)^2 + (x_3-Z)^2 + (x_4-Z)^2] \quad \text{---} \quad (20)$$

From the coupled terms of equation (20) energy can be dissipated from the collective degrees of freedom into intrinsic degrees of freedom represented by the four oscillators. With the normal coordinates the total Hamiltonian in equation (18) decouples into its normal modes of vibration. We obtain, in terms of the normal coordinates, the total Hamiltinian as

$$\hat{H} = -\frac{\hbar^2}{2}\left(\frac{1}{\mu_1}\frac{\partial^2}{\partial g_1^2} + \frac{1}{\mu_2}\frac{\partial^2}{\partial g_2^2} + \frac{1}{\mu_3}\frac{\partial^2}{\partial g_3^2} + \frac{1}{\mu_4}\frac{\partial^2}{\partial g_4^2} + \frac{1}{\mu_5}\frac{\partial^2}{\partial g_5^2}\right)$$

$$+ \tfrac{1}{2}(\mu_1\omega_1^2 g_1^2 + \mu_2\omega_2^2 g_2^2 + \mu_3\omega_3^2 g_3^2 + \mu_4\omega_4^2 g_4^2) \quad \text{-----} \quad (21)$$

where μ_i are the equivalent masses of the particles corresponding to the normal coordinates g_i , $i = 1,2,3,4,5$.

We now obtain solutions of the time-independent Schrodinger's equation with the decoupled Hamiltonian, $\hat{H}$, given by equation (21). The eigenvalue equation is given by

$$H\,\psi(g_1, g_2, g_3, g_4, g_5) = E\,\psi(g_1, g_2, g_3, g_4, g_5) \quad \text{---} \quad (22)$$

Since H describes a free translational motion of the centre of mass and decoupled harmonic oscillations in the $g_{\bar{1}}$, $g_{\bar{2}}$, $g_{\bar{3}}$ and $g_{\bar{4}}$ degrees of freedom, the eigenvalues and eigenfunctions are simply obtained as

$$E_{krstv} = \frac{\hbar^2 k^2}{2\mu_5} + \hbar\omega_1(r+\tfrac{1}{2}) + \hbar\omega_2(s+\tfrac{1}{2}) + \hbar\omega_3(t+\tfrac{1}{2})$$

$$+ \hbar\omega_4(v+\tfrac{1}{2}) \quad \text{-----} \quad (23)$$

$$\psi_{krstv}(g_1, g_2, g_3, g_4, g_5) = U_k(g_5)\psi_r(g_1)\psi_s(g_2)\psi_t(g_3)$$

$$.\psi_v(g_4) \quad \text{-----} \quad (24)$$

where $r,s,t,v = 0,1,2,$ --- and k is the wave number for the plane-wave motion. The plane-wave function for the centre of mass, normalized by means of the Dirac δ-function (Schiff, 1968) is given by

$$U_k(g_5) = \frac{1}{\sqrt{2\pi}}\exp(ikg_5) \quad \text{-----} \quad (25)$$

The normalized, bound state, wave functions of the harmonic oscillators are written as

$$\left.\begin{array}{lcl}\psi_r(g_1) & = & N_r\, e^{-\frac{1}{2}\alpha_1^2 g_1^2}.H_r(\alpha_1 g_1) \\ \psi_s(g_2) & = & N_s\, e^{-\frac{1}{2}\alpha_2^2 g_2^2}.H_s(\alpha_2 g_2) \\ \psi_t(g_3) & = & N_t\, e^{-\frac{1}{2}\alpha_3^2 g_3^2}.H_t(\alpha_3 g_3) \\ \psi_v(g_4) & = & N_v\, e^{-\frac{1}{2}\alpha_4^2 g_4^2}.H_v(\alpha_4 g_4)\end{array}\right| \qquad \text{-----}\quad (26)$$

where H_i denotes the Hermite polynomial of order i and the α_j's denote the inverse oscillator lengths given by

$$\alpha_j = (\mu_j\omega_j/\hbar)^{\frac{1}{2}} \qquad \text{-----}\quad (27)$$

$j = 1,2,3,4$. The normalization constants are given as

$$N_i = \left(\frac{\alpha_j}{\pi^{\frac{1}{2}}2^i i!}\right)^{\frac{1}{2}} \qquad \text{-----}\quad (28)$$

where i and j are as defined above.

3.2 The Eigenvalues of the Intrinsic Hamiltonian

For the intrinsic Hamiltonian defined in equation (20), we set

$$\left.\begin{array}{lcl}x_1' & = & x_1 - Z \\ x_2' & = & x_2 - Z \\ x_3' & = & x_3 - Z \\ x_4' & = & x_4 - Z\end{array}\right| \qquad \text{-----}\quad (29)$$

Substituting these values into equation (20), we obtain the eigenvalue equation,

$$\left\{-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_2'^2}+\frac{\partial^2}{\partial x_3'^2}+\frac{\partial^2}{\partial x_4'^2}\right)+\frac{1}{2}c(x_1'^2+x_2'^2+x_3'^2+x_4'^2)\right\}\phi_{n_1n_2n_3n_4} = \varepsilon_N\,\phi_{n_1n_2n_3n_4}(x_1',x_2',x_3',x_4') \qquad \text{---}\quad (30)$$

This gives,

$$\left.\begin{aligned} E_N &= \hbar\omega(N + 2) \\ \phi_{n_1n_2n_3n_4}(x_1',x_2',x_3',x_4') &= \psi_{n_1}(x_1')\psi_{n_2}(x_2')\psi_{n_3}(x_3')\psi_{n_4}(x_4') \end{aligned}\right\} \quad \text{--- (31)}$$

where $N = n_1 + n_2 + n_3 + n_4$ and

$n_1, n_2, n_3, n_4 = 0, 1, 2, ---$.

The normalized oscillator eigenfunctions are:

$$\left.\begin{aligned} \psi_{n_1}(x_1') &= N_{n_1} e^{-\frac{1}{2}\gamma^2 x_1'^2} H_{n_1}(\gamma x_1') \\ \psi_{n_2}(x_2') &= N_{n_2} e^{-\frac{1}{2}\gamma^2 x_2'^2} H_{n_2}(\gamma x_2') \\ \psi_{n_3}(x_3') &= N_{n_3} e^{-\frac{1}{2}\gamma^2 x_3'^2} H_{n_3}(\gamma x_3') \\ \psi_{n_4}(x_4') &= N_{n_4} e^{-\frac{1}{2}\gamma^2 x_4'^2} H_{n_4}(\gamma x_4') \end{aligned}\right\} \quad \text{----- (32)}$$

where the N_{n_j}'s are the normalization constants and γ is the inverse oscillator length. These are similarly given by

$$N_{n_j} = \left\{\frac{\gamma}{\pi^{\frac{1}{2}}\, 2^{n_j}\, n_j\,!}\right\}^{\frac{1}{2}} \quad \text{----- (33)}$$

and

$$\left.\begin{aligned} \gamma &= (m\omega/\hbar)^{\frac{1}{2}} \\ \text{where} \quad \omega &= (k/m)^{\frac{1}{2}} \end{aligned}\right\} \quad \text{----- (34)}$$

3.3 The Collective Wave Functions

The total wave function can be expanded in terms of the eigenfunctions of the intrinsic Hamiltonian since these form a complete set (Enge et al, 1981)

$$\psi_{krstv} = \sum_{n_1n_2n_3n_4} f^{krstv}_{n_1n_2n_3n_4}(Z)\; \phi_{n_1n_2n_3n_4}(x_1-Z,x_2-Z,x_3-Z,x_4-Z) \quad \text{----- (35)}$$

The expansion coefficients are the collective amplitudes and are obtained by multiplying equation (35) from the left by $\phi^{*}_{n_1n_2n_3n_4}$ and integrating. The result is

$$f^{krstv}_{n_1n_2n_3n_4}(z) = \iiiint_{-\infty}^{\infty} \phi^{*}_{n_1n_2n_3n_4}(x_1-Z,x_2-Z,x_3-Z,x_4-Z)\psi_{krst}\, dx_1dx_2dx_3dx_4 \quad \text{----} \quad (36)$$

Inserting the wave functions ψ_{krstv} and $\phi_{n_1n_2n_3n_4}$ given by equations (24) and (31) respectively, and taking the formulae in equation (13) for the normal coordinates, it is straight forward to evaluate the multiple integral of equation (36). This integral can be cast into the form

$$\begin{aligned} f^{krstv}_{n_1n_2n_3n_4}(z) = {} & \frac{1}{\sqrt{2\Pi}} N_{n_1} N_{n_2} N_{n_3} N_{n_4} N_r N_s N_t N_v \iiiint_{-\infty}^{\infty} \exp\Big[-\frac{\gamma^2}{8} \\ & \{(7+1/\sigma)(x_1^2+x_4^2)+(11+1/\sigma)(x_2^2+x_3^2)+ \\ & 16(1+1/\sigma)Z^2\}+\{Z\gamma^2(1+1/\sigma)+b\}(x_1+x_2+ \\ & x_3+x_4)+\frac{\gamma^2}{4}(3-1/\sigma)(x_1x_2+x_3x_4) - \\ & \frac{1}{4\sigma}\gamma^2(x_1x_3 + x_1x_4 + x_2x_4) + \frac{\gamma^2}{4}(4-1/\sigma)x_2x_3 \\ & + b\frac{M}{m}Z\Big].\ H_{n_1}(\gamma(x_1-Z))\, H_{n_2}(\gamma(x_2-Z)) \\ & .H_{n_3}(\gamma(x_3-Z))\, H_{n_4}(\gamma(x_4-Z))\, H_r(\frac{\gamma}{2\sqrt{\sigma}}(-x_1-x_2 \\ & -x_3-x_4+4Z))\, H_s(\frac{\sqrt{3}}{2}\gamma(x_1-x_2))\, H_t(\gamma(x_2-x_3)) \\ & H_v(\frac{\sqrt{3}}{2}\gamma(x_3-x_4))dx_1dx_2dx_3dx_4 \quad \text{-----} \quad (37) \end{aligned}$$

The constants σ and b are defined by

$$\left.\begin{aligned} \sigma &= [\frac{4m}{M}+1]^{\frac{1}{2}} \\ b &= \frac{ikm}{4m+M} \end{aligned}\right\} \quad \text{-----} \quad (38)$$

By a factorization method and the application of the addition theorem for Hermite polynomials (Morse and Feshbach, 1953),

$$H_n(x+y) = \frac{1}{2^{n/2}} \sum_{m=0}^{n} \binom{n}{m} H_m(\sqrt{2}\,x)\, H_{n-m}(\sqrt{2}\,y) \qquad ----- \quad (39)$$

the final expression for the collective amplitudes esults:

$$f^{krstv}_{n_1n_2n_3n_4}(Z) = F^{(\sigma)}_{n_1n_2n_3n_4rstv} \exp[ikZ - \tfrac{1}{8}(1+1/\sigma)d^2] \sum_{m_1}^{n_1} \sum_{m_2}^{r} \sum_{m_3}^{s}$$

$$\cdot \sum_{m_4}^{n_1-m_1} \sum_{m_5}^{n_2} \sum_{m_6}^{r-m_2} \sum_{m_7}^{s-m_3} \sum_{m_8}^{t} \sum_{m_9}^{n_1-m_1-m_4} \sum_{m_{10}}^{n_2-m_5} \sum_{m_{11}}^{n_3}$$

$$\cdot \sum_{m_{12}}^{r-m_2-m_6} \sum_{m_{13}}^{s-m_3-m_7} \sum_{m_{14}}^{t-m_8} \sum_{m_{15}}^{v} \sum_{m_{16}}^{n_1-m_1-m_4-m_9} \sum_{m_{17}}^{n_2-m_5-m_{10}}$$

$$\cdot \sum_{m_{18}}^{n_3-m_{11}} \sum_{m_{19}}^{n_4} \sum_{m_{20}}^{r-m_2-m_6-m_{12}} (-1)^{r+s+t+v-m_3-m_8-m_{15}}$$

$$\cdot 2^{-\frac{1}{2}(4n_1+4r+3s+3n_2+2t+2n_3+v+n_4-3m_1-3m_2-2m_3-2m_4-2m_5-2m_6-m_7-m_8 -m_9-m_{10}-m_{11}-m_{12})}$$

$$\cdot \binom{n_1}{m_1}\binom{r}{m_2}\binom{s}{m_3}\binom{n_1-m_1}{m_4}\binom{n_2}{m_5}\binom{r-m_2}{m_6}\binom{s-m_3}{m_7}\binom{t}{m_8}$$

$$\cdot \binom{n_1-m_1-m_4}{m_9}\binom{n_2-m_5}{m_{10}}\binom{n_3}{m_{11}}\binom{r-m_2-m_6}{m_{12}}\binom{s-m_3-m_7}{m_{13}}$$

$$\cdot \binom{t-m_8}{m_{14}}\binom{v}{m_{15}}\binom{n_1-m_1-m_4-m_9}{m_{16}}\binom{n_2-m_5-m_{10}}{m_{17}}\binom{n_3-m_{11}}{m_{18}}$$

$$\cdot \binom{n_4}{m_{19}}\binom{r-m_2-m_6-m_{12}}{m_{20}} \cdot I^{a_1a_2a_3}_{m_1m_2m_3} \cdot I^{a_4a_5a_6a_7a_8}_{m_4m_5m_6m_7m_8}$$

$$\cdot I^{a_9a_{10}a_{11}a_{12}a_{13}a_{14}a_{15}}_{m_9m_{10}m_{11}m_{12}m_{13}m_{14}m_{15}} \cdot I^{a_{16}a_{17}a_{18}a_{19}a_{20}a_{21}a_{22}a_{23}}_{m_{16}m_{17}m_{18}m_{19}m_{20}\ 21m_{22}m_{23}}$$

$$\cdot H_{n_1-m_1-m_4-m_9-m_{16}}(id) \cdot H_{n_2-m_5-m_{10}-m_{17}}\left(\frac{id}{\sqrt{2}}\right) \cdot H_{n_3-m_{11}-m_{18}}\left(\frac{id}{2}\right) \cdot H_{n_4-m_{19}}\left(\frac{id}{2\sqrt{2}}\right)$$

$$\cdot H_{r-m_2-m_6-m_{12}-m_{20}}\left(\frac{2id}{\sqrt{\sigma}}\right) \qquad ----- \quad (40)$$

$$\text{where} \quad \left.\begin{aligned} m_{21} &= s-m_3-m_7-m_{13} \\ m_{22} &= t-m_8-m_{14} \\ m_{23} &= v-m_{15} \end{aligned}\right\} \qquad ----- \quad (41)$$

CHAPTER FOUR

4. RESULTS AND DISCUSSIONS

In this section we present some graphical values of probability distributions for the intrinsic excitations calculated with the amplitudes given by equation (40). All parameters occuring in the final formula that enter into the numerical calculations are dimensionless quantities. These include the ratio of the oscillator mass m to the central particle mass M; the ratio of the energy of free motion E_k to the energy of oscillator spacing, $\tilde{E}_k = E_k/\hbar\omega = \hbar^2k^2/[2(4m + M)\hbar\omega]$; and the ratio of the intrinsic excitation energy to the energy of oscillator spacing, $\tilde{\varepsilon}_N = \varepsilon_N/\hbar\omega$. These quantities have been varied in order to study the numerical results systematically.

4.1 Probability Distributions as Functions of Energy in the Translational Degree of Freedom

In Fig. 1 we have plotted probability amplitudes, $2\Pi\,|f^{krstv}_{n_1n_2n_3n_4}|^2$, as functions of the energy of free motion, $\tilde{E}_k$, for various quantum number, n_1, n_2, n_3, n_4, r, s, t, v and for fixed ratio m/M = 1. Two sets of quantum numbers have been chosen: r = s = t = v = 0 and r = s = 2, t = v = 0 as illustrated in Figs. 1(a) and 1(b) respectively. In Fig. 1(a) we have taken (i) $n_1 = n_2 = n_3 = n_4 = 0$ or N = 0 ; (ii) $n_1 = n_2 = 1$, $n_3 = n_4 = 0$, or N = 2 ; (iii) $n_1 = n_2 = n_3 = n_4 = 1$, or N = 4 ; (iv) $n_1 = 2$, $n_2 = n_3 = n_4 = 1$, or N = 5 ; (v) $n_1 = 3$, $n_2 = 2$, $n_3 = n_4 = 1$ or N = 7 and (vi) $n_1 = 1$, $n_2 = 2$, $n_3 = 3$, $n_4 = 4$, or N = 10 .

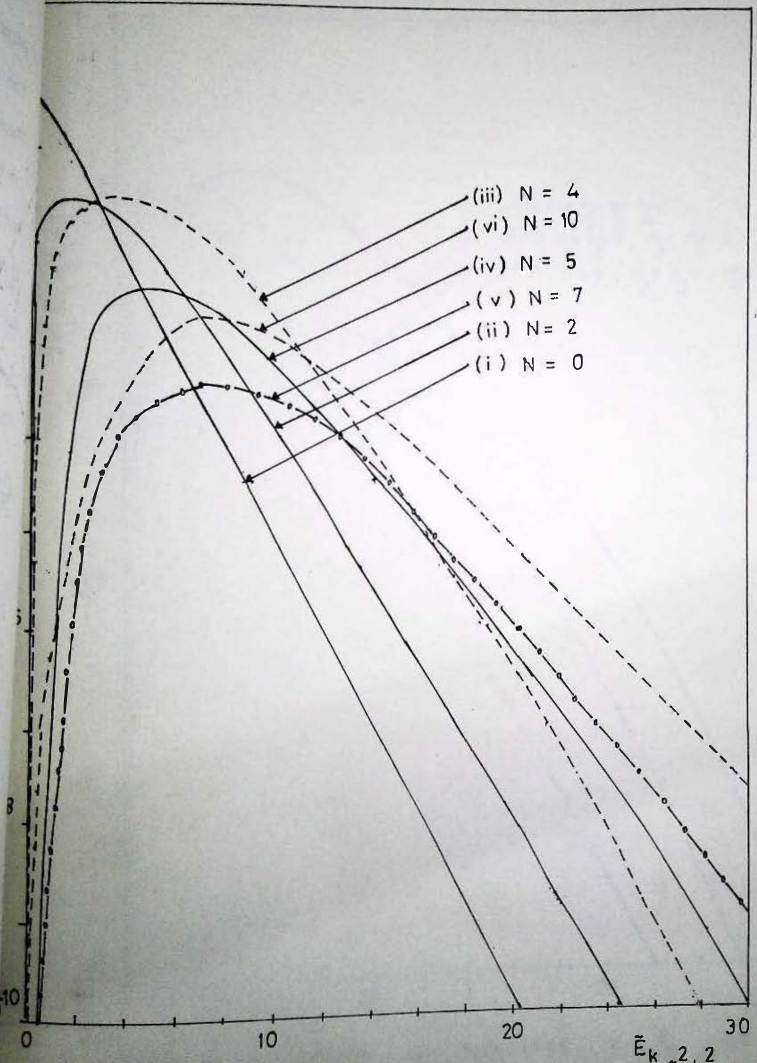


Fig. 1 (a) Probabilities as functions of $\tilde{E}_k = (\frac{\hbar^2 k^2}{2(4m+M)})/\hbar\omega$ for $m/M' = 1$ and $r = s = t = v = 0$

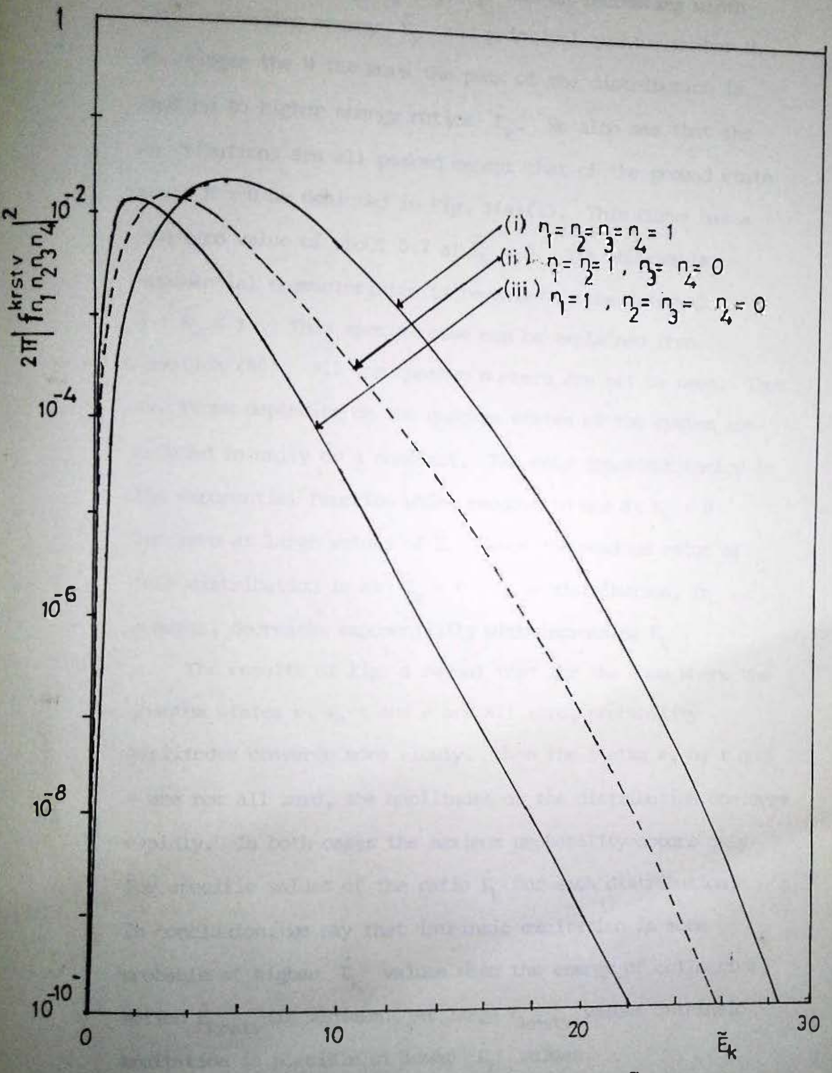


Fig. 1(b) Probabilities as functions of $\tilde{E}_k$ for $m/M = 1$ and $r = s = 2,\ t = v = 0$

We observe from the two figures that the excitation of intrinsic states (n_1, n_2, n_3, n_4) has an increasing width with increasing energy $\tilde{E}_k$ and principal quantum number N. The larger the N the more the peak of the distribution is shifted to higher energy ratios $\tilde{E}_k$. We also see that the distributions are all peaked except that of the ground state where N = 0 as depicted in Fig. 1(a)(i). This curve has a non-zero value of about 0.7 at $\tilde{E}_k = 0$. Its observable exponential characteristic is revealed in the interval $0 < \tilde{E}_k < 1$. This special case can be explained from equation (40): All the quantum numbers are set to zero. Thus all terms depending on the quantum states of the system are reduced to unity or a constant. The only dependent factor is the exponential function which reduces to one at $\tilde{E}_k = 0$ and zero at large values of $\tilde{E}$. Hence the maximum value of this distribution is at $\tilde{E}_k = 0$. The distribution, in general, decreases exponentially with increasing $\tilde{E}_k$.

The results of Fig. 1 reveal that for the case where the quantum states r, s, t and v are all zero, probability amplitudes converge more slowly. When the states r, s, t and v are not all zero, the amplitudes of the distribution converge rapidly. In both cases the maximum probability occurs only for specific values of the ratio $\tilde{E}_k$ for each distribution. In conclusion, we say that intrinsic excitation is more probable at higher $\tilde{E}_k$ values when the energy of collective motion $\tilde{E}_{krstv}$ is minimum. At large $\tilde{E}_{krstv}$ values intrinsic excitation is possible at lower $\tilde{E}_k$ values.

Figs. 2(a) and 2(b) show plots of probability distributions as functions of $\tilde{E}_k$ for fixed sets of quantum numbers and for

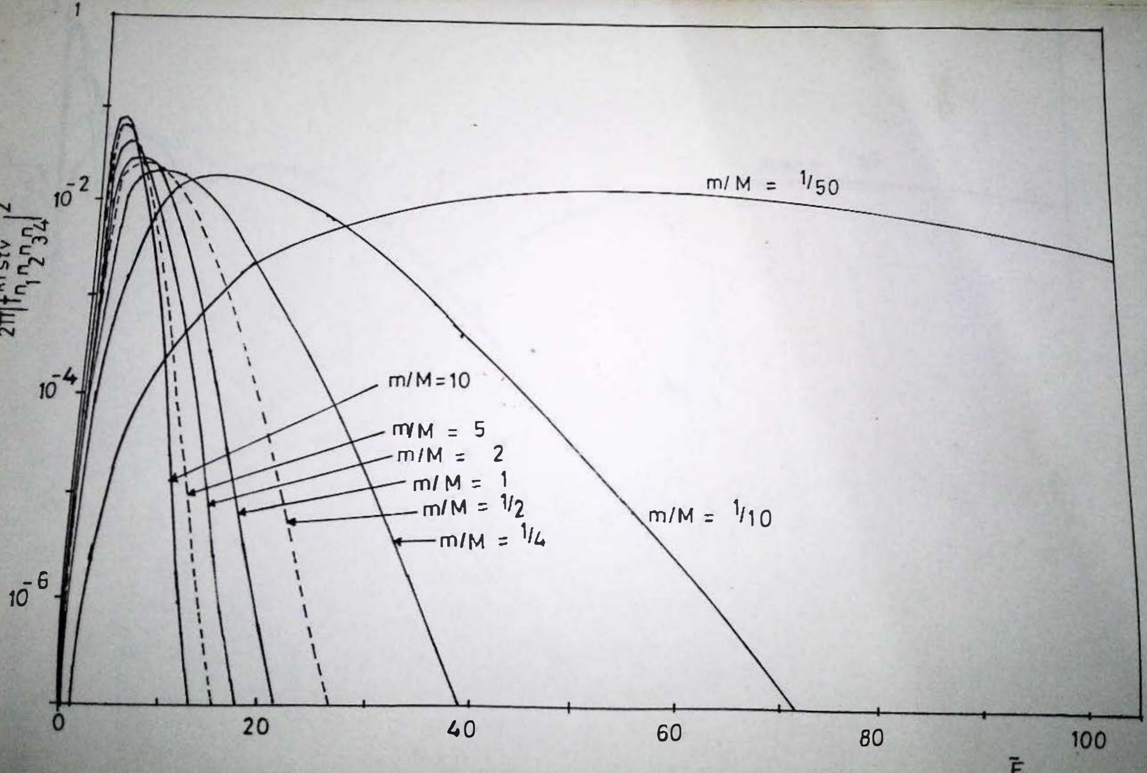
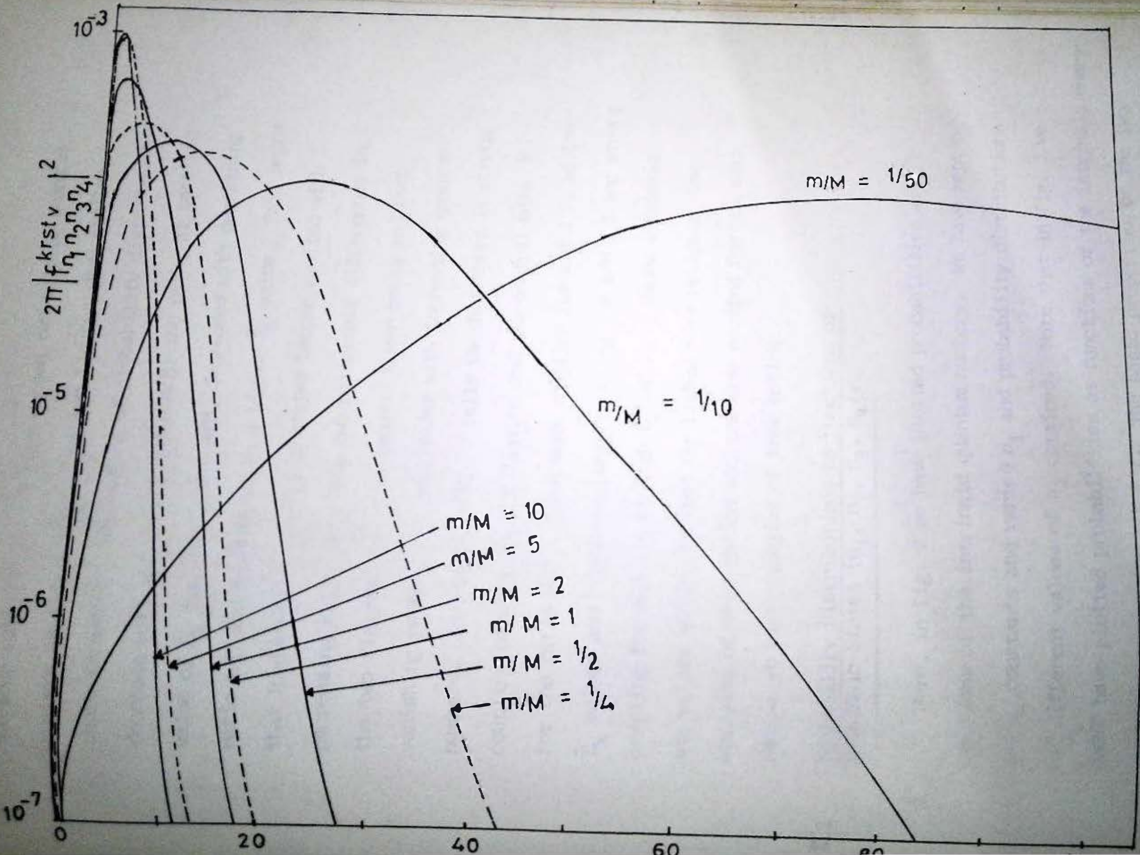


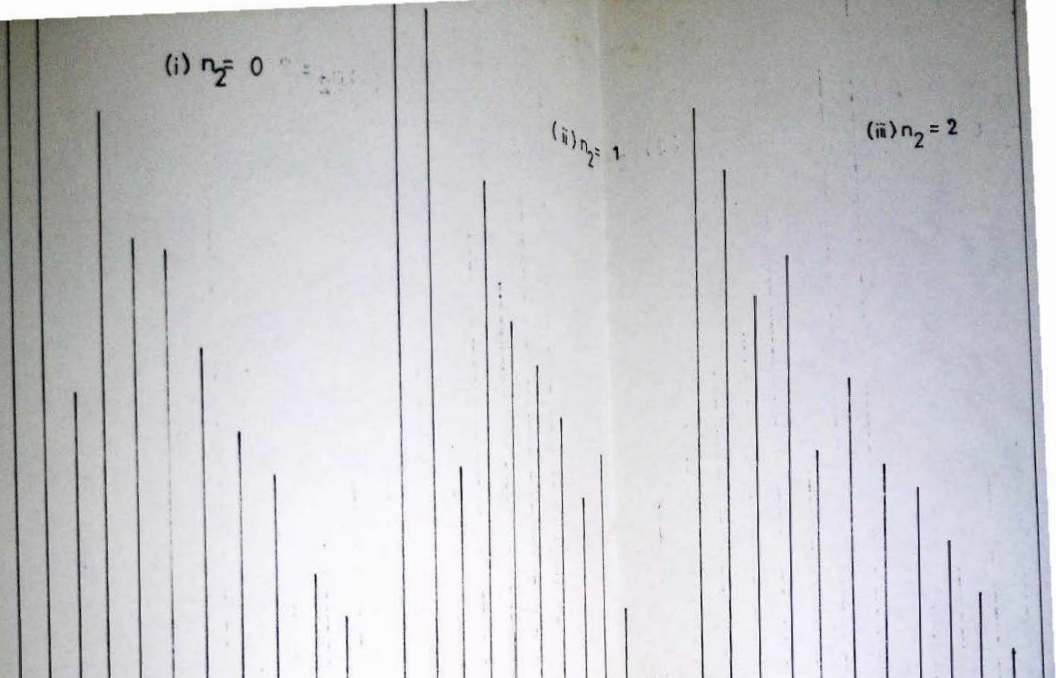
Fig. 2(a) Probabilities as functions of $\bar{E}_k$ for $n_1 = n_2 = n_3 = n_4 = 1$ and $r = s = t = v = 0$


$2\pi\left|f^{krstv}_{n_1n_2n_3n_4}\right|^2$
10^{-3}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
m/M = 1/50
m/M = 1/10
m/M = 10
m/M = 5
m/M = 2
m/M = 1
m/M = 1/2
m/M = 1/4

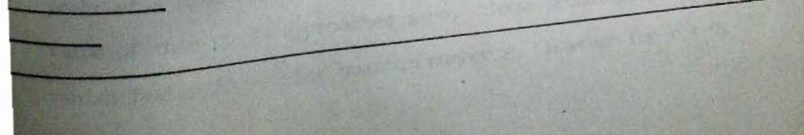
various ratios m/M. We have chosen two sets of quantum numbers as illustrated. The two figures show similar characteristics: The widths of probability distributions decrease with increasing ratio m/M; the peak, at specific value of $\tilde{E}_k$ for each ratio m/M, decreases with decreasing ratio m/M and is shifted to a higher $\tilde{E}_k$ value. This means that higher-lying intrinsic states become excited with decreasing ratio m/M. The two noticeable differences in the two figures are conspicuous: Lower peaks and fast vanishing probability amplitudes with increasing quantum numbers. In summary, Fig. 2 tells us that there is strong coupling between the oscillators and the central mass M for the ratio $m/M > 1$ and weak coupling for $m/M < 1$, at low $\tilde{E}_k$ values and conversely weak coupling for $m/M > 1$ and strong coupling for $m/M < 1$ at high $\tilde{E}_k$ values. Hence intrinsic states are easily excited for higher mass ratios at low energies of free motion and are also excited for low mass ratios at high energies of free motion.

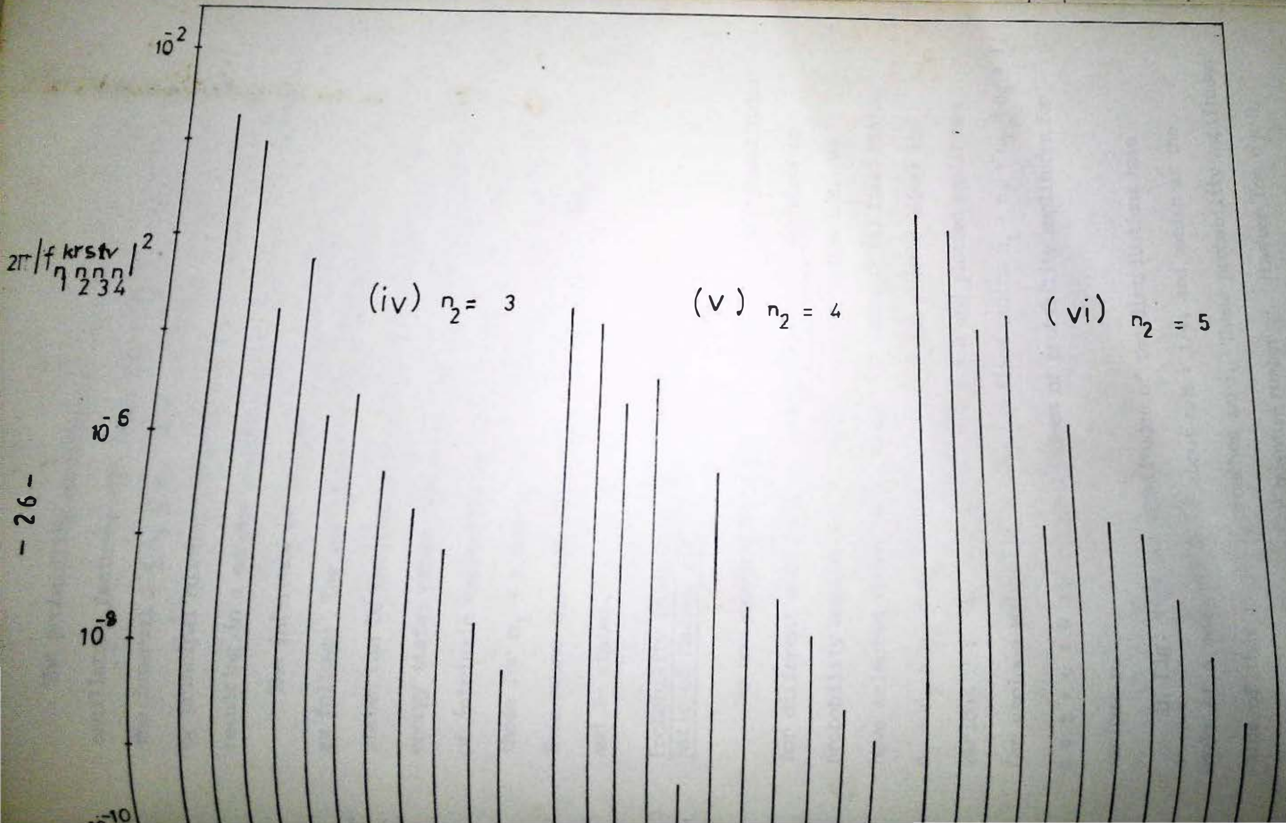
4.2 Probability Distributions as Functions of Intrinsic States (n_1, n_2, n_3, n_4).

Here, in Fig. 3 we have plotted probabilities as functions of the intrinsic quantum numbers. We have kept n_3 and n_4 constants and varied n_1 and probability distributions for different values of n_2 obtained. Note that in Fig. 3 we could have plotted probabilities as functions of the intrinsic energy $\tilde{\varepsilon}_N$, since $\tilde{\varepsilon} = \varepsilon_N/\hbar\omega = N + 2$. The basic properties of the two would then be the same. We can therefore discuss the plots of Fig. 3 in terms of intrinsic energy, $\tilde{\varepsilon}_N$.



ig. 3 Probabilities as functions of the intrinsic states n_1, n_2, n_3, n_4 for $n/M = 1$, $\tilde{E}_k = 1$, $n_3 = n_4 = 1$ and $r = s = t = v = 0 =$ [illegible]




10^{-2}
$2\pi|f^{krstv}_{n_1n_2n_3n_4}|^2$
10^{-6}
10^{-8}
10^{-10}
(iv) $n_2 = 3$
(v) $n_2 = 4$
(vi) $n_2 = 5$

The probability amplitudes as depicted in the figure have oscillatory features with large amplitudes of oscillation in the interval $1 \leq n_1 \leq 4$. With increasing intrinsic state n_1 , or principal quantum number N, the oscillations die out resulting in a smooth vanishing of the probability amplitudes.

The inference we can draw from the shapes of Fig. 3 is as follows: The oscillatory structure reveals that intrinsic states can be excited or de-excited in any higher-lying energy states other than zero. As an example, the probability of intrinsic excitation in the state $n_1 = 2$ is lower than those for $n_1 = 1$ and $n_1 = 3$ in each of the six distributions. This means that the state $n_1 = 2$ is more stable than its 1- and 3- states.

.3 Probability Distributions as Functions of Ratio of Masses m/M

We now examine the behaviour of the probability amplitudes for different mass ratios. Figs. 4(a) - 4(c) are plots or probability amplitudes as functions of the ratio m/M. We have selected three sects of quantum numbers: (a) fixed states $n_1 = n_2 = n_3 = n_4 = 0$, $s = t = v = 0$ and plotted values for various r ; (b) set $r = s = t = v = 0$ and plotted amplitudes for various values of N, and (c) fixed states $n_1 = n_2 = n_3 = n_4 = 1$, $s = t = v = 0$ and plotted values of probability amplitudes for various r.

In Fig. 4(a) the amplitudes of the distributions have peaks at a mass ratio of about m/M = 1/4, and vanish as the value of this ratio approaches zero. These probability amplitudes vanish faster for higher quantum number r. However for r = 0,

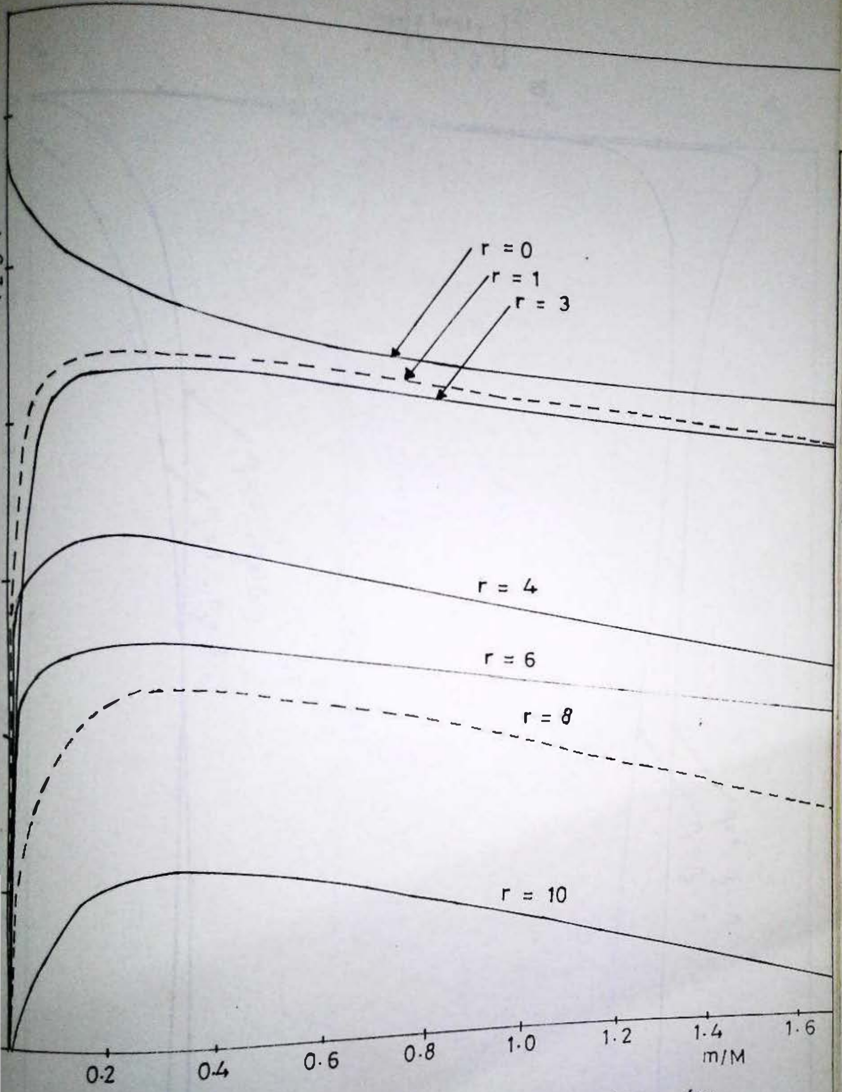
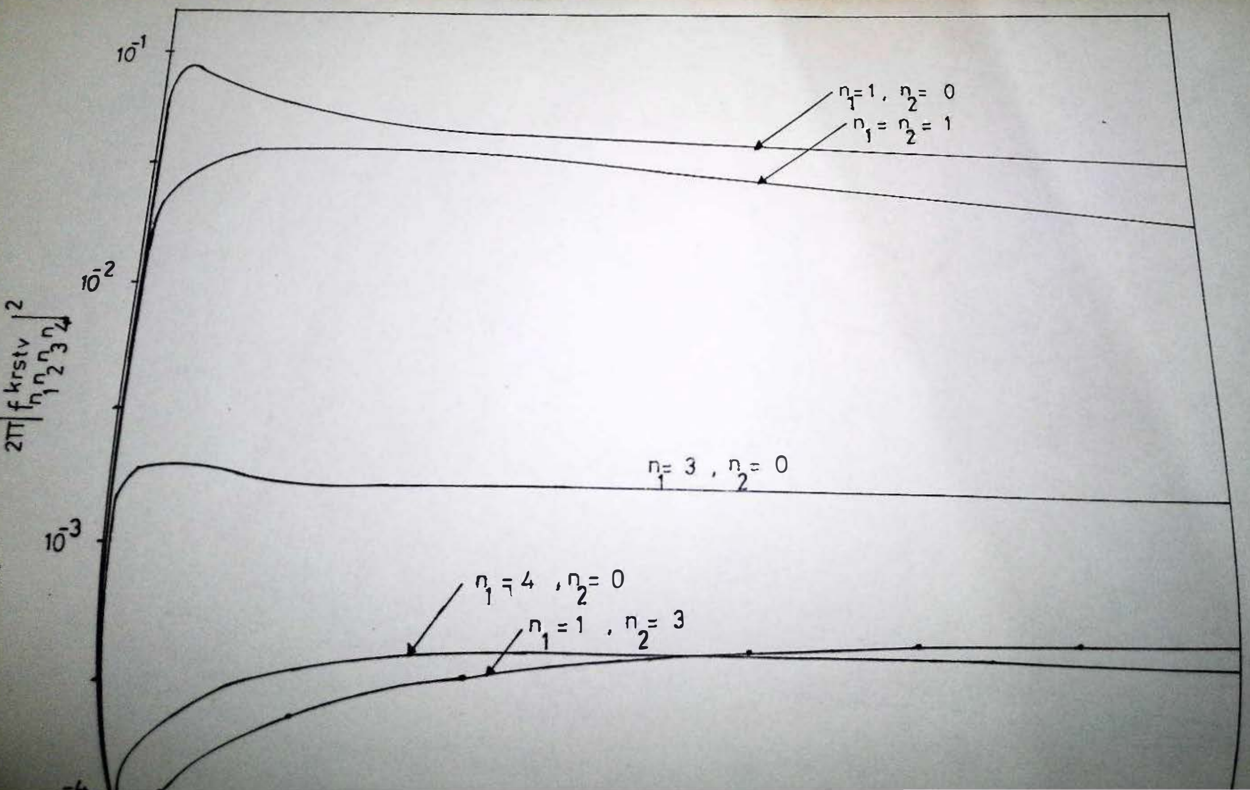


Fig. 4(a) Probabilities as functions of m/M for $\eta_1 = \eta_2 = \eta_3 = \eta_4 = 0$ and $s = t = v = 0$; $\tilde{E}_k = 1$


10^{-1}
10^{-2}
10^{-3}
$2\pi\left|f^{krstv}_{n_1 n_2 n_3 n_4}\right|^2$
$n_1 = 1,\ n_2 = 0$
$n_1 = n_2 = 1$
$n_1 = 3,\ n_2 = 0$
$n_1 = 4,\ n_2 = 0$
$n_1 = 1,\ n_2 = 3$

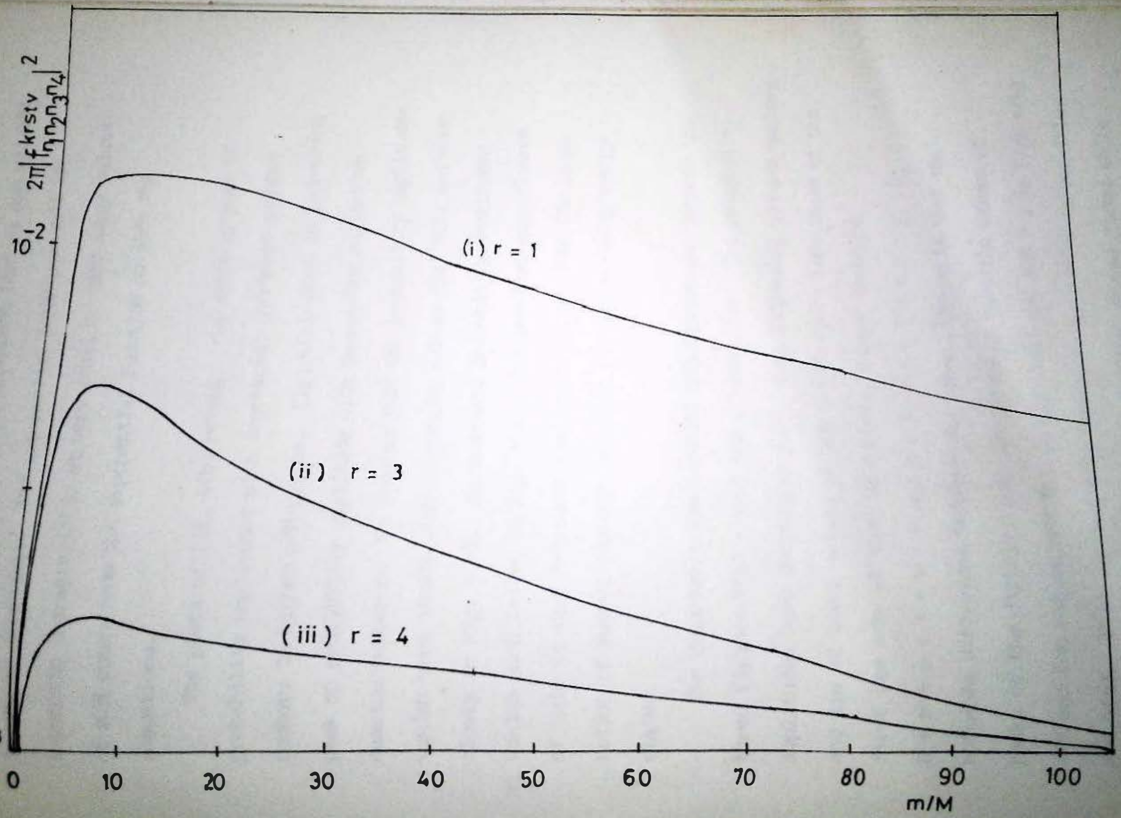


Fig.4(c) Probabilities as functions of m/M for $n_1 = n_2 = n_3 = n_4 = 1$

all quantum numbers are zero and it is evident from equation (40) that all the quantum state-dependent factors are reduced to one or constants. The exponential behaviour with an asymtotic characteristic at very small or very large values of m/M comes from the exponential function outside the summations.

The plots of Fig. 4(b) reveal the shift in peak of probability amplitudes with increasing intrinsic quantum numbers to higher m/M ratios. Fig. 4(c) shows an increasing peak of probability amplitude with decreasing collective quantum number r. We observe that the probability amplitudes vanish very slowly with increasing m/M in Fig. 4(b) and more slowly in Fig. 4(c). In contrast, probability amplitudes vanish much faster in Fig. 4(a). The physical significance of this is that intrinsic excitation is possible for large ratios of masses and also possible for higher-lying energy states.

The remarkable similarities and differences between the three figures 4(a) - 4(c) are illustrative: The probability amplitudes have decreasing peaks with increasing quantum numbers and lie at lower values of m/M ratios apart from those of the first few sets of quantum numbers, namely those of $N + r + s + t + v = 0$ and $N = 1$, $r = s = t = v = 0$. A noteable difference between the three figures is that the peaks of the distributions are found at specific values of m/M and for ratios $m/M < 1$ in Fig. 4(a) and $m/M > 1$ in Figs 4(b) and 4(c). This implies that intrinsic excitation is possible for small ratios of masses when in their ground states while excitation is possible for large ratios of masses when they are in states other than their ground states.

CHAPTER FIVE

CONCLUSIONS AND FUTURE DEVELOPMENT

A demonstration of the problem of energy dissipation from a collective coordinate into internal degrees of freedom has been presented using an exactly solvable model. The model which consists of four oscillators coupled to a fifth particle which has the freedom of under-going translational motion is described mathematically through both classical and quantum mechanics via the Lagrangian dynamics and Schrödinger's equation respectively. The amplitudes of a collective motion obtained as multiple integrals have been solved exactly. These amplitudes represent probabilities of intrinsic energy states being excited.

The probability distributions obtained in the previous chapter are the amplitudes for transitions into discrete energy levels. The results calculated and presented graphically show interesting features which are summarized as follows: The probability distributions are peaked at specific values of the function variables (energy of free motion, $\tilde{E}_k = E_k/\hbar\omega$; principal quantum number, $N = n_1 + n_2 + n_3 + n_4$; and the ratio of masses m/M). These indicate that the maximum probability of finding the intrinsic energy lies within a certain energy interval only. Peaks of probability amplitudes shifted to higher values of the energy of free motion with increasing quantum numbers indicate that excitation of intrinsic states is possible at high energy values when states are not all at their ground levels. A wide range of probability excitation is revealed by the increasing widths $\Delta\tilde{E}_k$ with increasing energy and the principal quantum number N. The intrinsic degrees of freedom, represented in our model by x_1 , x_2 , x_3 and x_4 are strongly coupled to the collective degree of freedom Z when the mass of intrinsic oscillator is greater than that of the free particle when in its low energies, while strong coupling occurs at $m/M < 1$ only at high energy values of the free particle. We have also seen that for small ratios of masses excitation is possible even at zero levels of the intrinsic states while for large ratios of masses excitation is possible only at non-zero intrinsic states. This is obvious since nothing goes for nothing.

The results obtained are in good agreement with the ones calculated from a previous work (Mshelia et al, 1981). Although our simulation here is based on a simple model, the formulation we have developed can be extended to the more realistic but more complex case of heavy-ion collisions, where energy dissipation plays a very important role in deciding whether the fusion of superheavy elements is possible or not.

In future developments investigations of a model of a chain of n linear harmonic oscillators can be carried out employing the same formalism. Secondly work should also aim at approaching the same or similar models in two and three dimensions using the same formalism or a modified form if necessary. Thirdly, application of the time-dependent Schrödinger's equation to this model is recommended ; in which case one can study the dissipation of energy from the collective degree of freedom to the intrinsic ones as a function of time and the initial energy in the translational degree of freedom.

APPENDIX

Further factors appearing in equation (40) are defined as follows:

$$F(\sigma)_{n_1n_2n_3n_4rstv} = \frac{1}{(\pi^5 2^{n_1+n_2+n_3+n_4+r+s+t+v} n_1!n_2!n_3!n_4!r!\,s!\,t!\,v!)^{\frac{1}{2}}}$$

$$\cdot\ 2(\sqrt{\sigma}/[5(\sigma + 1)])^{\frac{1}{2}}$$

$$d = \frac{k}{\gamma}\left(1 - \frac{1}{\sigma}\right) \qquad \text{-----} \quad (A.1)$$

$$\text{-----} \quad (A.2)$$

and the a_i's are defined in terms of the dimensionless quantity σ as follows:

$$a_1 = 4(\sigma/(7\sigma + 1))^{\frac{1}{2}}$$

$$a_2 = 2(3\sigma - 1)(2/((7\sigma + 1)(17\sigma + 6)))^{\frac{1}{2}},$$

$$a_3 = 4(2\sigma - 3)(3/((17\sigma + 6)(53\sigma + 33)))^{\frac{1}{2}},$$

$$a_4 = (3\sigma - 17)(2/[5(\sigma + 1)(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_5 = 2((7\sigma + 1)/(17\sigma + 6))^{\frac{1}{2}},$$

$$a_6 = 2(14\sigma - 3)(2/[3(17\sigma + 6)(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_7 = (7\sigma - 13)(1/[5(\sigma + 1)(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_8 = 4((17\sigma + 6)/[3(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_9 = (17\sigma - 3)(1/[10(\sigma + 1)(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_{10} = ((53\sigma + 33)/[20(\sigma + 1)])^{\frac{1}{2}},$$

$$a_{11} = 2((1/(7\sigma + 1)))^{\frac{1}{2}},$$

$$a_{12} = 10(2\sigma/[(7\sigma + 1)(17\sigma + 6)])^{\frac{1}{2}},$$

$$a_{13} = 36(3\sigma/[(17\sigma + 6)(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_{14} = 8(10\sigma/[(\sigma + 1)(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_{15} = 2(3\sigma/(7\sigma + 1))^{\frac{1}{2}},$$

$$a_{16} = 2(2\sigma + 1)(6/[(7\sigma + 1)(17\sigma + 6)])^{\frac{1}{2}},$$

$$a_{17} = 4(4\sigma + 3)(1/[(17\sigma + 6)(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_{18} = 4(3(\sigma + 1)/[10(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_{19} = 2((7\sigma + 1)/(17\sigma + 6))^{\frac{1}{2}},$$

$$a_{20} = 10(4\sigma + 3)(2/[3(17\sigma + 6)(53\sigma + 33)])^{\frac{1}{2}},$$

$$a_{21} = (10(\sigma + 1)/(53\sigma + 33))^{\frac{1}{2}},$$

$$a_{22} = 2((17\sigma + 6)/(53\sigma + 33))^{\frac{1}{2}},$$

$$a_{23} = 9(3(\sigma + 1)/[5(53\sigma + 33)])^{\frac{1}{2}} \qquad \text{-----} \quad (A.3)$$

The quantity $I^{a_1 a_2 \ldots}_{m_1 m_2 \ldots}$ in equation (40) is an integral defined by

$$I^{a_1 a_2 \ldots}_{m_1 m_2 \ldots} = \int_{-\infty}^{\infty} \exp[-x^2]\, H_{m_1}(a_1 x)\, H_{m_2}(a_2 x) \ldots dx \qquad \text{----} \quad (A.4)$$

It can be shown that the collective amplitudes, $f^{krstv}_{n_1 n_2 n_3 n_4}(Z)$, obtained in equation (40) satisfy the normalization conditions. Recall the assertion that the total wave-function $\psi_{krstv}(x_1, x_2, x_3, x_4, Z)$ can be expanded in terms of the complete orthonormal set of oscillator functions $\phi_i(x_1, x_2, x_3, x_4, Z)$. The normalization of ψ_{krstv} gives

$$\iiiint_{-\infty}^{\infty} \psi_\beta^{*} \psi_\beta dx_1 dx_2 dx_3 dx_4 = \sum_i \sum_j f_i^{*}(Z)\, f_j(Z) \iiiint_{-\infty}^{\infty} \phi_i^{*} \phi_j \cdot dx_1 dx_2 dx_3 dx_4$$

$$= \sum_i |f_i(Z)|^2 \qquad \text{-----} \quad (A.5)$$

since the ϕ_i's are orthonormal.

On the other hand, the normalization of ψ_{krstv} with respect to the variables g_1, g_2, g_3 and g_4 gives

$$\iiiint_{-\infty}^{\infty} \psi_\beta^{*} \psi_\beta dg_1 dg_2 dg_3 dg_4 = \psi_k^{*}(g_5)\, \psi_k(g_5) \int_{-\infty}^{\infty} \psi_r^{*}(g_1)$$

$$\cdot\, \psi_r(g_1) dg_1 \int_{-\infty}^{\infty} \psi_s^{*}(g_2)\, \psi_s(g_2) dg_2$$

$$\cdot \int_{-\infty}^{\infty} \psi_t^{*}(g_3)\, \psi_t(g_3) dg_3 \cdot \int_{-\infty}^{\infty} \psi_v^{*}(g_4)\, \psi_v(g_4) dg_4 \qquad \text{-----} \quad (A.6)$$

$$= \frac{1}{2\pi}$$

The relationship between the left hand sides of equations (A.5) and (A.6) is given by the transformation

$$\iiiint_{-\infty}^{\infty} \psi_\beta^*(g_1,g_2,g_3,g_4,g_5)\, \psi_\beta(g_1,g_2,g_3,g_4,g_5)\,dg_1dg_2dg_3dg_4$$

$$= \iiiint_{-\infty}^{\infty} \psi_\beta^*(x_1,x_2,x_3,x_4,Z)\, \psi_\beta(x_1,x_2,x_3,x_4,Z) \cdot \left| \frac{\partial(g_1,g_2,g_3,g_4)}{\partial(x_1,x_2,x_3,x_4)} \right| dx_1dx_2dx_3dx_4 \qquad \text{----- (A.7)}$$

When values are substituted the Jacobian is

$$\left| \frac{\partial(g_1,g_2,g_3,g_4)}{\partial(x_1,x_2,x_3,x_4)} \right| = 1 \qquad \text{----- (A.8)}$$

Comparing equations (A.5), (A.6) and (A.7) we have

$$\sum_i |f_i(Z)|^2 = \frac{1}{2\Pi} \qquad \text{----- (A.9)}$$

Thus the amplitudes in equation (40) satisfy the normalization conditions

$$\sum_{r,s,t,v} |f^{krstv}_{n_1n_2n_3n_4}(Z)|^2 = \sum_{n_1n_2n_3n_4} |f^{krstv}_{n_1n_2n_3n_4}(Z)|^2 = \frac{1}{2\Pi} \qquad \text{----- (A.10)}$$

REFERENCES

Bass R. (1980). Nuclear Reactions with Heavy Ions. Texts and Monographs in Physics. Berlin.

Beck R. and Gross D.H.E. (1973). Phys. Lett., 47B, 143.

Desloge E.A. (1982). Classical Mechanics, Volumes I and II. Wiley Interscience Publication.

Enge H.A., Wehr M.R. and Richards J.A. (1981). Introduction to Atomic Physics. Adison-Wesley.

Glas D. and Mosel U. (1974). Phys. Lett., 49 B, 301.

Goldstein H. (1978). Classical Mehcanics. Adison-Wesley.

Gross D.H.E. and Kalinowski H. (1978). Phys. Rep., 45, 175.

Hasse R. (1979). Nucl. Phys. A, 318, 480.

Hasse R.W. (1978). Rep. Prog., Phys., 41, 1027.

Hofmann H. and Siemens P.J. (1976). Nucl. Phys. A, 257, 165; (1977) 275, 464.

Kanai E. (1948). Prog. Theor. Phys., 3, 440.

Kerner E.H. (1958). Can. Journ. Phys., 36, 371.

Morse P.M. and Feshbach H. (1953). Methods of Theoretical Physics. New York.

Mshelia E.D., Hahn D. and Scheid W. (1981). Energy Dissipation in a Model of Coupled Oscillators. Nuovo Cimento A, 61, 28.

Mshelia E.D., Hahn D. and Scheid W. (1984). A Time-Dependent Formalism for Energy Dissipation Nuovo Cimento A, 81, 4.

Mshelia E.D., Scheid W. and Greiner W. (1975). Theory of Energy Dissipation in Heavy-ion Reactions. Nuovo Cimento A, 20, 589.

Pars L.A. (1965). A Treatise on Analytical Dynamics. London.

Schiff L.I. (1968). Quantum Mechanics, third edition. McGraw-Hill.

Senitzky I.R. (1960). Phys. Rev., 119, 670.

Stevens W.K. (1958). Proc. Phys. Soc., 72, 1027.

Swiatecki W.J. and Bjornholm S. (1972). Phys. Rep., 4, 325.