DEVELOPMENT OF AN IMPROVED MIMO CHANNEL ESTIMATION SCHEME

FOR MULTIPATH WIRELESS SYSTEM

BY

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A THESIS SUBMITTED TO THE DEPARTMENT OF ELECTRICAL ENGINEERING, BAYERO UNIVERSITY KANO.IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER DEGREE IN COMMUNICATION ENGINEERING

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Declaration

I hereby declare that the work in this thesis titled "**DEVELOPMENT OF AN IMPROVED MIMO CHANNEL ESTIMATION SCHEME FOR MULTIPATH WIRELESS SYSTEM**" was performed by me in the Department of Electrical Engineering, Bayero University Kano, under the supervision of Dr D.S. Shuaib. I also solemnly declare that to the best of my knowledge no part of this thesis has been submitted here or present somewhere else in previous application for award of a degree or masters at any institution. All sources of knowledge used have been duly acknowledged.

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AbdulrazakYakubu Date

Certification

This is to certify that this thesis titled “Development of an improved MIMO channel estimation scheme for multipath wireless system” byAbdulrazakYakubu (SPS/13/MEE/00036), meets the requirement and regulations governing the award of masters of engr. (communication) degree of Bayerouniwersitykano and I approve for its contribution to knowledge and literary presentation.

Supervisor: D.S.Shuaibu

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Signature: ---------------------------------

Date: ----------------------------------

Approval Page

This is to certify that the research work for this thesis entitled "Development of an Improved MIMO Channel estimation scheme for Multipath Wireless System" by Abdulrazak Yakubu (SPS/13/MEE/00036) has been examined and approved for the award of the degree of Master of Engineering (Communication Option) in Bayero University Kano.

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Dedication

This thesis is dedicated to my parents Mr. and late Mrs. Yakubu, who gave me good upbringing and proper foundation from the onset. May her soul rest in perfect peace (Amin).

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Abbreviation and Symbols

AWGN: Additive White Gaussian Noise

BER: Bit Error Rate

BPSK: Binary Phase Shift Keying

CP: Cyclic Prefix

CSI : Channel State Information

Eb: Energy per Bit

H : Channel Matrix

ICA: Independent Component Analysis

ICI: Inter Carrier Interference

IID: Independent Identically Distributed

ISI: Inter-symbol Interference

ITU: International Telecommunication Union

MAI: Multiple Access Interference

MIMO: Multiple Input Multiple Output

MISO: Multiple Input Single Output

ML: Maximum Likelihood

MLSE: Maximum likelihood Sequence Estimation

MMSE: Minimum Mean Square Error

ND: No Diversity

No: Noise Power Density

OFDM: Orthogonal Frequency Division Multiplexing

QPSK: Quadrature Phase Shift Keying

SIMO: Single Input Multiple Output

SIC ZF: Successive Interference Cancellation Zero Forcing

SIC MMSE: Successive interference Cancellation Minimum Mean Square Estimation

SISO: Single Input Single Output

SNR: Signal to Noise Ratio

STBC: Space Time Block Code

X:Transmitted Symbols

Y:Received Symbols

ZF: Zero Forcing

Abstract

In wireless communication system Multiple-Input Multiple-Output (MIMO) technique has been recognized as a key to modern technology for future wireless communication system. MIMO system uses multiple antenna to increases the link liability, high data rate and efficiency. Some of the major problems encountered in the wireless communication systems are channel impairments, multipath propagation fading, inter-symbol interference etc. This thesis work analyzes and compares five different MIMO channel algorithms (Zero Forcing, ZF Successive Interference Cancellation, Minimum Mean Square Error, MMSE-SIC and Maximum Likelihood) based on the bit error rate using Mat-lab software. The result shows an improvement to the ZF and MMSE using SIC to the two linear algorithms. As a result of reduction in the amplified noise of ZF channel estimation system, ZF SIC was formed with an improvement of 3 percentages. Also, as a result of noise reduction in the content of MMSE channel system, MMSE SIC was formed with an improvement of 4.6 percentages.

CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

Efficient communication system is a major backbone for the growth and development of any society or nation at large. With the increase in wireless technological development, there is a need for innovative approach designed to integrate features such as high data rates, high quality of service delivery and multimedia in the existing communication network. In wireless systems, radio signals are corrupted due to channel fading, distortion, dispersion, inter-symbol interference and noise. In order to combat these channel effects, modern systems employ various techniques including multiple-antenna transceivers using spatial diversity (Zheng and Tse, 2003). Initially, multi-antenna systems were proposed only for point-to-point communication but have now extended to point-to-multi point communication.

Multiple-Input Multiple-Output from multiple transmitters and multiplereceivers' antenna can reduce the effects of multipath propagation fading and noise in the channel (Idris and Syed, 2008). MIMO signals are transmitted from different antennas at the transmitter using the same frequency and multiplexed in space. Received signal in MIMO system is usually distorted by multipath propagation fading, in order to recover the original transmitted signal correctly, channel effect must be compensated for and repaired at receivers' side. Various channel estimation schemes are employed in order to mitigate the physical effects of the medium present.

Hence,MIMOsystems utilize space multiplex by using array of antennas for enhancing the efficiency of the wireless signals at particular utilized bandwidth (Goldsmith, 2005). MIMO technology has being proposed in the near future for many modern wireless systems in many different ways to combat effects ofmultipath propagation fading, noise and to improveon wireless system performance. Basically, these techniques transmit different data streams on different transmit antennas simultaneously. By designing an appropriate processing architecture to handle these parallel streams of data, the data rate and/or the Signal-to-Noise Ratio (SNR) performance can be increased (Zheng and Tse, 2003). The mean square error of the large scale MIMO channel estimator algorithm is derived as a closed form function of the doppler frequency shift, forgetting factor, channel dimensions and the length of the training sequences. The performance of MIMO channel is optimize by using different channel estimation scheme such as Zero Forcing (ZF), Minimum Mean Square Error (MMSE), Maximum Likelihood (ML) and Maximal Ratio Combining (MRC) techniques (Kumar and Foschini, 2011). The space time code used is Alamouti space time block code which gives very good performance and is quite simple to implement(Parveen and Nazia, 2012).

The MIMO system is best suitable for the Wireless Metropolitan Area Networking technology. MIMO-Orthogonal Frequency Division Multiplexing fulfills the high data rate requirement through spatial multiplexing gain and improved link reliability due to antenna diversity gain (Idris and Ali, 2014). Wireless communication is the fastest growing segment of the communication industry. Cellular systems have experienced geometric growth over the last decade and there are currently around three billion users worldwide. Wireless means of communication has become a critical business tool and are part of everyday life in most developed and developing countries and are gradually replacing the wired line system of communication in many countries.

In MIMO system digital modulation techniques such as the M-arry modulation schemes deliver higher data rate effectively in multipath fading channels, as they achieve better bandwidth efficiency and give higher data rate compared to other digital transmission systems (Suthar el at, 2010). When digital modulation is combined with MIMO system a blindfold of spectral efficiency is attained.However, in mobile radio communications, the radio channel puts limits on the performance of communication system due to various impairment such as multipath propagation, time dispersion and fading. These impairments introduce errors like inter-symbol interference (ISI), and other distortion into the signals transmitted over the wireless channel. To combat theseerrors,channel estimation scheme, filters, convolution coding and other schemes are used. The channel estimation scheme combats the effect of inter-symbol interference whereas the convolution coding enables for detection and correction of errors in a digital mobile communication systems.

1.2 Problem Statement

Channel impairments, such as, noise and multipath propagation fading are some of the major problems encountered in wireless communication channel. Transmission of multiple signals using multiple antennas at the transmitter results into overlapping of the signals in the channel. This causes time dispersion which leads to inter-symbol interference. Also, noise introduce into the transmitted signal affect the system performances by increasing the bit error rateand degrade the signal quality. Some of the approaches employed in combating inter-symbol interference in multipath channel are through the use of OFDM and channel estimation scheme. However, the channel estimation scheme is much complex in diversity, since the channel need to be optimized both in space and time. This poses a major challenge to wireless communication designers in selecting an optimal channel estimator with better performance and low complexity to support future high data rate wireless applications. There is also need to understand the potentials and limitations of the channel estimation scheme when used in wireless communications systems characterized by very high data rates, high mobility and presence of multiple antennas.

1.3Thesis Aim and Objectives

The aim of this thesis is to develop an improved MIMO channel estimation scheme for multipath wireless system. This aim of the thesis can be achieved through the following objectives:

1. To determine the effective way of minimizing channel impairments in MIMO channels
2. To develop an improved MIMO channel estimation scheme for successive interference cancellation.
3. To carry out a comparative analysis performance of various channel estimation scheme on the basis of bit error rate on energy per bit/noise density.

1.4Methodology

1. To adopt a model for multipath channel estimation scheme antenna based on spatial diversity in a Rayleigh flat fading channel through:

- A random binary sequence of 1's and 0's was generated as the data input
- The generated data is then modulated using BPSK modulation
- The modulated data is then grouped into pairs of symbols and sent in on time slot
- The signals are then passed through a multipath fading channel with Additive White Gaussian Noise
- After passing through a multipath channel, equalization (Zero Forcing) is performed on the received signal
- The received symbols are Demodulate using BPSK demodulation
- The Bit Error Rate is then calculated by comparing the transmitted signal to the received signal.

- Simulation of the model parameters using Mat-lab Software

1.5 Significance of the Thesis

High data rate broadband wireless access is demanded by various users with different applications. In order to meet such user requirement, various technologies such as the MIMO and spatial diversity are introduced for capacity and spectrum efficiency. However, due to the nature of the wireless environment, especially the wideband where there is presence of large signals scattered along the propagation path, inter-symbol interference occurred, as a result error introduced into the transmitted signal. To mitigate this effect channel estimations are employed to compensate for the channel induced interference due to multi-path propagation and inter-symbol interference.

1.6 Contribution to Knowledge

By introducing successive interference cancellation algorithm into zero forcing estimation, there was 3% improvement and bit error rate reduction.Also,by introducing successive interference cancellation algorithm into minimum mean square error estimation, there was 4.7%improvement and bit error rate reduction.The knowledge of the channel estimation scheme is used to model the theoretical BER. The model scheme was derived as a function of energy per bit per noise density (E_b/N_o) and the bit error rate (BER).

1.7 Limitation of the Thesis

The channel estimation algorithms considered in this thesis depends on the condition whereby the channel information is been known at the receiver. In order meet this requirement, the channel must be estimated at the receiver side. Hence perfect channel estimation is of utmost important. Therefore a critical issue in this thesis is ensuring perfect channel estimation especially at the receiver side.

1.8 Thesis Outline

This thesis is organized as follows:

The first chapter briefly introduced the thesis work and it contains the thesis aim, objectives, statement of problems, scope of the thesis and the thesis limitation. Chapter two gives a brief review of a related work on the thesis topic and also the origin of MIMO technology. Chapter three discusses the methodology adopted in achieving the thesis objectives. Chapter four gives results obtained from the methodology. Chapter Five discusses the limitations, conclusion and recommendations for further work.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter presents a fundamental concepts and review of prior works done similar to the study area. The principle of MIMO transmission system and it gain has been presented, MIMO information theory, digital modulation techniques and channel coding has been discussed.

2.2 HISTORY OF MIMO

MIMO is often traced back to 1970s research papers concerning multi-channel digital transmission systems and interference (crosstalk) between wire pairs in a cable bundle: AR Kaye and DA George (1970),Branderburg and wyner (1974), and W. van Etten (1975,1976). Although these are not examples of exploiting multipath propagation to send multiple information streams, some of the mathematical techniques for dealing with mutual interference proved useful to MIMO development. In the mid-1980s Jack Sals at Bell Laboratories took this research to a step further, investigating multi-user systems operating over “mutually cross coupled linear networks with additive noise sources” such as time division multiplexing and dually-polarized radio system.These methods were developed to improve the performance of cellular radio networks and enable more aggressive frequency reuse in the early 1990s. SpaceDivision Multiple Access (SDMA) uses directional or smart antennas to communicate on the same frequency with users in different locations within range of the same base station. An SDMA system was proposed by Richard Roy and Bjorn Ottersten, researchers at array Communication in 1991.At this time, however, the processing power necessary to handle MIMO signals was too expensive to be practicable. Later on, in 1985, Jack Winters and Jack Salz published a paper on beam forming techniques describing a way of sending data from multiple users on the same frequency/time channel using multiple antennas at the transmitter and receiver (Salz, 1985). This ground breaking research opened ways for researches in the field of MIMO.Winters (1987) studied the fundamental limits on the data rate of multiple antenna systems in a Rayleigh fading environment. With multiple (M)transmit and multiple (M)receive antennas, up to multiple independent channels can be established in the same bandwidth. The results showed the potential for large capacity in systems with limited bandwidth. The research can only be viewed as an attempt to increase system capacity for systems with limited bandwidth using multiple transmit and multiple received antennas. There was no insight into the extent to which capacity growth is attained. This led Foschini and Gans (1998), to look into the capacity of MIMO systems using multi-element array technology, which is processing the spatial dimension to improve wireless capacities in certain applications. The result showed that capacity increased linearly with increase in number of transmitter and receiver antenna rather than logarithmically as in the case of Multiple-Input Single-Output (MISO) or Single-Input Multiple-Output (SIMO). The drawback for this research was that, the linear relationship only holds true for independently identically distributed (i.i.d) flat Rayleigh fading channel and did not held true for all cases. Also for large number of antennas packed into small volumes, the linear relationship collapsed due to the effect of antenna correlation.

Apart from capacity gain, also using multiple antenna system can provide diversity by transmitting multiple copies of the same information across multiple fading channels to achieve higher reliability. Winters (1998) studied the ability of transmit diversity to provide diversity benefit to a receiver in a Rayleigh fading environment. The results showed that for 20-30 antennas, transmit diversity can achieve diversity gain of about 0.1dB for receive diversity. This implied that, the same diversity benefit can be obtained at remote and base station using multiplebase-station antennas only. The main drawback of the research was the fact that an ideal Maximum Likelihood Sequence Estimation with perfect channel estimation was considered. But in practice, when increasing the number of antennas, at some point the degradation due to channel estimation error may become greater than the increase in diversity gain.

Other studies have shown that MIMO systems provide a considerable increase in throughput and link range reliability without additional power or bandwidth augmentation (Li et al, 2002). This made it suitable for high data-rate multimedia application transmission. However, as the MIMO channel bandwidth became larger relative to the channel's multipath delay spread, the channel suffered from Inter-symbol interference(ISI) similar to the case of Single-Input Single-Output (SISO) (Goldsmith, 2005). Researchers proposed two approaches in dealing with ISI in MIMO channels. These included the use of a channel equalizer and/or a multicarrier modulation techniqueOFDM.

Tarik and Samir (2005) addressed the problem of soft equalization and detection of coded MIMO symbols transmitted over a frequency selective fading channel. They developed a low complexity MMSE-based approach to iteratively equalizing MIMO-ISI channels where both interference cancellation and computation of the soft information were performed in a successive fashion. The approach decoupled ISI equalization and Multiple Access Interference (MAI) suppression, and allowed the ISI cancellation part to benefit from the soft output of the MAI resolution stage. Monte-Carlo simulations demonstrated attractive performance even under the presence of the proposed method of imprecise channel state information (CSI) at the receiver. The major drawback of the research was the fact that it was based on Minimum Mean Square

Error (MMSE) equalization only, other equalization types such as Zero Forcing(ZF), Maximum Likelihood(ML) etc. was not considered.

Jiang et al (2005) analyzed the performances of the ZF and MMSE equalizers as applied to multiple transmitter and receiver antennas in wireless MIMO systems. In terms of output Diversity-Multiplexing (D-M) and gain tradeoff, the noise power density and the bit-error rate reduces, as the number of the transmitting and receiver antennaincreases. They showed that there was a gap between the output Signal-to-Noise Ratio of ZF and MMSE equalizers. However, this work compared only two types of equalization algorithms ZF and MMSE. Other equalization schemes like ML DF and Blind equalizers were not considered.

Gurpreet and Pardeep(2008) presented a paper which focused on equalization techniques, for Rayleigh flat fading. In their presentation equalizers wereused to combat the effect of ISI. In this paper they discussed different types of equalizers such as ZF, MMSE, decision feedback and ML. In this paper they compare different equalizers with different modulation techniques like BPSK, QPSK,16-QAM and found out which modulation techniques is better than the others and then compared ZF, MMSE, ML with best modulation techniques at BER=0.001. In their comparison between the different detectors ML had a better performance, while ZF had the worst performance. However, this work did not specify the number of transmitting antennas and the number of receiving antennas and also compared just three equalization schemes, without considering other equalization such as Decision Feedback, Turbo Code equalization, and Blind equalizers.

Shaodan and Tung-Sang (2008) presented a semi-blind time-domain equalization technique for general (MIMO-OFDM) systems. In the proposed technique, the received OFDM symbols are shifted by more than or equal to the cyclic prefix (CP) length, and a blind equalizer is designed to completely suppress both inter-carrier interference (ICI) and ISI using second-order statistics of the shifted received OFDM symbols. Simulations showed that, the proposed technique outperformed the existing techniques, and it was robust against the number of shifts in excess of the CP length. However, the authors assumed perfect time and frequency synchronization for the research which cannot be achieved in practical terms.

Bara'u and Xiaoheng (2011) studied the effect of ZF and MMSE equalizers in MIMO System. They simulated a 2X2 MIMO system using MATLAB to determine the effect of these equalizers in combating ISI. They found out that MIMO transmission with MMSE equalization offers greater performance over ZF equalization in the region of about $3dB$ which ultimately helped in reducing the effect of ISI in MIMO communication systems. The drawback for the research includes;(i) only two equalizers were compared which were both linear in nature and not optimal (ii) the 2X2 MIMO system model was too basic as a better performance could be achieved by increasing the number of antennas.

Sathish and Shankar (2011) extended the work done by introducing an optimal equalizer which was non-linear in nature. The non-linear equalizer introduced was the ML equalizer. The result showed that, the ML equalizer outperformed the two other linear equalizers (ZF and MMSE) although it had more complexity in design than the linear equalizers. However, the complex structure of this equalizer made it difficult to use, especially with growing number of delay taps constituting a drawback.

Agrawal et al, (2012) investigated on improving the BER performance of MIMO receiver using ZF, MMSE and ML equalization. QPSK modulation was employed at the transmitter to modulate the symbols. Three types of antenna configurations were used for the simulation: 2X2, 3X3 and 4X4 respectively. The simulation result showed that higher MIMO configuration

4X4gave better BER performance. However, the drawbacks for the research include (i) QPSK modulation has low BER performance in multipath Rayleigh fading channel and hence can affect the overall system performance. (ii) The research considered only the case of equal number of transmits and receive antenna. A better performance could be achieved if the number of receiving antennas were more than the transmitting antennas because more receiving antennas translate to high diversity gain,(winter) 1998.

Ananya et al. (2012) present in details analysis of the ZF, MMSE and Successive Interference Cancellation (SIC) equalizers using Vertical Bell Labs Layered Space-Time (V-BLAST) techniques. Here they study the SIC receiver, incorporating ZF and MMSE receivers which gives improved performance with less complexity. Also the Bit Error Rate (BER) is compared using the above mentioned detectors and find that ZF-SIC and MMSE-SIC performance are superior to conventional ZF and MMSE detectors, this paper does not include ML and feedback, feed forward equalizer and only used 2 transmitter and 2 receiver antennas.

Shreedhar. A. Joshi et al. (2012) provide a general multiple antenna system MIMO with the Vertical-Bell Laboratories Layered Space Time (V-BLAST) technique using several detectors (MMSE, ML, ZF) and conclude that the performance is limited by error propagation. They show the benefits of ordering strategy over SIC cancellation methods. However , the drawback of Bell Laboratories Layered Space Time algorithms is the propagation of decision errors, unequal diversity advantage for each symbol; and the major drawback of this paper is that the number of transmit and receive antennas are not specified and other equalization technique was not considered such as turbo code equalization and decision feedback equalizer.

Lanjewar Rajesh Krushnarau et al(2015) Performance analysis ofMIMO OFDM systems for wireless communication system using adaptive modulation technique to offer a simple and high performance system for high channel capacity and data rate. Although the OFDMA concept is simple in its basic principle, but it suffers one of the most challenging issues, which is synchronization error that introduces the ISI, thus degrades the signal performance. The goal of their paper is to provide a method to mitigate ISI by employing equalizers at the receiver end and using Space Time Block Codes (STBC) to improve the BER performance and to achieve a Maximum diversity order in MIMO-OFDMA by using simulation based on the platforms of MATLAB software. However, the method employed in the research was based on STBC only. Space time trellis code which is another Space-Time code types was not considered which had much more coding gain than STBC.

Paresh and Nisha (2015) Investigate comparative performance of MIMO channel estimation techniques such as ZF, MMSE and STBC (Alamouti code) for MIMO system and presented in Mat-lab using modulation techniques like BPSK and QPSK. These techniques are compared effectively to estimate the channel in MIMO systems. STBC gives better performance in terms of bit error rate and signal to noise ratio.

To overcome some of the limitations mentioned in the reviewed works of **Jiang et al (2005), Tarik and Samir (2005), Bara'u and Xiaoheng (2011), Agrawal et al, (2012)**; used (i) Less number of transmit and received antennas (ii) Non-optimal performance nature of ZF and MMSE .

2.3 Overview of Digital Mobile Communication

As time passes, the digital communications becoming more dominant over analogue communication. The main factors for this situation are the increasing performance and the decreasing cost of digital communication equipment. (Wong and Lok, 2004).Electrical communication which involves the processing, sending and receiving of information by electrical means encompasses three basic stages which are the source transmitter, the propagation medium (channel) and the destination (receiver) as shown in figure 2.1. Each stage is characterized by various processing technique of the information signal (Frenzel, 2012). The processes at the transmitting end are sampled (which involves obtaining samples of the analogously varying signal), quantization (breaking the analogously varying signal into set of discrete values) encoding (representation of information in bits of zeros and ones) and lastly modulation (transforming the information into suitable signal waveforms that can be carried in the channel dedicated for communication).

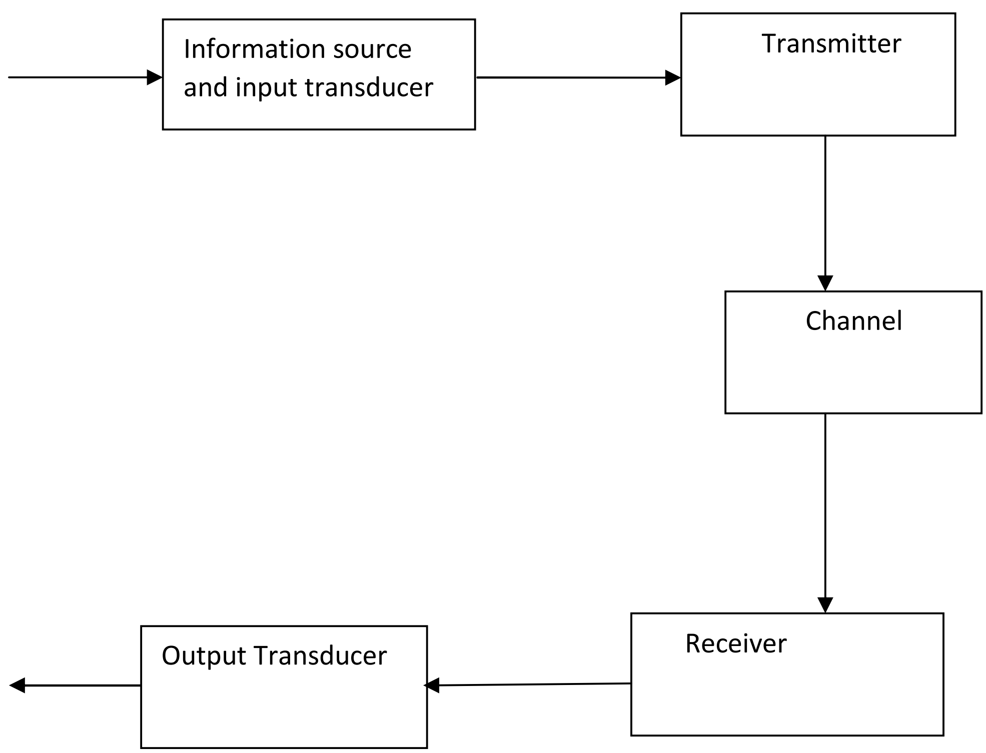


Figure 2.1 Functional block diagram of a communication system

Source of information can be either natural or human made. An example of information source is human speech. Digital communication systems represent information, irrespective of its type or origin, by discrete set of allowed symbols. It is this alphabet of symbols and the device or mechanism which selects them for transmission that is usually regarded here as the information source (Wong and Lok, 2004).

2.3.1 Wireless Channel Modelling

Wireless communication is one of the most active areas oftechnologydevelopment and has become an ever-moreimportant and prominent part of everyday life. Simulation of wireless channels accurately is very important for the design andperformance evaluation of wireless communication systems andcomponents (Harada and Prasad, 2002). Fading or loss of signals is a phenomenon that affects wireless communications system. Hence, this thesis tries to describe thefading patterns in different environments and conditions.Although, no model can perfectly describe an environment,they strive to obtain as much precision as possible. The better amodel can describe a fading environment, the better can it becompensatedwith other signals, so that, on the receiving end,the signal is error free or at least close to being error free; thiswould mean higher clarity of voice and higher accuracy of data transmitted over wireless medium. An important issue inwireless application development is the selection of fadingmodels (Foschini and Gans, 19989).

2.4Fading and Multipath

Fading refers to the distortion that a carrier-modulatedtelecommunication signal experiences over certain propagationmedia. In wireless systems, fading is due to multipathpropagation and is sometimes referred to as multipath inducedfading (Babu and Rao, 2011). To understand fading, it is essential to understandmultipath. In wireless telecommunications, multipath is thepropagation phenomenon that results in radio signals' reachingthe receiving antenna by two or more paths (Haque, 2010). Causes of multipathinclude atmospheric ducting, ionospheres reflection andrefraction, and reflection from terrestrial objects, such asmountains and buildings.The effects of multipath includeconstructive and destructive interference, and phase shifting ofthe signal.This distortion of signals caused by multipath isknown as fading. In other words it can be said that in the real-world, multipath occurs when there is more than one pathavailable for radio signal propagation. The phenomenon of reflection, diffraction and scattering all gives rise to additionalradio propagation paths beyond the direct Line of Sight path between the radio transmitter and receiver (Foschi and Gans,1998).

A Fading Channel is known as communications channel which has to face different fading phenomenon's, during signaltransmission. In real world environment, the radio propagationeffects combine together and multipath is generated by thesefading channels. Due to multiple signal propagation paths,multiple signals will be received by receiver and the actualreceived signal level is the vector sum of all the signals. Thesesignals incident from any direction or angle of arrival.Inmultipath, some signals aid to direct path and some otherssubtract it (Goldsmith and Jafar, 2005).

2.5Causes of Fading

Fading is caused by different physical phenomenon which includes reflection, diffraction scattering and Doppler shift. These causes are explained as follows:

2.5.1Doppler Shift

When a mobile is moving at a constant velocity Valong a path,Vsis the velocity of the source, $F'$is the observed frequency and *F*is the emitted frequency (Fumuyaki, 1996). All these terms will be related by theFollowing equation:

$$F' = \left(\frac{V}{(V+Vs)}\right) \times F \qquad (2.1)$$

The above equation shows that the detected frequency increasesfor objects moving towards the observer and decreases when the source moves away. This phenomenon is known as the Doppler-Effect.

2.5.2Reflection:

When a propagating electromagnetic wave impinges on objectwhich has generated large dimensions wave length, compared to wavelength of the propagating wave, then reflection will occurred. Actually we know that if the planewave is incident on a perfect dielectric, part of the energy istransmitted and part of the energy is reflected back into themedium. If the medium is a perfect conductor, all the energy isreflected back. Reflections occur from the surface of the earthand from buildings and walls. In practice, not only metallicmaterials that cause reflections, but dielectrics also cause thisphenomenon (Amit et al, 2013).

2.5.3 Diffraction:

The sharp irregularity edges of a surface between transmitter and receiver that obstructs the radio path then diffraction occurs. The bending waves around the obstacle, even when a LOS does not exist between transmitter and receiver the secondary waves will be spread over the space. Diffraction lookslike a reflection at high frequencies depends on the amplitude,phase and polarization of the incident wave and geometry of theobject at the point of diffraction.

2.5.4Scattering:

The wave travels through the medium consists of smallerdimension objects compared to the wavelength and havinglarger volumes of obstacles per unit volume, then scattering willoccurred. Due to rough surfaces, small objects and irregularitiesin the channel scattered waves are produced. In practice, inmobile communications, electrical poles and street signs etc. Induces scattering (Babu and Rao, 2011) in communication.

2.6 Principle of MIMO Transmission System

MIMO system uses multiple transmitters and receivers antennas. A digital source in the form of binary bits when fed to transmitting block for coding and mapping using modulation format of Binary Phase Shift Keying. This produces several separate symbol streams in which each is then mapped onto one multiple transmit antennas. The signals are then launched into the wireless channel. At the receiver, the signals are captured by the multiple antennas and demodulation, decoding and de-mapping operations are performed to recover the information.

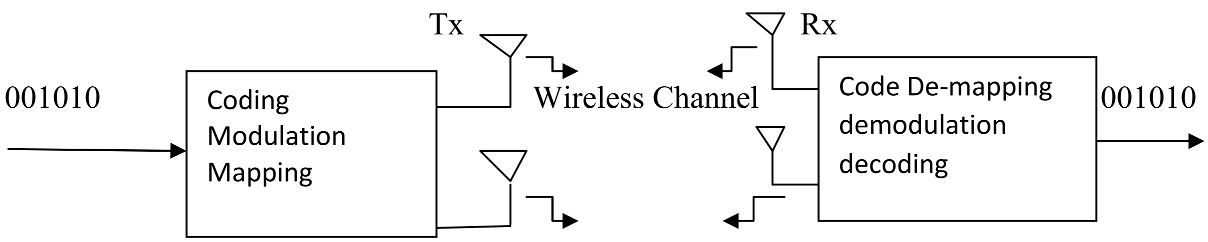


Figure 2.2: A MIMO wireless transmission system

However, in MIMO systems two types of gain can be achieved, spatial multiplexing gain and diversity gain.

2.6.1 Multiplexing Gain

Spatial diversity can be achieved with the help of multiple transmit antenna (transmit diversity) or multiple receive antenna (receive diversity) with sufficient spacing between the antennas. The main idea of transmit diversity is to provide a diversity and/or coding gain by sending redundant signals over multiple transmit antennas. At the receiver we use a combining scheme which eliminates spatial interference from other antennas. In all these schemes it is assumed that the channel state information is known to the receiver.

Spatial multiplexing in MIMO system offers a linear increase in capacity as the number of transmitting and receiving antenna pair increases in the transmission rate or capacity (Penchong and Hong, 2009) for the same bandwidth and with no additional power expenditure. Multiple independent spatial channels are created in order to provide multiplexing gain. Independent information sequences can be transmitted on each channel at the same time. The maximum number of channels is the number of transmitter antennas (Tx) and the number of receiver antennas (Rx). As both Tx and Rx increases, the number of spatial channels increase linearly. The system capacity also increases linearly (Penchong and Hong), 2009.Using singular value decomposition (SVD) from matrix theory, a matrix can be decomposed as follows;

$$H = USV^H \tag{2.2}$$

Where U and V are unity matrix of size $M \times M$ and $N \times N$ respectively, and the superscript H stands for Hermitan transform. Unity means M×M = I

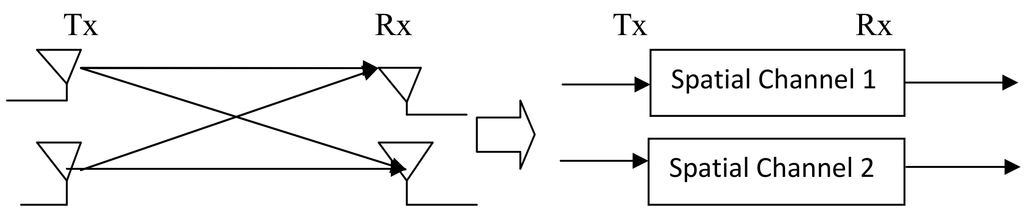


Figure 2.3 MIMO multiplexing

And S is a diagonal matrix. The signal is processed by multiplying Uat the transmit side and V at the receive side, then the equivalent channel is given by

$$H'=U^H HV = (U^H U)S(V^H V) = \begin{pmatrix} S_{11} & 0 \\ 0 & S_{22} \end{pmatrix} \tag{2.3}$$

H' is a diagonal matrix, with gains and. The channel equation will become

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}=\begin{pmatrix} S_{11}.X1 \\ S_{22}.X2 \end{pmatrix} \tag{2.4}$$

This indicates that two independent channels are created when considering 2×2 antenna systems

2.6.2 Diversity Gain

Multipath fading is a significant problem incommunications. In a fading channel, signalexperiences fade (i.e they fluctuate in theirstrength).When the signal power dropssignificantly, the channelis said to be indeep fade. This gives rise to high BER.Thediversity is used to combat fading. This involvesproviding replicas of thetransmitted signal over time,frequency, orspace. In MIMO systems, each pair of transmitting and receiving antennas provide a signal path from the transmitter to the receiver and each path carries the same information simultaneously, the signal achieved in the receive antenna is more reliable and the fading can be effectively decreased. For instance, instead of two independent information sources, we can send some processed representation of a single information source in order to achieve the diversity. The four channels can be seen as independent faded branches. So the MIMO channel now has a diversity order of 2 × 2 = 4. For the generalized MIMO channel, the diversity order is transmitterandreceiverantenna(Bara'u and Tan Xiaoheng 2011).

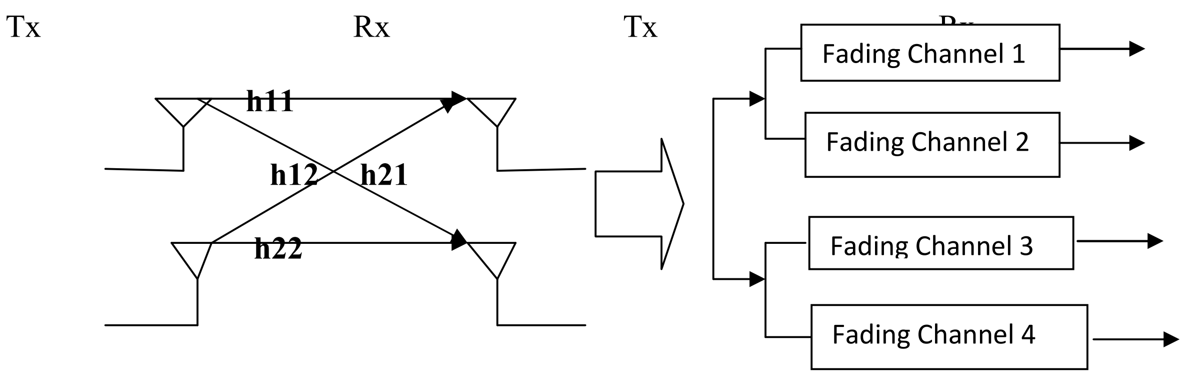


Figure 2.4 MIMO Diversity

2.7 MIMO Information Theory

For a SISO system, the maximum mutual information rate corresponding to the maximum data rate that can be transmitted over the channel with arbitrarily small error probability is given by the Shannon's formula (Goldsmith et al, 2002).Which is given as follows

$$C = log_2 1 + p \,|\quad|^2 \ bps/Hz \qquad (2.5)$$

Where h is the normalized complex gain of a fixed wireless channel or that of a particular realization of a random channel. ρ is the SNR at any receiver antenna.

As the number of receiving antennas increased, the statistics of capacity improve and with transmitter and receiverantennas, SIMO system is formed. The expression for capacity is now given by:

$$C = log_2(1 + \rho \sum_{m=1}^{n} | \quad _m |^2) bps/Hz \quad (2.6)$$

Where $_m$ is the gain for receiverantenna? It is worth mentioning that in the previous formula, increasing the value of N_R only results in a logarithmic increase in the average capacity (Gesbert et al, 2003). Now, for the case of Multiple-input Single-output (MISO), where there are transmit antennas for transmit diversity, in this common case the transmitter does not have the channel knowledge. So the capacity is given by (Foschini and Gans, 1998)

$$C = \log_2(1 + \rho / NT \sum_{n=1}^{Nr} | \quad _n |^2) \; bps/Hz \quad (2.7)$$

C is the channel capacity, $_n$ is the transmitter antenna gain according to (Gesbert et al.2003), in a SIMO and MISO system the capacity increases logarithmically as the number of transmit and receive antenna increases.

2.7.1 MIMO System Capacity

Now, let us consider the use of diversity at both transmitter and receiver for a MIMO system. For multiple transmitter and receiverantennas, the equal power capacity increases according to (Foschini and Gans, 1998).

$$C_{EP} = log_2 \left[det \left(I_M + (\frac{p}{N_r}) \right) HH' \right] \text{bps/Hz} \quad (2.8)$$

Where det. denotes the determinant of a matrix,I_m is a square identity matrix, ρ is the average received SNR, and H' is the complex conjugate transpose of H. (Foschini,1998) and Telatar,(1995) both demonstrated that capacity in MIMO systems with equal power grew linearly rather than logarithmically as in MISO and SIMO systems. The characteristics of H can be investigated by performing singular value decomposition (SVD) of H to diagonalizable H and find the eigenvalues. An SVD expansion of any matrix H ($N_R \times N_T$) can be written as (Telatar, 1995).

$$H = UDV' \tag{2.9}$$

Where, U number of receiver and transmitter antenna and are unitary matrices, which means that UU'=VV'= I

2.8Modulation

This is the process of varying one or more properties of periodic waveform (Frequency, amplitude or Phase) called the carrier signal, with a modulating containing the information to be transmitted. In telecommunications, modulation is the process of conveying a message signal, for example a digital bit stream or an analog audio signal, inside another signal that can be physically transmitted. Thisalteration is called modulation, and it is the modulated signalthat is transmitted. The receiver then recovers the original signalthrough a process called demodulation.Modulation techniques are expected to have three positiveproperties

(Frenzel, 2012):

- Good Bit Error Rate Performance: Modulation schemesshould achieve low bit error rate in the presence offading, Doppler spread, interference and thermal noise.
- Power Efficiency: Power limitation is one of the criticaldesign challenges in portable and mobile applications.Nonlinear amplifiers are usually used to increase powerefficiency. However, nonlinearity may degrade the bit error rate performance of some modulation schemes.Constant envelope modulation techniques are used to prevent the re-generation of spectral side lobes during nonlinear amplification.
- Spectral Efficiency or the bandwidth efficiency refers to the information rate that can be transmitted over a given bandwidth in a specific communication system.

2.9 Digital Modulation

Digital modulation schemes transform digital signals intowaveform that are compatible with the nature of thecommunications channel. One category uses a constantamplitude carrier and the other carries the information in phase or frequency variations Frequency Shift Keying, Phase Shift Keying. A major transition from thesimple amplitude modulation and frequency modulation to digital techniques such as Quadrature Phase Shift Keying, FSK, MSK and QAM (Frenzel, 2005).

2.10 Quadrature Amplitude Modulation:

QAM is the encoding of the information into a carrier wave byvariation of the amplitude of both the carrier wave and a 'quadrature' carrier that is 90^0 out of phase with the main carrierin accordance with two input signals (Araora et al.2011),that is, the amplitude andthephase of the carrier wave are simultaneously changedaccording to the information you want to transmit.

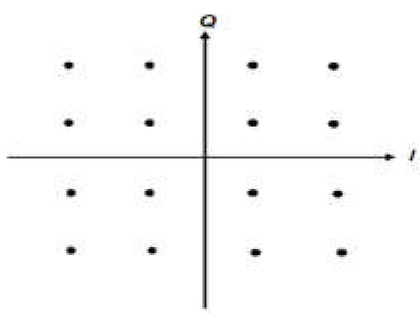


Figure 2.5 16-QAM Constellation (source Frenzel, 2005)

2.11(64-Quadrature Amplitude Modulation):

64-QAM is the same as 16-QAM except it is 64 possible signalcombinations with each symbol represent six bits (2^6 =64). 64-QAM is a complex modulation technique but gives highefficiency.

This digital frequency modulation technique isprimarily used for sending data downstream over a coaxial cablenetwork. 64QAM is very efficient, supporting up to 28bps (Yahong et al,, 2003). Peak transfer rates over a single 6-MHz channel. But 64QAM'ssusceptibility to interfering signals makes it unsuitable to noisyupstream transmissions.

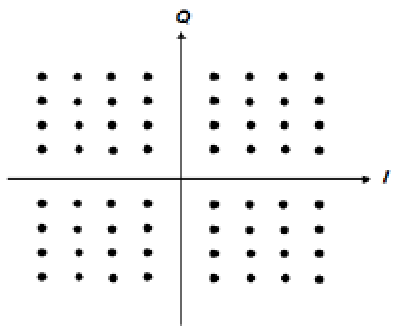


Figure 2.6 Constellation Diagram for 64-QAM (Source Frenzel, 2005)

2.12 Differential Phase Shift Keying:

Differential phase shift keying (DPSK), a common form ofphase modulation conveys data by changing the phase of carrierwave. In Phase shift keying, High state contains only one cycle.

But DPSK contains one and half cycle. DPSKis a modulation technique that codesinformation by using the phase difference between twoneighboring symbols. In the transmitter, each symbol ismodulated relative to the previous symbol and modulatingsignal, for instance in BPSK 0 represents no change and 1 represents 180 degrees. In the receiver, the current symbol isdemodulated using the previous symbol as a reference. Theprevious symbol serves as an estimate of the channel. A nochange condition causes the modulated signal to remain toremain at the same 0 or 1 state of the previous symbol.Differential modulation is theoretically 3 dB poorer than coherent. This is because the differential system has 2 sources oferror: a corrupted symbol, and a corrupted reference.In DPSK, the transmitter, each symbol is modulated relative tothe phase of the immediately preceding signal element and thedata being transmitted. In this paper, we choose 16-DPSK and64-DPSK schemes to analyze BER and SNR in different fadingchannels.

2.13 Binary Phase Shift Keying

In Phase Shift Keying the phase of the carrier is varied to represent binary 0 or 1. Both peak amplitude and frequency remain constant as the phase changes. For example, if a phase of 0 degree represents binary 0, then the phase can be changed to 180 degrees to send binary 1. The phase of the signal during each bit duration is constant and its value depends on the bit 0 or 1 (Harada and Prasad, 2002).

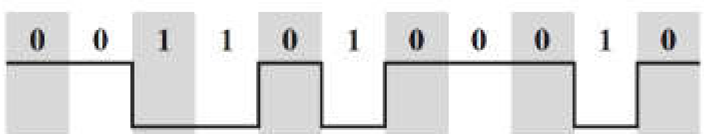


Figure: 2.7 Phase Shifting Keying Scheme(Source Harada and Prasad

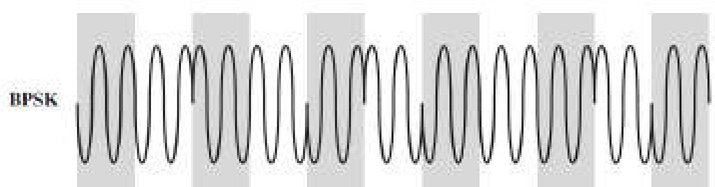


Figure: 2.8 Binary Phase Shift Keying (Harada and Prasad, 2002)

The phase shift keying above is known as 2-PSK or binary phase shift keying, because two different phases (0 and 180) are used.

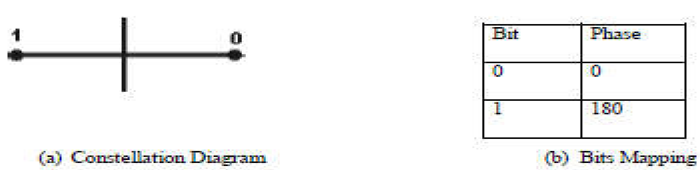


Figure: 2.9 BPSK Constellation (a) Constellation Diagram (b) Bits Mapping

PSK is not susceptible to the noise degradation as ASK, nor the bandwidth limitation of FSK. This means that smaller variations in the signal can be detected reliably by the receiver. The probability of error or BER for 1x1 systems with BPSK modulation in Additive White Gaussian Noise and Rayleigh fading channel is given by equations 2.10. (Harada and Prasad, 2002)

$$P_b = \frac{1}{2}\left(1 \quad \frac{1}{\sqrt{1-\frac{1}{\frac{E_b}{N_o}}}}\right) \qquad (2.10)$$

P_b is Probability of error in the system, E_b is the energy per bit, and N_o is the noise power density

2.14 Rayleigh Fading Channel

The delays associated with different signal paths in a multipath fading channel changes in an unpredictable manner and can only be characterized statistically. When there are a large number of paths, the central limit theorem can be applied to model the time-variant impulse response of the channel as a complex-valued Gaussian random process. When the impulse response is modeled as a zero mean complex-valued Gaussian process, the channel is said to be a Rayleigh fading channel. Rayleigh fading is most applicable when there is no dominant propagation along a line of sight between the transmitter and receiver (Tse and Viswanath, 2005).

2.15 Channel Estimation

Channel estimation is an important technique especially in mobile wireless network systems where the wireless channel changes over time, usually caused by transmitter and/or receiver being in motion at vehicular speed. Mobile wireless communication is adversely affected by the multipath interference resulting from reflections from surroundings, such as hills, buildings and other obstacles (Naik and Nisha, 2015). In order to provide reliability and high data rates at the receiver, the system needs an accurate estimate of the time-varying channel. The different channel estimation techniques used in this paper are discussed below. The block diagram of channel estimation as shown below:

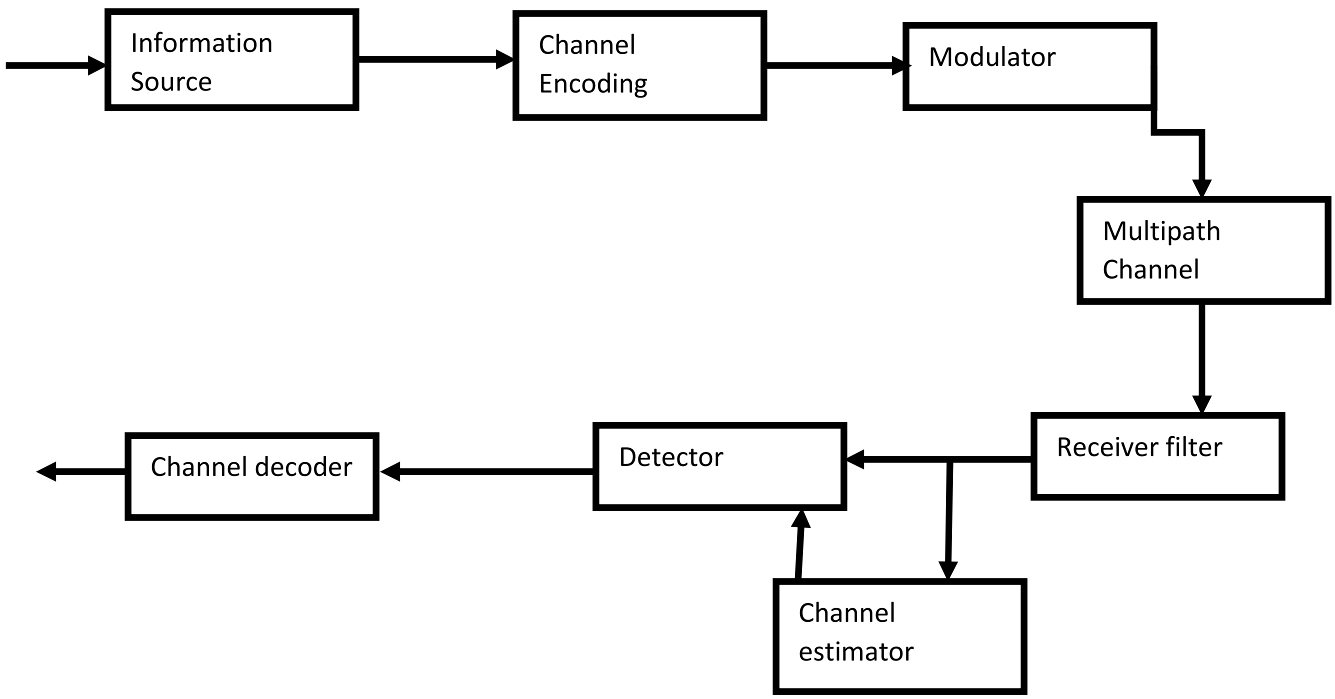


Figure 2.10: Block diagram of channel estimation

2.16 Channel Estimation Equalization

Equalization is defined as a signal processing technique used at the receiver to alleviate the problem of ISI caused by channel distortions (Goldsmith, 2005). This is achieved by the use of an adaptive digital filter (equalizer) at the receiver output. The channel frequency response of practical wireless channels is not flat due to multipath and other channel distortions. Hence, applying an equalizer flattens the channel frequency response and hence reduces the effect of ISI (Li et al, 2006). It was clear that the channel delay spread plays a vital role in causingISI and hence contributes to an irreducible error floor. Several techniques were proposedin the past in order to mitigate the effects of ISI. Equalization is one such techniqueemploying signal processing methods at the receiver side to alleviate the effects of ISI.Equalization can also be implemented at the transmitting side, but receiverimplementation is most common given the diversity of the channel (Singh and Sharma, 2008). Delay spread controlmeasures can also be provided through antenna solution. Due to the limited scope of thisthesis, we concentrate only on signal processing techniques employed at receiver level.When RMS delay spread is greater than the channel symbol time, an irreducible errorfloor is formed. Digital communications involving high data rate applications usuallyrequire high performance equalizers. Mitigating the effects of delay spread is consideredone of the major hurdles in designing a high data rate digital communication system.Whenever a good equalizer is designed, a balance has to be maintained by not enhancingthe noise power in the received signal (Amit et al, 2013) in the process of mitigating ISI. Noise powerenhancement is a common problem in equalizer design; hence, a good equalizer designshould not enhance the noise power in the received signal.

Channel estimation equalizer fall into two broad categories: linear and nonlinear. Linear equalizers suffer more from noise enhancement than the non-linear equalizers, butthe later has higher complexity. Moreover, the equalizer should adapt itself to fluctuatingchannel conditions by understanding the channel impulse so as to reduce the effects of ISI (Yusuf et al 2008).

Among the equalizer types, linear equalizers suffer from noise enhancement or frequencyselective fading and are, therefore not used in wireless communication systems.Equalizers can also be classified as symbol-by-symbol (SBS) detectors or sequenceestimators (SE). All linear equalizers as well as Decision Feedback Equalizers (DFE)belong to the SBS category where ISI is eliminated at symbol level by detecting eachsymbol individually. On the other hand, SE equalizers detect sequences of symbols;sothe effect of ISI is a part of the estimation process. MMLSE belongs to this category and is an optimal equalization technique. The main draw back with MLSE is that the complexity grows exponentially with increase in channel memory length (Penchong and Hong, 2009).

2.17 Linear Channel Estimation Equalizers

Linear equalizers usually consist of a transversal filter with coefficients $C_i$chosen toovercome the effects of the channel impediment. The coefficients are applied to versions of the received signals delayed by the symbol interval T, resulting in a symbol-spaced equalizer. The output of the filter is given by (Saunders and Zavala, 2007) as;

$$\widetilde{U} = \sum_{i=m}^{m} C_i . y(k \quad 1) \qquad (2.11)$$

There are two types of linear channel estimation equalizers: ZF and MMSE which shall be discussed shortly. The former equalizer cancels all ISI, but can lead to considerable noise enhancement. The latter technique minimizes the expected mean squared error between the transmitted symbol and the symbol detected at the equalizer output, thereby providing a better balance between ISI mitigation and noise enhancement.

Table 2.1: Showing the literature review of the previous works done on the topic, pros and cons

Author Name and Year	Works	Pros	Cons
Jiang et al 2005	Analyzed the performance of ZF and MMSE channel estim ation equalizer applied multiple transmitter and receiver antennas in wireless MIMO system.	The ZF Channel estimation is suitable for a noiseless channel and MMSE provide a trade-off between the transmitted signal and the receive	The ZF Channel estimation equalizer leads to the amplification of noise and the

		signal	MMSE can lead to Zero Forcing at high noise
Bara'u and Xiaoheng (2011)	Studied the effect of ZF and MMSE channel estimation in MIMO system using 2×2 antennas	The two channel estimation scheme used are less complex and also lead to Maximum Diversity	They are both linear channel estimation which shows that they are not optimal
Sathish and Shankar (2011)	They include an optimal channel estimation equalizer which was nonlinear in nature	It involves both linear and nonlinear channel estimation equalizer	The optimal channel estimation equalizer can lead to increase in complexity of the system
Agrawal et al (2012)	Investigated and improvement in BER performance of MIMO receiver using ZF,MMSE and ML channel estimation equalizer and modulation format of QPSK	In this work the performance of the signal at the receiver increases with increase in number of transmitting and receiving antenna	The modulation format of QPSK is not suitable in multipath Rayleigh channel and it

	using 2×2, 3×3, 4×4, etc		only consider equal number of both transmitting and receiving antenna
Azlina et al (2014)	They analyzed combination of multiple input multiple output and orthogonal frequency division multiple access to offer a simple and high performance system to increase the channel capacity	This work leads to increase in the data rate of the transmitted signal and it does not requires channel estimation equaliz ers	Absence of the various channel equalizers will affect the efficiency of the synchronization error that introduce the inter-symbol interference, this degrade the signal performance

2.18 Conclusion

In this chapter, brief history of MIMO system and prior works were reviewed and presented. Digital mobile communication, wireless channel modeling, fading in multipath channel, MIMO information theory and fundamental concept of MIMO transmission system was presented, channel estimation scheme equalizers was also presented, and finally the table showing the previous works their pros and cons was also presented.

CHAPTER THREE

METHODOLOGY

3.1 INTRODUCTION

This chapter provides the detailed description of the material and methodology adopted for the thesis. First channel estimation scheme (equalization) was presented. A MIMO system with different configuration was also modeled and simulated with the three different equalization scheme, an improved equalization scheme for the three algorithms were also simulated using Mat-lab Communication tool box. Finally, the output obtained was compared to analyze the effect of the different equalization schemes employed.

3.2Zero Forcing Channel Estimation Equalizer

Zero Forcing equalizer is a form oflinear equalization algorithm which applies the inverse ofthe channel frequency response to the received signal, tore-instate the signal back to its original form in the channel. This form of equalizer was first proposedby Robert Lucky. It has many useful applications. Forexample, it is applied for IEEE 802.16e (Mobile WiMAX)in MIMO, where knowing the channel allows recovery ofthe two or more streams which will be received on top ofeach other on each antenna. The name ZF corresponds to bringing down the ISI to zero and will be useful when ISI is significantcompared to noise.

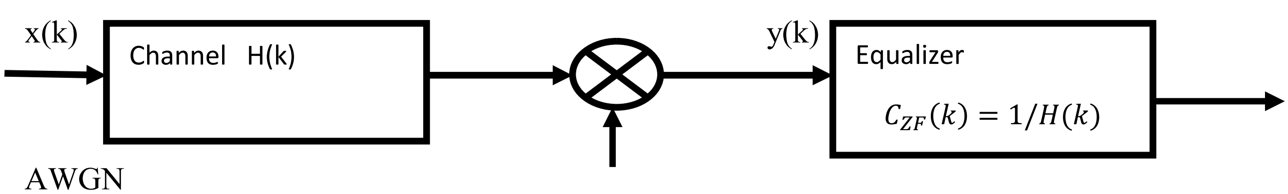


Figure 3.1: Block diagram of Zero-Forcing Channel Estimation equalizer

The above figure 3.1 is an example to show that there are different parameters used in Zero Forcing equalization. Let $C_{ZF}(k)$ be the equalizing circuit filter. $C_{ZF\,(k)}$ is considered to be the ZF equalizer, that can be realized by multiplying the MIMO received signal as in Equation (3.1) with the vector $\frac{1}{H_k}$ which produces the Equation (3.2). In thiscase, the equalizer filter compensates for the channelinduced ISI as well as the ISI, but this is not eliminatingall ISI because the filter is of finite length.

$$y_{(k)} = x_{(k)}.H_{(k)} + w_{(k)} + I_{(k)} \quad (3.1)$$

$$C_{ZF}(k) = 1/H_{(k)} \quad (3.2)$$

$$C_{ZF}(k).H_{(k)} = 1 \quad (3.3)$$

From the above equation,

y(k) represent the receiver signal vector, x(k) transmitted signal vector, H(k) propagation channel parameters, W(k) white Gaussian noise, I(k) signal interferences in the channeltherefore, the combination of channel and equalizer gives a flat frequency response and linear phase. The ZF equalizer is simple to implement but suffers from the problem of noise amplification hence not suitable for channels with high noise characteristics.

3.3 Mathematical Modeling of ZF Channel Estimation Equalization Matrix

Now consider a 2×2 MIMO channel estimation for simplicity, in the first time slot, the received signal on the first receive antenna is shown below in figure 3.2:

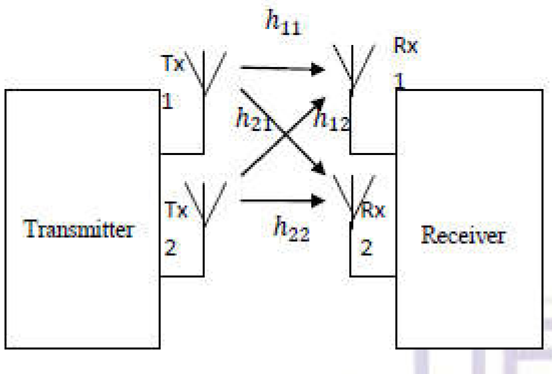


Figure: 3.2 MIMO 2×2 system model

The received symbol on the first receiver is given by

$$y_1 = {}_{11}x_1 + {}_{12}x_2 + n_1 \tag{3.4}$$

The received signal on the second receive antenna is given as:

$$y_2 = {}_{21}x_1 + {}_{22}x_2 + n_1 \tag{3.5}$$

Where;

$y_1, y_2$are the received symbol on the first and second antenna respectively,

${}_{11}$the channel from 1st transmit antenna to 1st receive antenna,

${}_{12}$the channel from 2nd transmit antenna to 1st receive antenna,

${}_{21}$the channel from 1st transmit antenna to 2nd receive antenna,

${}_{22}$the channel from 2nd transmit antenna to 2nd receive antenna,

$x_1, x_2$are the transmitted symbols and

n_1, n_2 are the noise on 1st and 2nd receive antennas

These two above equation can also be written as

$$\begin{pmatrix} y1 \\ y2 \end{pmatrix} = \begin{pmatrix} 11 & 12 \\ 21 & 22 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \end{pmatrix} + \begin{pmatrix} n1 \\ n2 \end{pmatrix} \quad (3.6)$$

It is clear from the above equation that if h_{11}, h_{12}, h_{21}, h_{22} and y_1,y_2 is known then it is easier for the receiver to compute the x_1 and x_2.

$$y = Hx + n \quad (3.7)$$

y = Received Symbol Matrix, H = Channel matrix, x = Transmitted symbol Matrix, n = Noise Matrix

From here it is clear that in order to find x from above equation, we need to find out the matrix which is inverse of matrix H. If W represents the inverse of H then it must satisfy the property

$$WH = I \quad (3.8)$$

Where I is the identity matrix. The matrix W which satisfy the above mentioned property is known as the zero forcing linear detector and is computed by following equation

$$W = (H^H H)^{-1} H^H \quad (3.9)$$

In this equation the matrix $H^H H$ is given by

$$H^H H = \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} = \begin{bmatrix} |\ 11|^2 + |\ 21|^2 & 11\ \ 21 + \ 21\ \ 22 \\ 12\ \ 11\ + \ 22\ \ 21 & |\ 12|^2 + |\ 22|^2 \end{bmatrix} \quad (3.10)$$

This model can be extended to $M \times N$ antenna configuration

From this matrix it is clear that the off diagonal terms arenon-zero and hence zero forcing equalizer cancel out the interference signal. It is reasonably simple and easy to implement but its main drawback is that it tends to amplify the noise and hence gives noisy output.

3.4. Minimum Mean Square Error (MMSE) Channel Estimation Equalizer.

The main feature of MMSE equalizer is that it does not usually eliminate ISI completely but, tries to provide a tradeoff between ISI mitigation and noise enhancement by minimizing the total power of the noise and ISI components in the output.This type of channel estimation equalizer uses the squared error as performance measurement. The receiver filter is designed to fulfill the minimum mean square error criterion. Main objective of this method is to minimize the error between target signal and output obtained by filter.

3.5 Mathematical Modeling of Channel estimation MMSE Equalization Matrix

To extract the two symbols which interfere with each other in the case 2x2 MIMO configuration, the received signal for MMSE algorithm computes the coefficient of matrix W which minimize the condition

$$\mathrm{E}\{[W_y \quad x][W_y \quad x]^{\wedge}H \qquad (3.11)$$

Solving above equation gives

$$W = (H^H H + N_O I)^{-1} \qquad (3.12)$$

H is the channel matrix, N_o is the noise power density I is the identity matrix. From the above equation it is clear that this equation is different from the equation of ZF equalizer. If the channel is noisy, the equation of MMSE equalizer becomes ZF equalizer. This method can be extended to $M \times N$ antenna configuration.

3.6 Non-Linear Channel Estimation Equalizer

Non-Linear equalizers are implemented using a lattice structure. The lattice filter employed in this type of equalizer has a more complex recursive structure than the transversal filter used by the linear equalizers. In exchange for this increased complexity relative to transversal structures, lattice structures often have better numerical stability and convergence properties and greater flexibility in changing their length.

3.7 Maximum Likelihood (ML) Channel Estimation Equalizer

The problem with most equalizers especially linear equalizers is the fact that they are not optimal in terms of minimizing the average symbol error probability. Because of the fact that the effect of a symbol is spread to other symbols, it is intuitive that the optimal receiver should observe not only the segment of received signal concerning the desired symbol, but the whole received signal instead. Thus, the ML equalizer provides the optimal solution to equalization problems by minimizing the probability of error over the entire sequence. The ML equalizer chooses the transmitted input sequence that maximizes the log-likelihood of the received signal sequence at the receiver when the channel information is been known at receiver side. The algorithm was first proposed by Forney and later implemented with Viterbi algorithm to reduce computational complexity.

3.8Mathematical Modeling of (ML) Channel Estimation Equalizer

The ML equalization approach determines the estimate of the transmitted signal vector x as:

$$K = |y \quad Hx|^2 \tag{3.13}$$

Expanding,

$$K = \left|\begin{pmatrix} y1 \\ y2 \end{pmatrix} \quad \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right|^2 \tag{3.14}$$

The possible values of x_1 is +1 or -1. Similarly x_2 also take values +1 or -1. In finding the Maximum Likelihood solution, there is the need to find the minimum from the all four possible combinations of x_1 and x_2

$$K_{+1,+1} = \left|\begin{pmatrix} y1 \\ y2 \end{pmatrix} \quad \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} \begin{pmatrix} +1 \\ +1 \end{pmatrix}\right|^2 \tag{3.15}$$

$$K_{+1,-1} = |\begin{pmatrix} y1 \\ y2 \end{pmatrix} \quad \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} \begin{pmatrix} +1 \\ -1 \end{pmatrix}|^2 \tag{3.16}$$

$$K_{-1,+1} = |\begin{pmatrix} y1 \\ y2 \end{pmatrix} \quad \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} \begin{pmatrix} -1 \\ +1 \end{pmatrix}|^2 \tag{3.17}$$

$$K_{-1,-1} = \left|\begin{pmatrix} y1 \\ y2 \end{pmatrix} \quad \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} \begin{pmatrix} -1 \\ -1 \end{pmatrix}\right|^2 \tag{3.18}$$

The estimate of the transmit symbol is chosen based on the minimum value from the above four values i.e

If the minimum is K+1, +1 that is [1 1]

If the minimum is K+1, -1 that is [1 0]

If the minimum is K-1, +1 that is [0 1]

If the minimum is K-1, -1 that is [0 0]

This analysis can be applied to a MIMO M×N case.

3.9 Decision Feedback Equalizer(DFE) consists of a Feedforward filter and a Feedback filter. The DFE determines the ISI contribution from the detected symbols by passing them through the feedback filter that approximates the combined discrete equivalent baseband channel. The resulting ISI is then subtracted from the incoming symbols. However, the feedback filter of the DFE does not suffer from noise enhancement because it estimates the channel frequency response rather than its inverse. For channels with deep spectral nulls, DFEs generally perform much better than linear equalizers.The detection process in the DFE relies on decisions being correct, however; when detection error is made, the error keeps on propagating which give catastrophically wrong results.

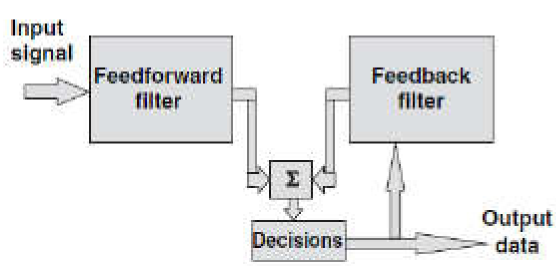


Figure: 3.3 Decision feedbacks Channel Estimation Equalization Structure (Saunder, 2007)

3.10 Other Channel Estimation Equalization Methods

Apart from the linear and non-linear equalizers discussed so far, other equalization method does exist such as the Turbo Equalizers, Adaptive Equalizers and Blind Equalizers. The Turbo equalizer has turbo decoding principles applied to the equalizer design. It iterates between a Map equalizer and a decoder to determine the transmitted symbol. On the other hand, the Adaptive Equalizer uses training and tracking mechanism incorporated into the Equalizer design. During training the coefficients of the equalizer are updated at time t based on a training sequence that has been sent over the channel. The tracking mechanism is used to adjust the channel estimate inherent in the equalizer. Blind equalizers do not use training: they learn the channel response via the detected data only. However, this research will focus on improvement on ZF, and MMSE.

3.11 Successive Interference Cancellation (SIC)

If the symbols are detected successively and the outcome of the previous detected symbol is used to cancel the interference of the next symbol. It leads to then decision directed detection algorithm called Successive Interference Cancellation.

SIC algorithm mainly consists of three parts

- Ordering: to find out the transmitter stream with lowest error variance
- Interference Nulling: estimate the strongest signal from the transmitter stream by nulling out the weaker signals
- Interference Cancellation: subtracting the contribution of the detected symbol from the rest received signal vector and go back to the ordering step.

3.12 Zero Forcing With Successive Interference Cancellation (ZF-SIC)

In this method, first of all the zero forcing equalizer find the estimated symbol x_1 and x_2 then one of the estimated symbol is subtracted from received symbol to compute the equalized symbol by applying maximum ration combining(MRC). If x_1 and x_2 are the estimated transmitted symbol then

$$\binom{x1}{x2} = (H^H H)^{-1} H^H \binom{y1}{y2} \quad (3.19)$$

By subtracting one of the estimated symbol (say x_2) from the received signal y_1 and y_2

$$\binom{r1}{r2} = \begin{pmatrix} y1 & 12x2 \\ y2 & 22x2 \end{pmatrix} = \begin{pmatrix} 12 & x1 + n1 \\ 21 & x1 + n2 \end{pmatrix} \quad (3.20)$$

$$\binom{r1}{r2} = \binom{h11}{h21} x1 + \binom{n1}{n2} \quad (3.21)$$

Or

$$r = \ x_1 + n \quad (3.22)$$

By applying maximum ratio combining(MRC), the equalized symbol is given by

$$x_1 = \frac{h^h r}{h^h h} \quad (3.23)$$

3.13 Successive Interference Cancellation Using Optimal Ordering

In the previous successive interference cancellation method, estimation symbol is chosen arbitrarily and then its effect is subtracted from received symbol y_1 and y_2. A better result can be obtained if we choose estimated symbol whose influence is more than other symbol. For this, the power of both the symbol is computed at the receivers and then the symbol having higher power is chosen for the subtraction process.

The power of transmitted symbol x_1 is given by

$$P_{x1} = | \ _{11}|^2 + | \ _{21}|^2 \quad (3.24)$$

The received power at both the antennas corresponding to the transmitted symbol x_2

Similarly the power of transmitted symbol $x_2$is given by

$$P_{x2} = | \ _{12}|^2 + | \ _{22}|^2 \quad (3.25)$$

If$P_{x1} > P_{x2}$, then the receiver decides to remove the effect of $\hat{x}_1$ from the received vector $y_1$and $y_2\hat{x}_2$. Else if $P_{x1} \leq P_{x2}$ then the receiver decide to subtract the effect of $\hat{x}_2$from the received vector y_1 and y_2, and then re-estimate $\hat{x}_1$

$$\begin{pmatrix} r1 \\ r2 \end{pmatrix} = \begin{pmatrix} y1 & 11x1 \\ y2 & 12x1 \end{pmatrix} = \begin{pmatrix} 12x1 & +n1 \\ 22x1 & +n2 \end{pmatrix} \quad (3.27)$$

$$\begin{pmatrix} r1 \\ r2 \end{pmatrix} = \begin{pmatrix} h_{12} \\ h_{22} \end{pmatrix} x_2 + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \quad (3.28)$$

Once the effect of either $\hat{x}_1$ or $\hat{x}_2$ is removed the new channel estimation becomes optimally equalizer called the maximum combining ratio.By applying maximum ratio combining (MRC), the equalized symbol is given by

$$x_2 = \frac{h^h r}{h^h h} \quad (3.29)$$

3.14 Minimum Mean Square Equalization Successive Interference Cancellation

In order to do OSIC with MMSE, then the resulting algorithm is as follows: At the first step of the method, a wiener equalization of matrix H is performed by the matrix

$G_1 = (H^H H + \sigma^2 I)^{-1} H^H$(3.30)

The carrier signal enjoy highest Signal to Interference Noise Ratio (SINR)

3.15 Choice of Modulation Scheme

In this thesis, PSK modulation scheme was chosen because it provides better bandwidth efficiency with less bit energy to noise power requirement than other modulation schemes like QAM, ASK etc. That is to say they are more immune to channel noise and are easy to demodulate in the presence of noise.BPSK has the best BER performance at lower carrier noise ratio (dB). Hence, the choice of BPSK modulation scheme is to be employed in this thesis.

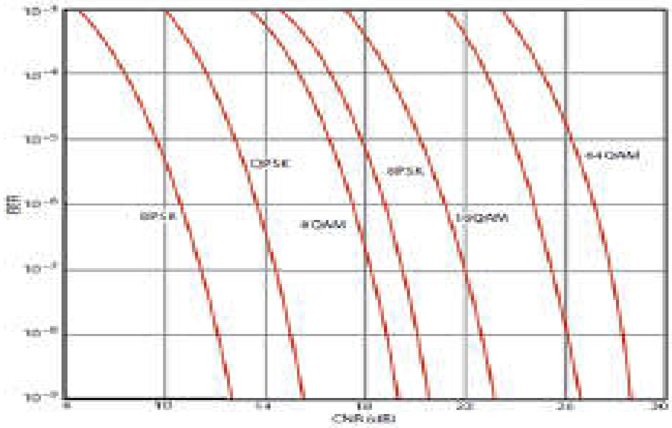


Figure 3.4 Comparison of different Modulation Scheme (Source, Frenzel, 2012)

3.16 Simulation to Determine the Performance of MIMO Equalizers

In order to determine the performance of the equalizers in reducing the effect of ISI experienced by the signal due to multipath propagation, in a MIMO N X M equalization system was modeled and simulated using MATLAB version 13 (R2013a) Software.This new version of Mat-lab has an updatecommunication tools than the latter version Mat-lab Software.Mat-lab provides graphic user interface for simulating dynamic systems. Mat-lab was chosen as simulation tool since it has a communication toolbox optimized for handling MIMO system simulation by just using simple mathematic expression, matrix and vector manipulation. The results obtained can easily be displayed by the help of spectrum scopes and time scopes during the runtime thus giving a feel of the actual communication system.

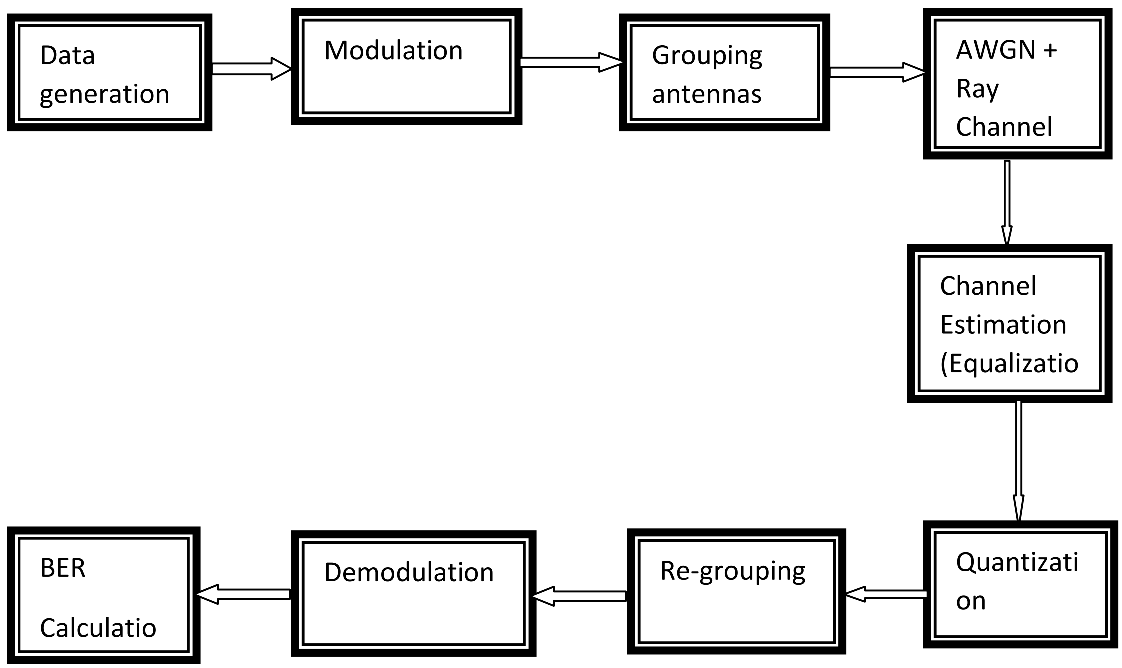


Figure 3.5MIMO Channel Estimation (Equalization) Simulation Block Model

3.17 Data Generation

Data generation block is used to generate a serial random binary data in form of 1's and 0's. This binary data stream models the raw information that is going to be transmitted. The data generated is then fed into the modulation block.

3.18 Modulation

In this block, binary data streams are mapped using BPSK modulation format. Each symbol is mapped to a phase angle of 0 and 180 degrees. The use of phase shift keying produces a constant amplitude signal and was chosen for its simplicity and to reduce problems with amplitude fluctuations due to fading. Coding and other forward error control mechanisms could also be implemented at this stage.

3.19 Grouping

The modulated data streams are now grouped into pairs of symbols depending on the number of available transmit antennas and then sent over the transmission channel.

3.20 Transmission Channel

The transmission channel consists of the Additive White Gaussian Noise (AWGN) channel and Rayleigh fading channel. In AWGN channel model, the only impairment is the linear addition of wideband or white noise with a constant spectral density (expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. The model does not account for the phenomena of fading, frequency selectivity, interference, nonlinearity or dispersion unlike the Rayleigh fading channel model

3.21Channel Estimation(equalization)

In this block channel estimation is applied to the received transmitted symbols in order to mitigate the effect of ISI present due to multipath propagation on the frequency selective fading channel (Rayleigh fading noise). The channel estimation (equalizers) was modeled using equations the various channel estimation scheme respectively.

3.22 Quantization

This is the conversion of analog waveform into digital signal; the process involves mapping a large set of input values to a countable smaller set. The simplest way to quantize a signal is to choose the digital amplitude value closest to the original analog amplitude. The received symbols from the equalizer are quantized and passed to the receiving antennas for re-shaping.

3.23 Regrouping and Reshaping

The quantized pairs of symbols are re-grouped and reshaped into serial data streams by the multiplereceivers' antennas to resemble the original bit of data streams that was sent. The reshape signal is free from channel impairments and will be re-constructed at the receiver sides for modulation.

3.24 Demodulation

This is the process of extracting the original information signal from a modulated carrier wave. A demodulator is an electronic circuit that is used to recover the information content from the modulated carrier wave. The re-shape signal from the receiver antenna is completely re-constructed at the receiver side.

3.25Bit-error Rate (BER) Calculation.

Thebit error rate (BER) is the number of bits errors per unit time. The bit error ratio is the number of bit errors divided by the total number of transferred bits during a studied time interval, the BER is a unit less performance measure, often expressed as a percentage.After decoding and demodulation, the received signal is then compared to the original transmitted signal to calculate the bit error rate (BER).

3.26 MIMO System with Zero Forcing Channel Estimation Equalizer

A MIMO $M \times N$ system was simulated using BPSK modulation and passed through a multipath fading channel with AWGN. The ZF was then applied to reduce the effect of ISI due to multipath propagation. The bit-error rate of the system is then determined. The simulation was carried out using Mat-lab software script.

The procedure used to actualize this simulation is given as follows;

The parameter used in this simulation are listed in Table 3.1

Table 3.1 showing the various parameters used in the simulationnbits

PARAMETER	VALUE
Number of bits or symbols	1,000,000 bits or symbols
Number of transmitters (M)	2-4
Number of Receivers (N)	2-4
Eb/No	0:5:25
Modulation	BPSK
Channel Estimation(equalization)	ZF/MMSE/ML and SIC ZF/MMSE/ML
Channel	Rayleigh channel+ AWGN channel
Data rate	1Mbps

3.27MIMO $M \times N$ System using BPSK with MMSE Channel Estimation Equalization

In this simulation, the effect of MMSE equalizer on MIMO signal transmitted using BPSK modulation was investigated. The MMSE equalizer though does not remove the effect of ISI but in essence provides a trade-off between ISI cancellation and noise amplification. The simulation was carried out using Mat-lab Software script.

The following procedure was used for this simulation

(i) A random binary sequence of 1's and 0's was generated as the data input

(ii) The generated data was encoded using BPSK encoding scheme

(iii) The modulated data is then grouped into pair of symbols and sent in time slot

(iv)Multiply the symbols with the channel and then add white Gaussian noise.

(v) The received symbols are then equalized using MMSE equalization

(vi) BPSK demodulation is then performed after equalizing the received signal

(vii) The bit error rate of the system is then calculated

The parameters used in this simulation are all the same, except for the equalization scheme used.

3.28 MIMO System using BPSK in Maximum Likelihood Channel Estimation Scheme

A MIMO $M \times N$ system was simulated using BPSK modulation and passed through a multipath fading channel with AWGN. The MLE was applied to reduce the effect of ISI due multipath propagation. The bit-error rate of the system is then determined. The simulation was carried out using Mat-lab Software

The simulation process consist the following steps;

(i) A random binary sequence of $1'$s and $0'$s was generated as the data input

(ii) The generated data is then Modulated using BPSK modulation

(iii) After modulation, the data is grouped into pair of symbols and send time slot

(iv)The symbols are then multiplied with the channel and then white Gaussian noise is added.

(v) The minimum among the possible transmit symbol combinations is then determined

(vi) Based on the minimum the estimate of the transmit symbol is chosen

(vi) Hard decision BPSK demodulation is applied on the received symbols

(vii) The bit error rate of the system is then determined.The flow chart for the simulation of the three different channel estimation scheme employed is depicted as flow.

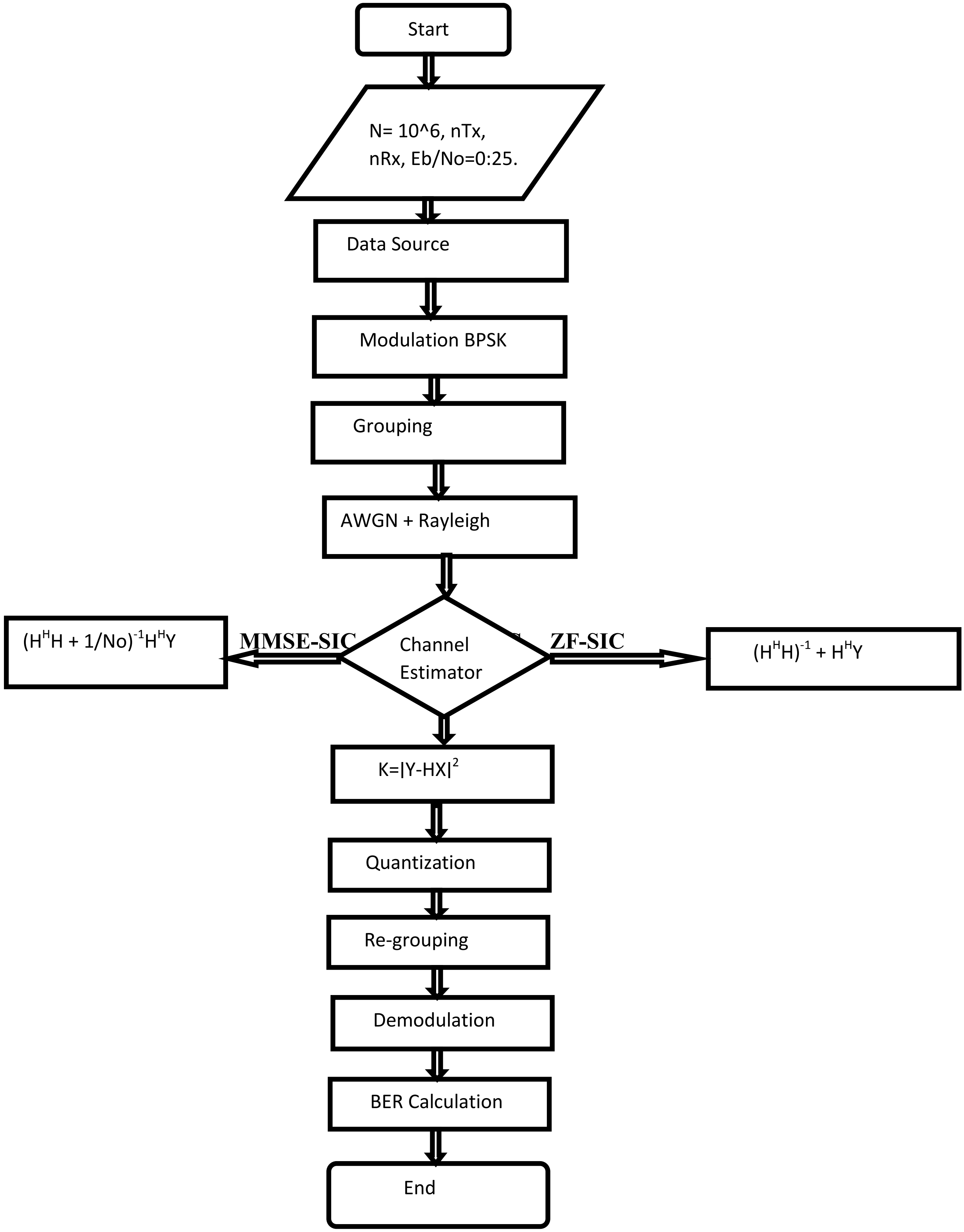


Figure 3.5: Flow chart showing Simulation Process for an Improved Channel Estimation

3.29 Conclusion

In this chapter, the methodology adopted for this thesis was presented. A channel estimation scheme was discussed, a MIMO system with different configuration was modeled and simulated using Mat-lab software for different equalization algorithms scheme and the improved equalization algorithm was also presented to show how it reduces the effect of inter-symbol interference and the effects of multipath propagation fading, which introduce error to the transmitted data.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 INTRODUCTION

In this chapter, the simulation results wereanalyzed and discussed. Graph of BER as a function of the energy per noise density (Eb/No) was computed using Mat-lab Software. The simulation results are plotted in terms of the performance of system BER as a function of the energy per bit verses noise density(1/SNR). The performance of the various channel estimation algorithms were compared for different MIMO system configuration, and the improved channel estimation algorithm (equalizers) was also presented.

4.2 Performance Measure

In digital transmission, the BER is the number of received bits that have been altered due to noise, interference and distortion, divided by the total number of transferred bits during a studied time interval. BER is an importance performance measurement quantity often expressed as a percentage number. Mathematically, BER is giving as:

$$BER = \frac{Number\ of\ Errors}{Total\ number\ of\ bits\ sent} \tag{4.1}$$

The BER is used to determining the strength of a signal and the energy of each bit in a transmission to the strength of the background noise. The Eb/No allows for comparing the BER performance of digital modulation schemes without taking bandwidth into account. The BER for voice transmission over internet protocol is 10^{-3}, this is the minimum acceptable BER value for good quality of service to be attained.International Telecommunication Union recommended BER Benchmark.

Table 4.1 BER for Voice and Data service.

S/N	Bit Error Rate Value	Application
1.	10^{-3}	Voice
2.	10^{-6}	Data

The bit error probability(P_e)is the expectation value of the BER. The BER can be considered as an approximation estimate of the bit error probability. This estimate is accurate for a long studied time interval and high number of bit errors.

4.3 MIMO with Zero Forcing (ZF) Channel Estimation Equalization

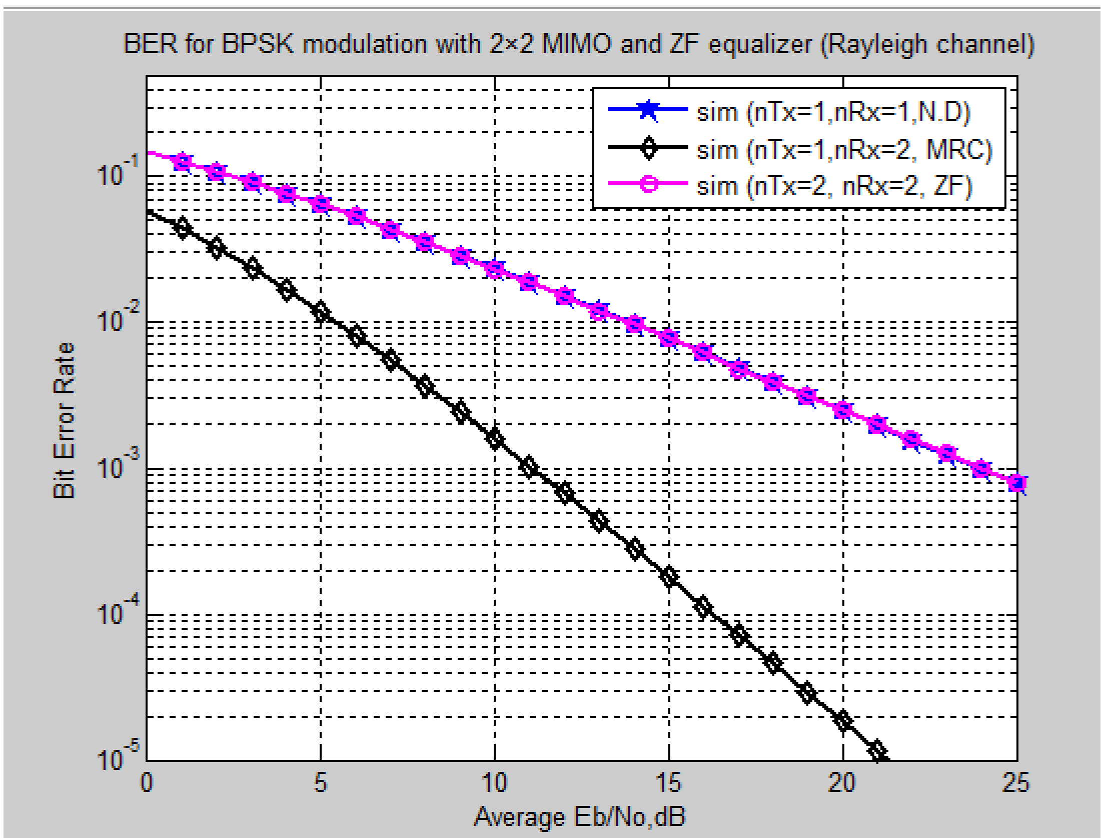


Figure 4.1 Plot of BER for MIMO 2 × 2 Zero Forcing Channel Estimation Equalization

In figure 4.1 above the graph indicates a plot showing the BER performance of Single-Input Single-Output, Single-Input Multiple-Output and MIMO $M \times N$ with ZF in a Rayleigh fading channel. The number of the transmitting antenna and that of the receiving antenna for SISO are equal (1×1) which shows that there is no diversity as a result the signal may get corrupted along the Rayleigh channel as a result of path loss, in the case of SIMO that is (1×2) the number of receiving antenna is greater than that of the transmitting antenna, this increases the throughput performance of the signal at the initial stage. For MIMO system that is (2×2), at BER point 10^{-3}; with 2×2 the Eb/No of ZF is $24dB$. While for the SIMO system at the reference point of the BER with 1×2 the Eb/No $12dB$. At point 15dB using multiple antennas. That is transmit antenna = 2, receive antenna =2, BER for ZF=0.007, BER for MRC = 0.00056

$$\% \; of \; BER \; improvement = \frac{ZF\;(BER) - MRC(BER)}{ZF\;(BER)} \qquad (4.2)$$

Where % is percentage, ZF is Zero Forcing, MRC is Maximum Ratio Combining, and BER is Bit Error Rate

$$\% \; of \; BER \; improvement = \frac{(0.007 \quad 0.00056) \times 100}{0.007}$$

$$\% \; of \; BER \; improvement = \frac{(0.00644) \times 100}{0.007} = 92\%$$

$$\% \; of \; BER \; improvement = 92\%$$

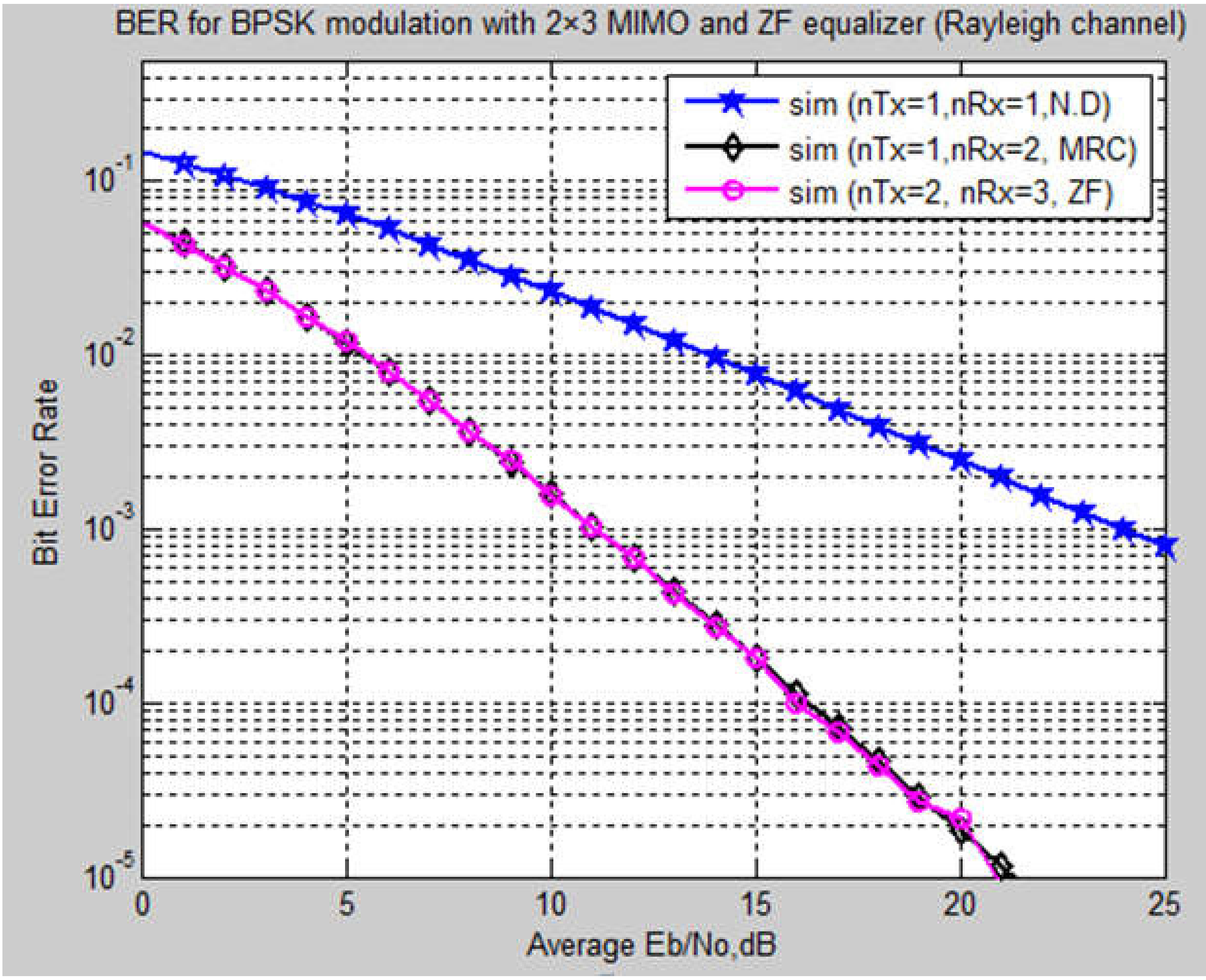


Figure 4.2Plot of BER for MIMO 2 × 3 Zero Forcing Channel Estimation Equalization

Figure4.2 above shows the results of a plot of BER against the E_b/N_o, the number of the receive antenna has increased from 2 to 3. The BER also reduced from 0.1 to 0.05 as against the 2 × 2. At BER point10^{-3}; with 2×3 transmitting and receiving antenna the E_b/N_o of ZF is 12dB. This show that in a SISO system(consisting of 1 transmitting and 1 receiving antenna) there is no diversity so the BER is high and the E_b/N_o is also high. This is not a good design for communication systems. Choosing E_b/N_o at random point of $15dB$ using multiple antenna. That is transmit antenna = 2, receive antenna =3, BER for ZF = 0.008, BER for ND = 0.00055,

$$\% \; of \; BER \; improvement = \frac{ZF(BER) \quad ND(BER)}{ZF(BER)} \tag{4.3}$$

ZF (BER) is zero forcing bit error rate channel estimation

N.D (BER) is no diversity bit error rate channel estimation

$$\% \; of \; BER \; improovement = \frac{(0.008 \quad 0.00055) \times 100}{0.008}$$

$$= \frac{0.00745 \times 100}{0.008}$$

$$= 93.1\%$$

As results of increase in the receiving antenna in the case of MIMO system, there is improvement and reduction of bit error rate that is increases from 92% to 93.1%

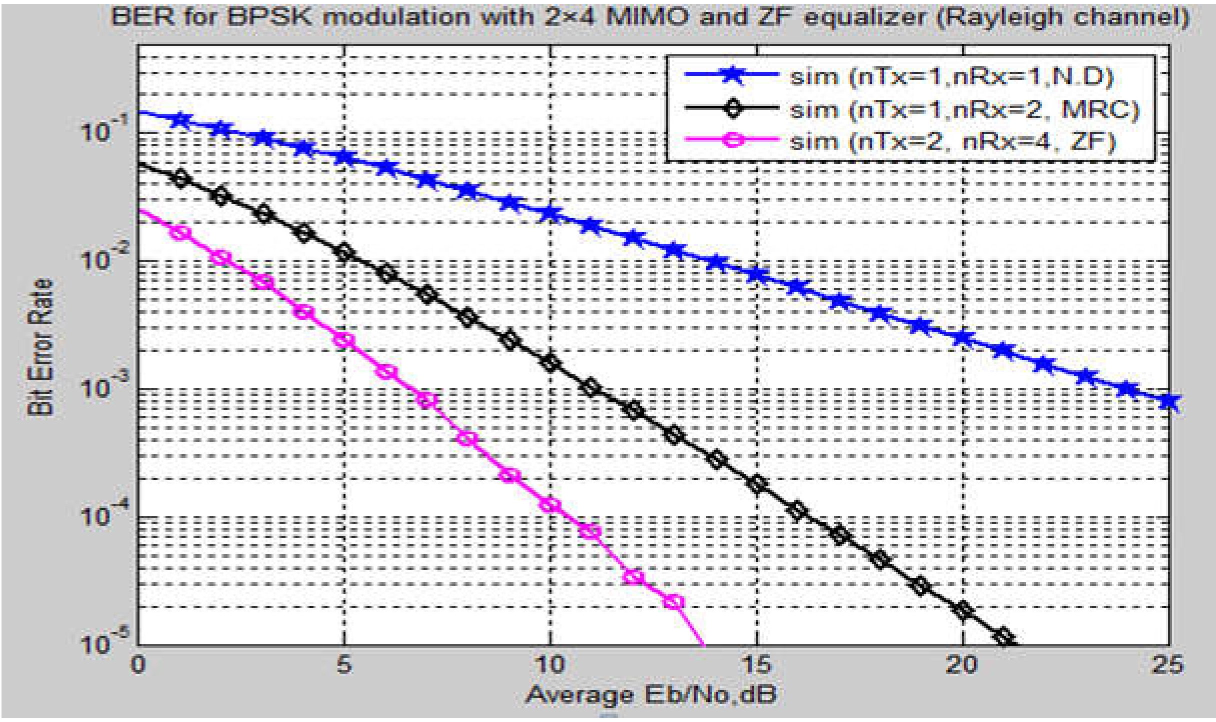


Figure 4.3Plot of BER for MIMO 2 × 4 Zero Forcing Equalization

Figure 4.3 above shows the results of a plot of BER against the E_b/N_o, the number of the receive antenna has increase from 3 to 4. From the figure above it shows that as the number received antennas increase from 3 to 4 the Bit Error Rate reduce from 0.05-to- 002; at BER point of 10^{-3} the E_b/N_o of ZF reduced to $7dB$. This indicate that as the number of receiving antenna keep on increasing the gain in the quality of signal deliver at the receiver side also increases.Choosing E_b/N_o at random point of$10dB$.transmit antenna = 2, receive antenna = 4,

BER for MRC = 0.002,

BER for ZF= 0.00013,

$$\% \; of \; BER \; improvement = \frac{MRC(BER) \quad ZF(BER) \times 100}{MRC(BER)} \tag{4.4}$$

MRC (BER) is maximum ratio combining bit error rate

ZF (BER) is the zero forcing bit error rates

$$\% \; of \; BER \; improvement = \frac{(0.002 \quad 0.00013) \times 100}{0.002}$$

$$\% \; of \; BER \; improvement = \frac{(0.00187) \times 100)}{0.002}$$

=93.5%

As a result of increasing the receiving antenna in the case of MIMO system, there is improvement in the % of bit error rate, it increases from 92% to 93.5%

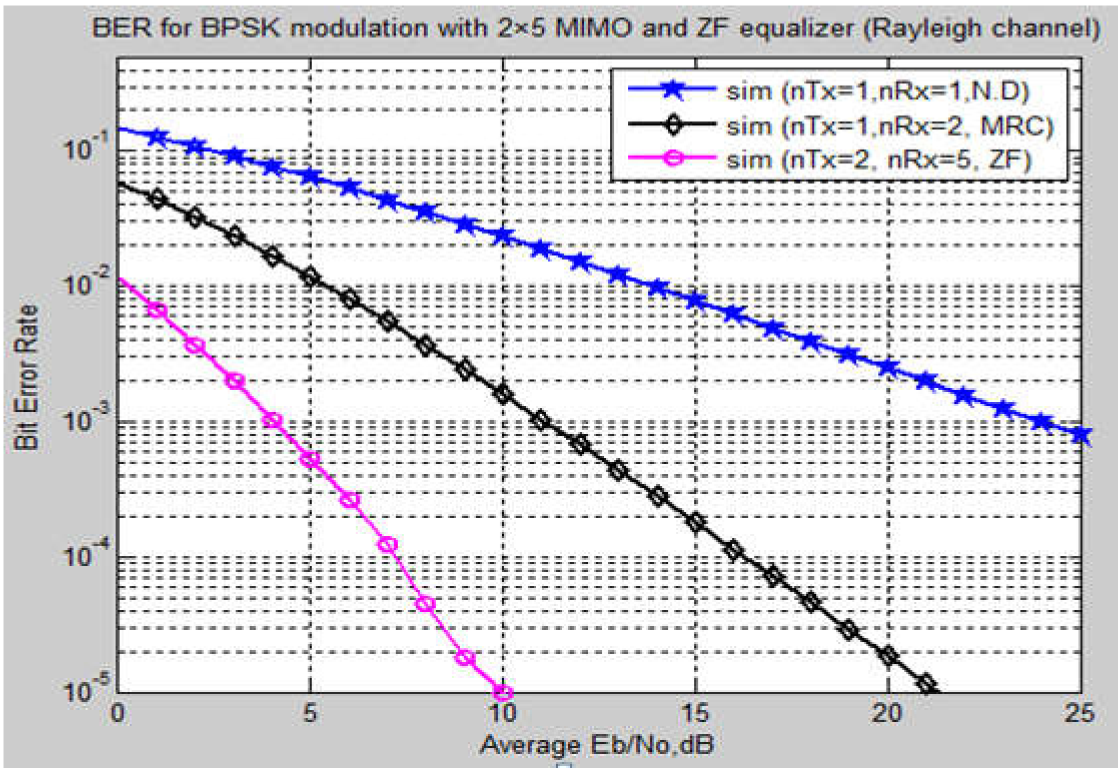


Figure 4.4 Plot of BER for MIMO 2 × 5 Zero Forcing Equalization

Figure 4.4 above shows the results of a plot of BER against the E_b/N_o, the figure shows that as the number of receivingantennas increase from 4 to 5 the BER reduce from 0.03-to- 001; at BER point of $10^{-3}$the E_b/N_o of ZFreduce to $4dB$. This indicate that as the number of receiving antenna keep on increasing there is again in the quality of signal deliver at the receiver side. At random point of $E_b/N_o 5dB$ using multiple antennas, that is transmit antenna= 2, receive antenna = 5, BER for MRC = 0.01, BER for ZF= 0.0006:

$$\% \; of \; BER \; improvement = \frac{MRC(BER) \quad ZF(BER) \times 100}{MRC(BER)} \tag{4.5}$$

$$\% \; of \; BER \; improvement = \frac{(0.01 \quad 0.0006) \times 100}{0.01}$$

$$= \frac{(0.0094)}{0.01} = 94\%$$

As a results of increase in the number of receiving antenna in the case of MIMO system, there is improvement in the % of bit error rate, it increases from 92% to 94%

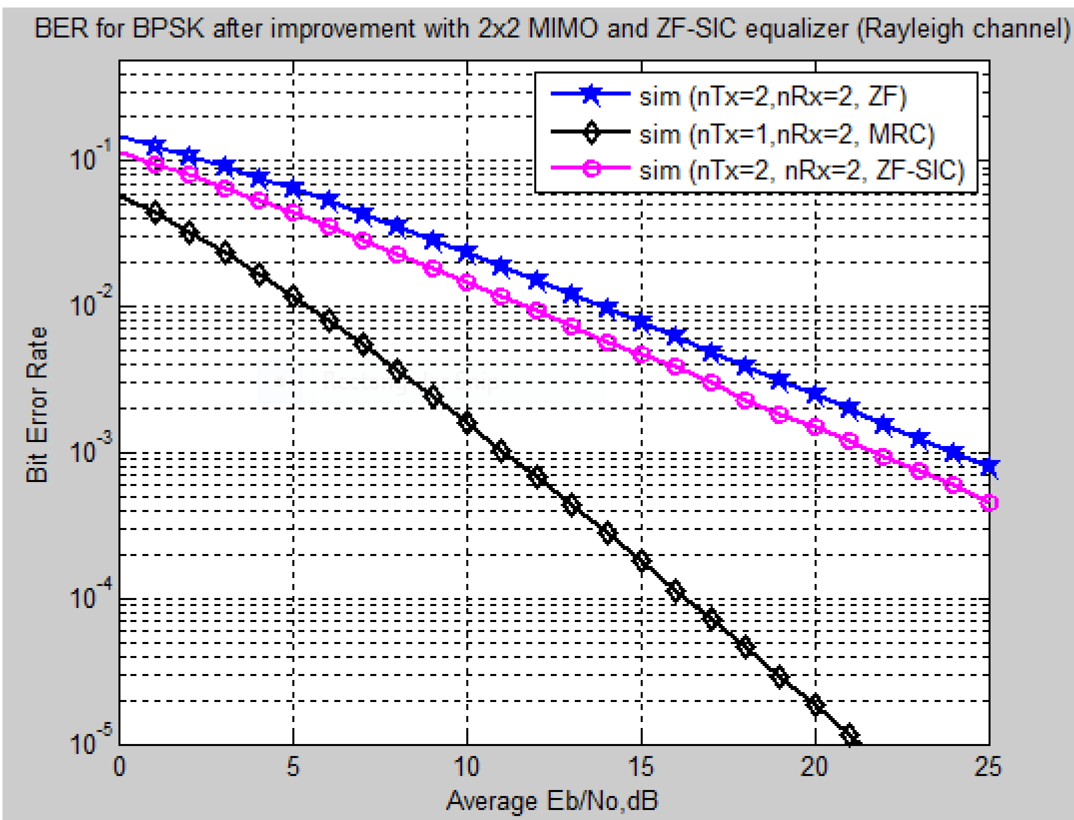


Figure 4.5 Plot of BER for MIMO 2 × 2 (SIC) Zero Forcing Equalization

Figure 4.5 above shows the results of a plot of BER against the E_b/N_o, the number of the transmit and receive antenna is 2×2. From the figure above it shows that the BER of ZF-SIC is less than the ZF equalizer; at BER point of 10^{-3} the E_b/N_o, of ZF-SIC is$22dB$.This is less thanthe ZFthat is $24dB$.At random point of$15dB$ using multiple antennas, that is transmit antenna= 2, receive antenna =2, BER for ZF SIC = 0.004, BER for MRC= 0.0002:

$$\%\ of\ BER\ improvement = \frac{ZF\ SIC(BER) \quad MRC(BER) \times 100}{ZF\ SIC} \qquad (4.6)$$

$$\%\ of\ BER\ improvement = \frac{(0.004 \quad 0.0002) \times 100}{0.004}$$

$$= \frac{(0.0038)}{0.004} = 95\%$$

Reduction in the amplified noise ofZF and increase number of receiving antenna, ZF SIC was formed, there is an improvement of 3% of bit error rate, which increases from 92% to 95%

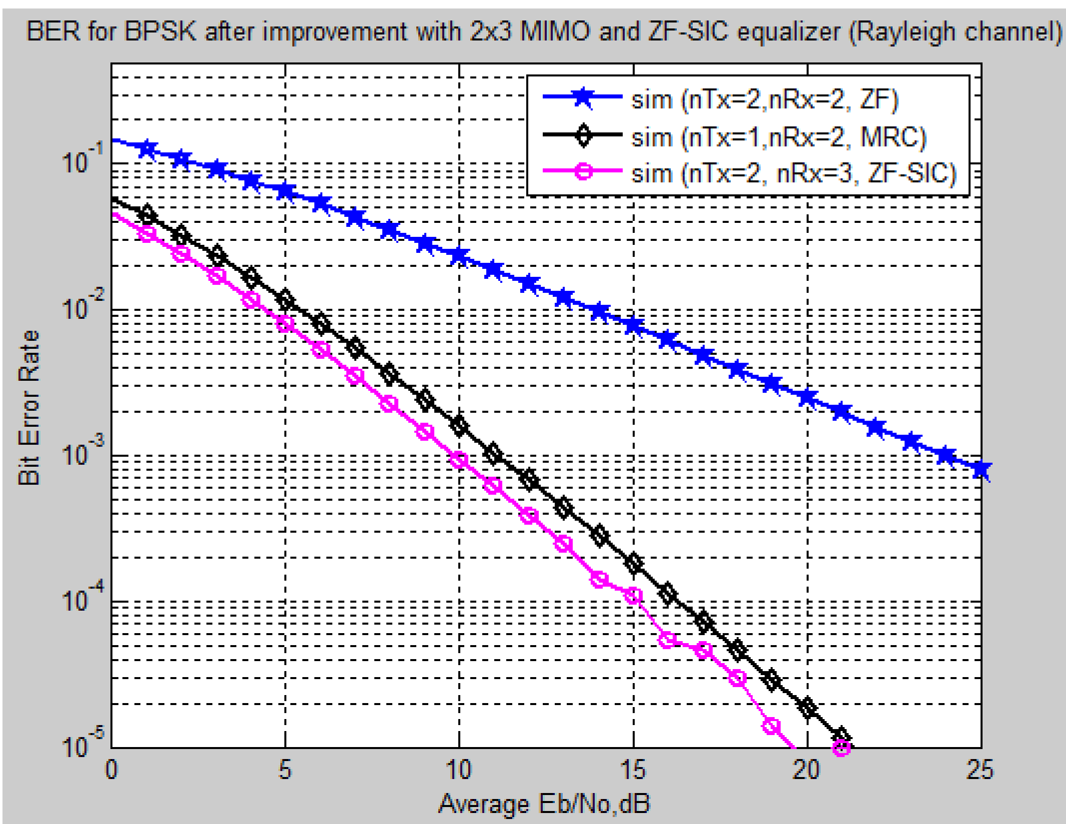


Figure 4.6 Plot of BER for MIMO 2 × 3 (SIC) Zero Forcing Equalization

Figure 4.6 shows the results of a plot of BER against the E_b/N_o, the number of transmit antenna is 2 and receive antenna is 3. From the figure above it shows that the BER of ZF-SIC is less than the ZF; at BER point of $10^{-3}$the E_b/N_o, of ZF-SIC is $10dB$. Which is less than the ZF which was $22dB$?At a random point of$15dB$ using multiple antennas, that is transmit antenna = 2, receive antenna =3, BER for ZF = 0.008, BER for ZF SIC= 0.00026:

$$\% \; of \; BER \; improvement = \frac{MRC(BER) \quad ZF \; SIC(BER) \times 100}{MRC(BER)} \tag{4.7}$$

$$\% \; of \; BER \; improvement = \frac{(0.008 - 0.00026) \times 100}{0.008}$$

$$= \frac{(0.00774) \times 100}{0.008} = 96.75\%$$

As a results of reducing the amplified noise from the ZF MIMO system using interferometer filter to form SIC ZF there is an improvement in the 4.75% of bit error rate, which increases from 92% to 96.75%.

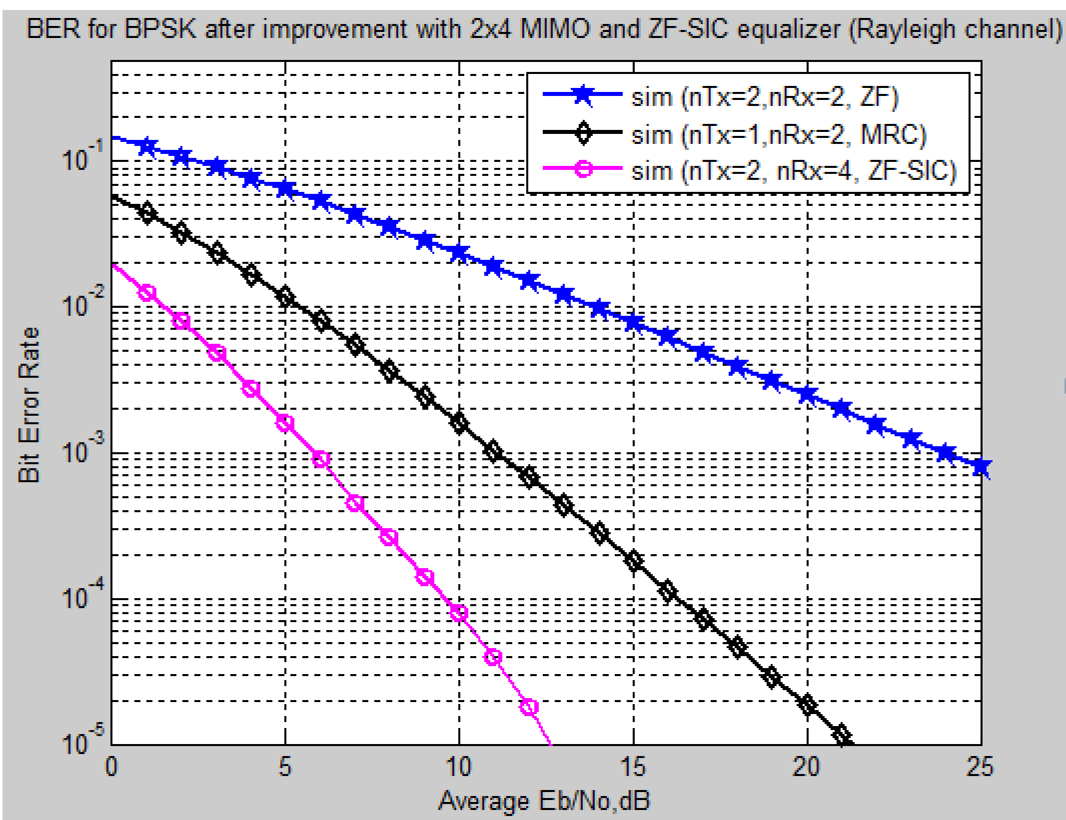


Figure 4.7 Plot of BER for MIMO 2 × 4 (SIC) Zero Forcing Equalization

Figure 4.7 shows the results of a plot of BER against the E_b/N_o, the number of transmit antenna is 3 and receive antenna is 4. From the figure above it shows that the BER ZF-SIC is less than the ZF; at BER point of $10^{-3}$the E_b/N_o, ofZF-SIC is 7dB. This is less than the ZF.At a random point of$10dB$ using multiple antennas, that is transmit antenna = 2, receive antenna =4, BER for MRC = 0.002, BER for ZF SIC= 0.00003:

$$\%\ of\ BER\ improvement = \frac{MRC(BER)\quad ZF\ SIC\ (BER)}{MRC(BER)} \tag{4.8}$$

$$\%\ of\ BER\ improvement = \frac{(0.002 \quad 0.00003) \times 100}{0.002} = \frac{(0.00197) \times 100}{0.002}$$

$$= 98.5\%$$

As a result of reducing the amplified noise from the ZF MIMO system, to form SIC ZF there is an improvement of6.5% of bit error rate, which increases from 92% to 98.5%

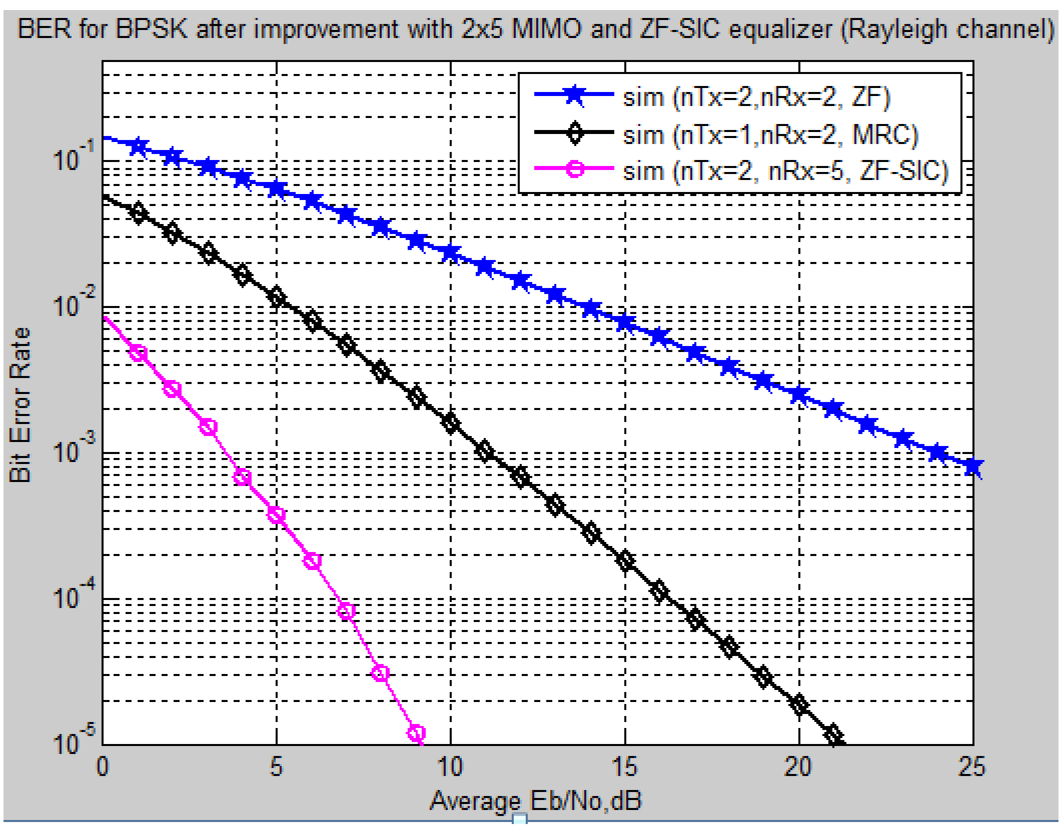


Figure 4.8 Plot of BER for MIMO 2 × 5 (SIC) Zero Forcing Equalization

Figure 4.8 shows the results of a plot of BER against the E_b/N_o, the number of receive antenna is increase from 2 to 5. From the figure above it shows that the BER of ZF-SIC is less than the ZF equalizer; and also at BER point of $10^{-3}$the E_b/N_o of ZF-SIC is $3..5dB$. This is less than the ZF at the same receiving antenna.At a random point of $5dB$ using multiple antennas, that is transmit antenna = 2, receive antenna =5, BER for MRC = 0.01, BER for ZF SIC= 0.00012:

$$\% \; of \; BER \; improvement = \frac{MRC(BER) \quad ZF \; SIC(BER) \times 100}{MRC(BER)} \tag{4.9}$$

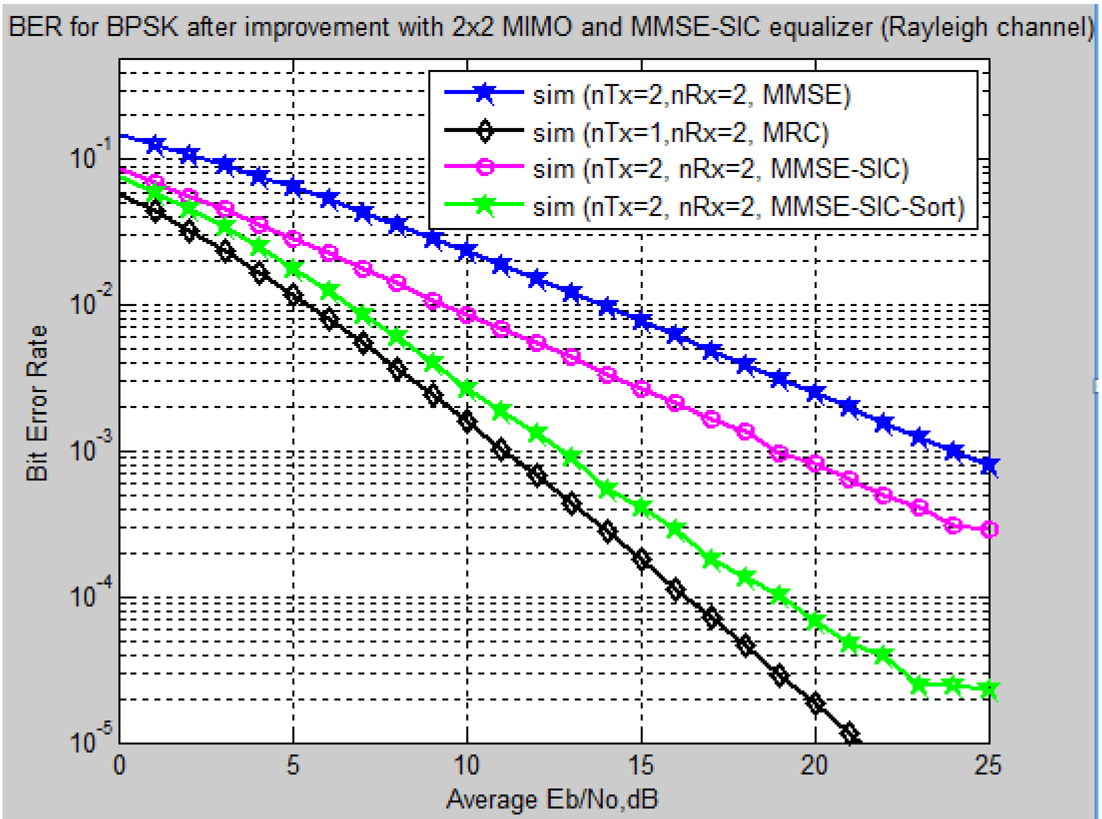
$$\% \; of \; BER \; improvement = \frac{(0.01 \quad 0.00012) \times 100}{0.01} = \frac{(0.00988) \times 100}{0.01}$$

$$= 98.8\%$$

As a result of reducing the amplified noise from the ZF MIMO system, to form SIC ZF there is an improvement in the 6.8% of bit error rate, which increases from 92% to 98.8%

4.4MIMO with Successive Interference Cancellation (MMSE) Equalization

This section presents a plot showing the BER performance of MIMO $M \times N$ with SIC MMSE equalization in a Rayleigh fading channel. As depicted in Figure 4.5, there is an improvement in the BER performance of the system when the antenna configuration is increase from 2×2, 2×3, 2×4, 2×5. This is because due to the fact that, (SIC) MMSE provides a trade-off between ISI cancellation and the noise amplification. Unlike the ZF equalizer which suffers from noise amplification while nulling out the ISI component present.



Figure; 4.9 plot of BER for MIMO 2 × 2 with (SIC) MMSE equalization

Figure 4.9 shows that the BER for SIC MMSE in MIMO system is lower than that of the MMSE using the same number of transmitting and receiving antenna; at BER of $10^{-3} E_b/N_o$,of MMSE (SIC) is given as $18dB$ and also the E_b/N_o of MMSE-SIC-SORT is $13dB$ that of MSE at $10^{-3}$is$24dB$ at that bench mark.At random point of$15dB$ using multiple antennas, that is transmit antenna = 2, receive antenna=2, BER for MMSE SIC = 0.003, BER for MRC= 0.0002:

$$\%\ of\ BER\ improvement = \frac{MMSE\ SIC(BER)\quad MRC(BER) \times 100}{MMSE\ SIC\ (BER)} \tag{4.10}$$

MMSE SIC (BER) is minimum mean square error bit error rate

MRC (BER) is the maximal ratio combining bit error rate

$$\%\ of\ BER\ improvement = \frac{(0.003 \quad 0.0002) \times 100}{0.003}$$

$$= \frac{(0.0028) \times 100}{0.003} = 93.33\%$$

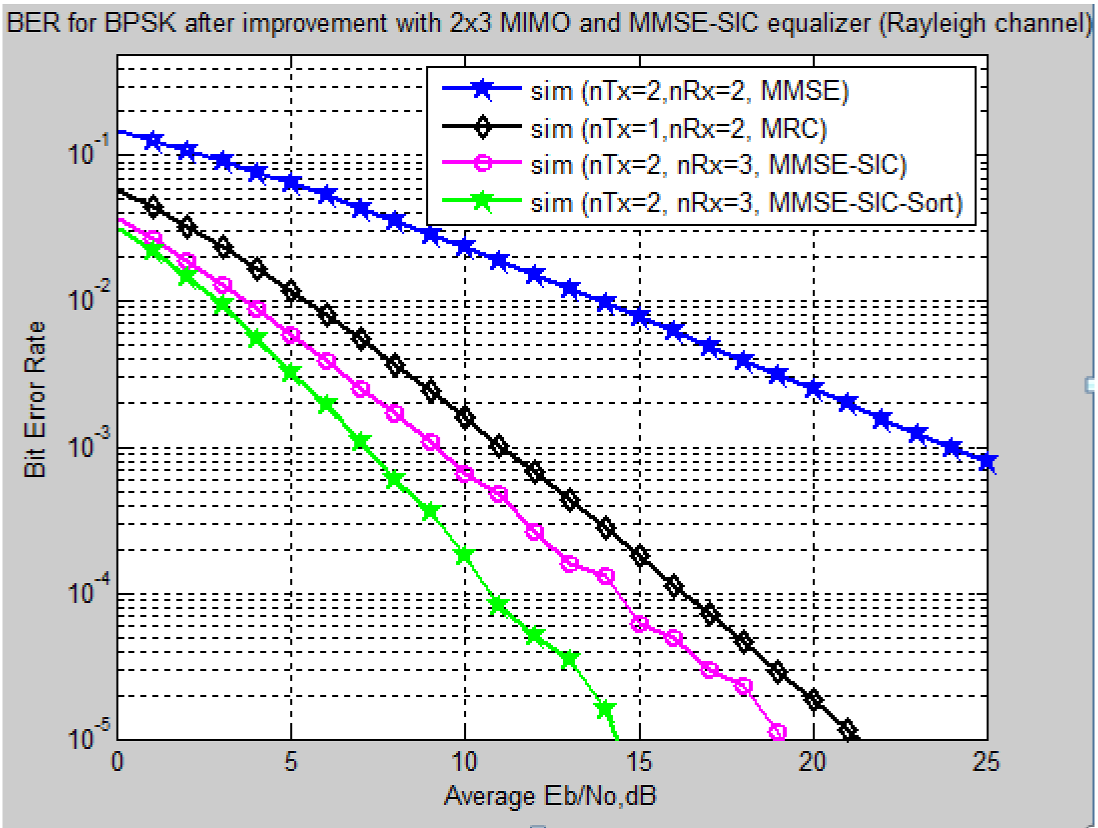


Figure 4.10 Plot of BER for MIMO 2 × 3 with (SIC) MMSE equalization

From the figure 4.10 above at BER of 10^{-3} for the SIC MMSE is $9dB$; at the reference point of voice data which is 10^{-3} the E_b/N_o,of MMSE-SIC-SORT is $7dB$. Also the receive antenna increase from 2-to-3, this show an improvement in the quality of signal at the receiver sides. This improvement is indicated by reduction inE_b/N_o. At a random point of$15dB$ using multiple antennas, that is transmit antenna = 2, receiver antenna =3, BER for MRC= 0.0002, BER for MMSE SIC = 0.00001,

$$\% \; of \; BER \; improvement = \frac{MRC(BER) \quad MMSE \; SIC \; (BER) \times 100}{MRC(BER)} \tag{4.11}$$

$$\% \; OF \; BER \; improvement = \frac{(0.0002 \quad 0.00001) \times 100}{0.0002}$$

$$= \frac{(0.00019) \times 100}{0.0002} = 95\%$$

Reducing the noise content from the MMSE and increase the number of receiver antenna, to form SIC MMSE there is an improvement in the 1.7% of bit error rate, which increases from 93.3% to 95%

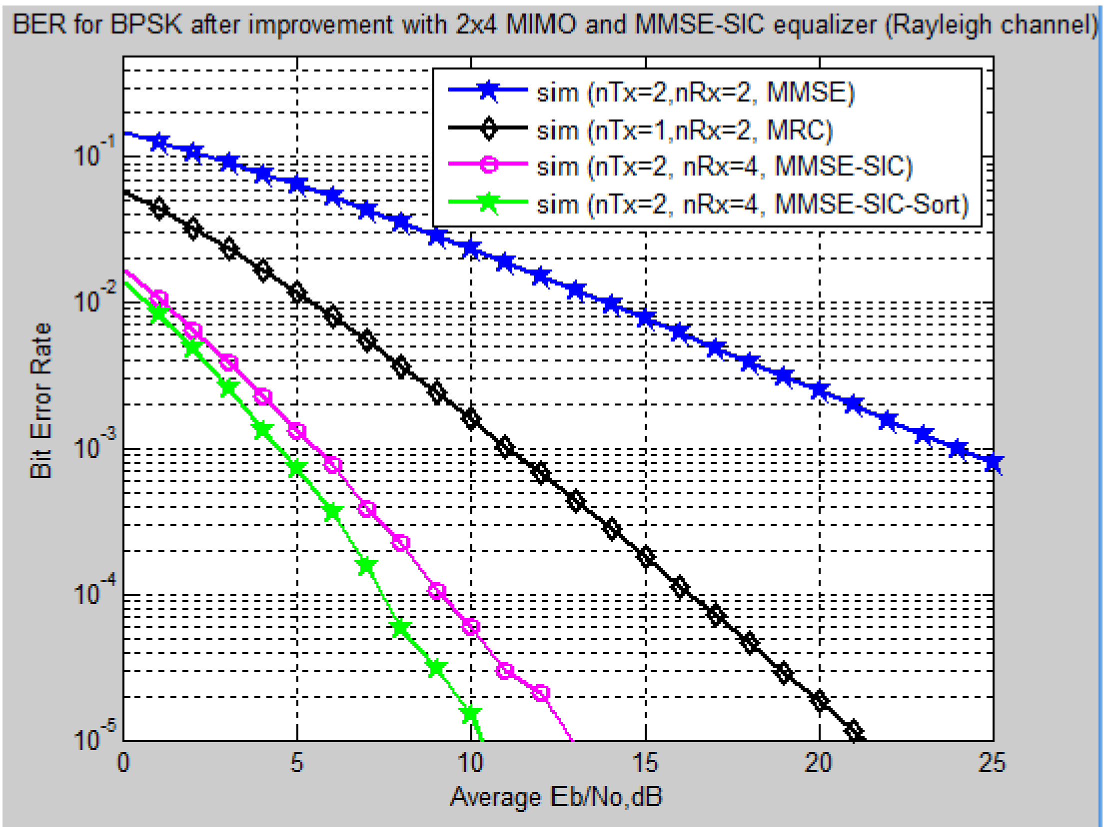


Figure 4.11 Plot of BER for MIMO 2 × 4 with (SIC) MMSE equalization

Figure 4.11 shows the BER and the E_b/N_o when the number of the transmitting antenna is 2, and the receiving increase from 3-to-4. At a BER point of$10^{-3}$the E_b/N_o for MMSE-SIC is $5..5dB$, which indicate an improvement in the signal quality as result of reduction in BER.At a random point of 10dB using multiple antennas, that is transmit antenna = 2, receive antenna =4, BER for MRC= 0.002, BER for MMSE SIC = 0.00005,

$$\%\ of\ BER\ improvement = \frac{MRC(BER) \quad MMSE\ SIC(BER) \times 100}{MRC(BER)} \tag{4.12}$$

$$\%\ of\ BER\ improvement = \frac{(0.002 \quad 0.00005) \times 100}{0.002}$$

$$= \frac{(0.00195) \times 100}{0.002} = 97.5\%$$

Reduction in the BER and also improvement of 2% of to the system performance, which increases from 95% to 97%.

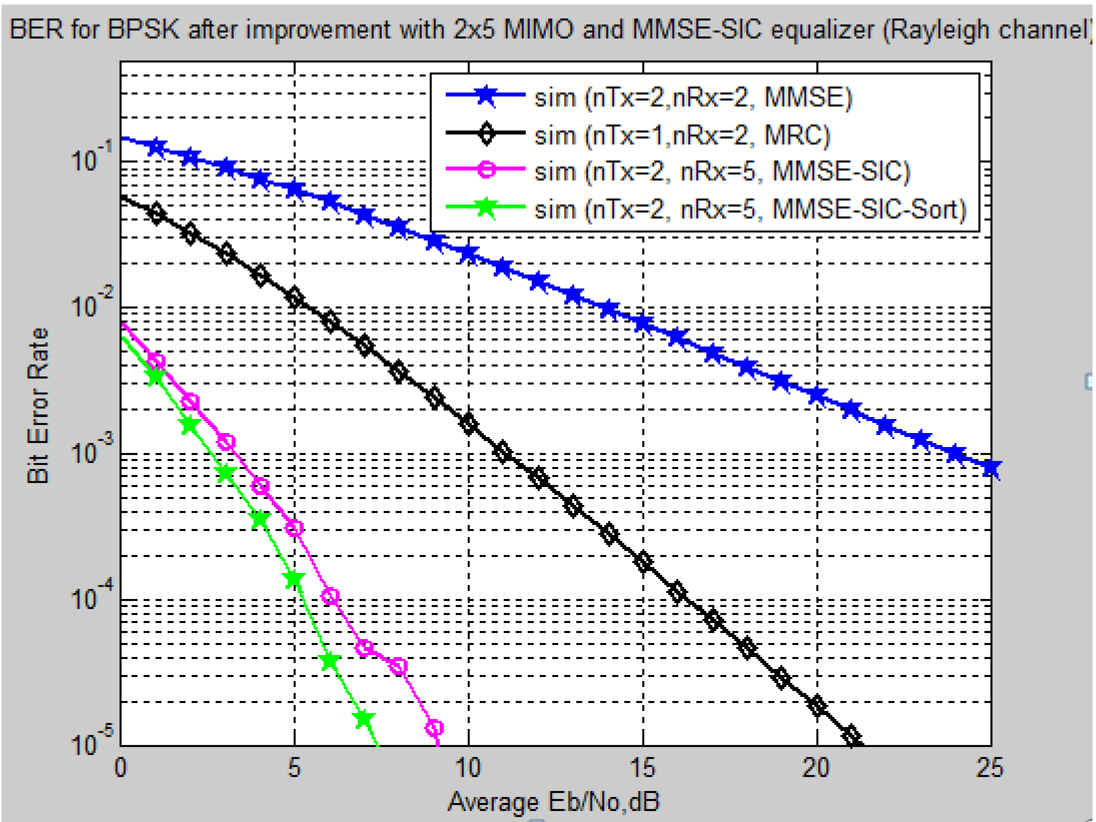


Figure 4.12 Plot of BER for MIMO 2 × 5 with (SIC) MMSE equalization

The figure4.12shows the BER and the E_b/N_o, when the number of the transmitting antenna is 2, and the receiving antenna is increase from 4-to-5. At BER of10^{-3}which the reference point for voice data transmission, E_b/N_o, is $3.5dB$, which indicate an improvement in the signal quality as result of reduction in BER.

At $10dB$ using multiple antennas, that is transmit antenna = 2, receive antenna =5, BER for MRC=0.01, BER for MMSE SIC = 0.0002,

$$\% \ of \ BER \ improvement = \frac{MRC(BER) \quad MMSE \ SIC(BER) \times 100}{MRC(BER)} \tag{4.13}$$

$$\%\ OF\ BER\ omprovement = \frac{(0.01 \quad 0.0002) \times 100}{0.01} = 98\%$$

As a result of reduction in the noise content of MMSE MIMO system, to form SIC MMSE there is an improvement in the BER by3% of bit error rate, which increases from 95 to 98%

4.5 MIMO with Maximum Likelihood (ML) Channel Estimation Equalization

The results of MIMO 2×2, 2×3, 2×4, 2×5 with ML equalization shows a high gain and good error performance is achieve when the number of both transmit antenna is 2, while the receiving antenna is increased. Although the gain in dB is not constant for multiple configurations as in MMSE and ZF but there is a remarkable gain as the configuration of the receive antennas is increased.

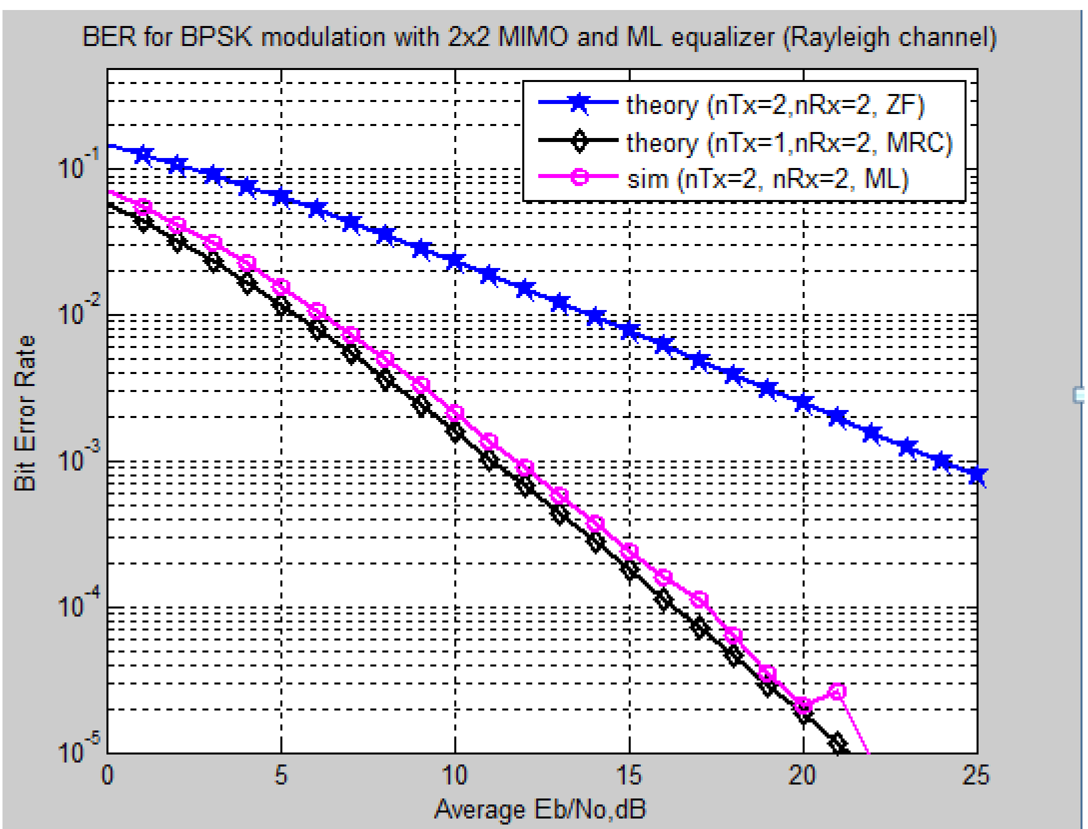


Figure 4.13 Plot of BER for MIMO 2 × 2 ML Channel Estimation equalization

The Maximum likelihood is an optimal MIMO equalization channel estimation that minimizes the probability of error over the entire sequence. Figure 4.13 shows the result of MIMO 2×2 with ML equalization, a high gain and good error performance is achieve when the number of both transmit and receive antenna is increase. Though the gain in dB is not constant for multiple configuration as in MMSE, but there is a gain as the configuration of receive antenna is increased. At reference point of BER of 10^{-3}; with 2×2 the E_b/N_o,the ML is$12dB$. At random point of$15dB$ using multiple antennas, that is transmit antenna= 2, receive antenna=2, BER for ZF= 0.008, BER for ML = 0.0003

$$\%\ of\ BER\ improvement = \frac{ZF(BER) \quad ML(BER) \times 100}{ZF(BER)} \tag{4.14}$$

$$\%\ 0F\ BER\ improvement = \frac{(0.008 \quad 0.0003 \times 100}{0.008} = 96.25\%$$

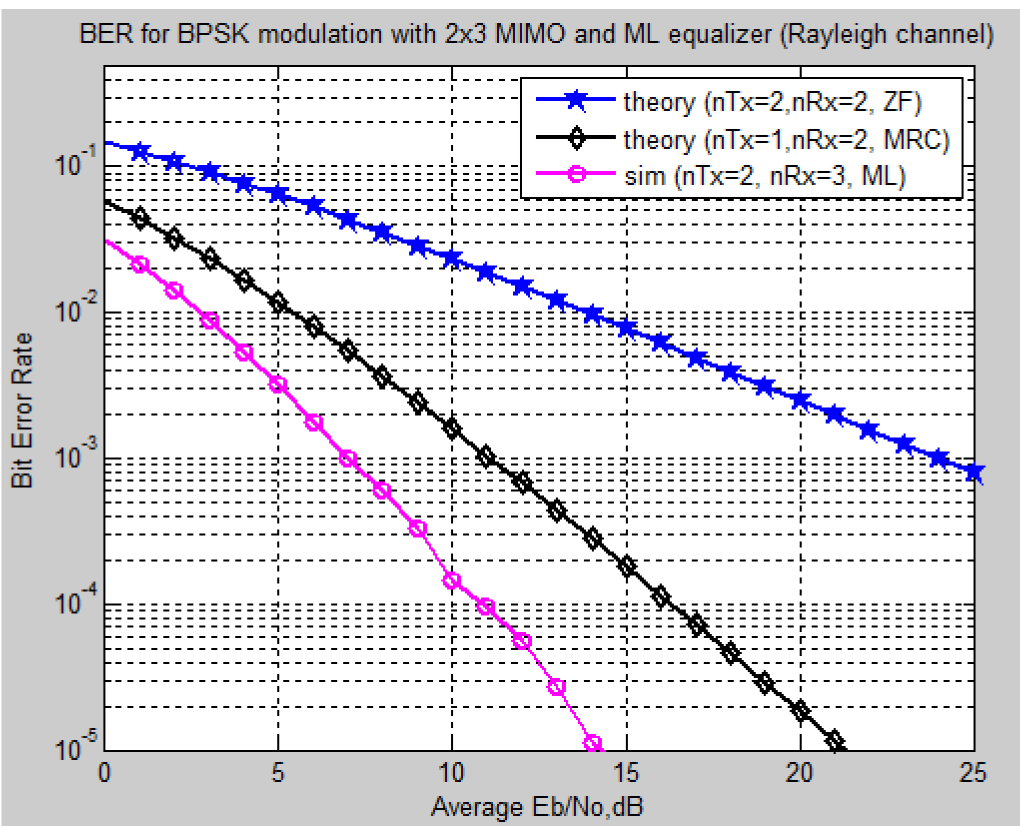


Figure 4.14 Plot of BER for MIMO 2×3 Maximum Likelihood equalization

As the number of receiving antenna increase from 2-to-3 the BER reduces and E_b/N_o, reduces also. At BER point of 10^{-3} which is the ITU reference point of voice signal transmission, the E_b/N_o of ML is 7dB.At random point of 10dB using multiple antennas, that is transmit antenna = 2, receive antenna =3, BER for ZF= 0.002, BER for ML = 0.0001,

$$\%\ of\ BER\ improvement = \frac{ZF(BER) \quad ML(BER)}{ML(BER)} \tag{4.15}$$

$$\%\ of\ BER\ improvement = \frac{(0.002 \quad 0.0001) \times 100}{0.002} = 97\%$$

As a result of varying the receiving antenna from 2-to-3 the ML MIMO system improvement increases by0.75% of BER, which increases from 96.25to 97%

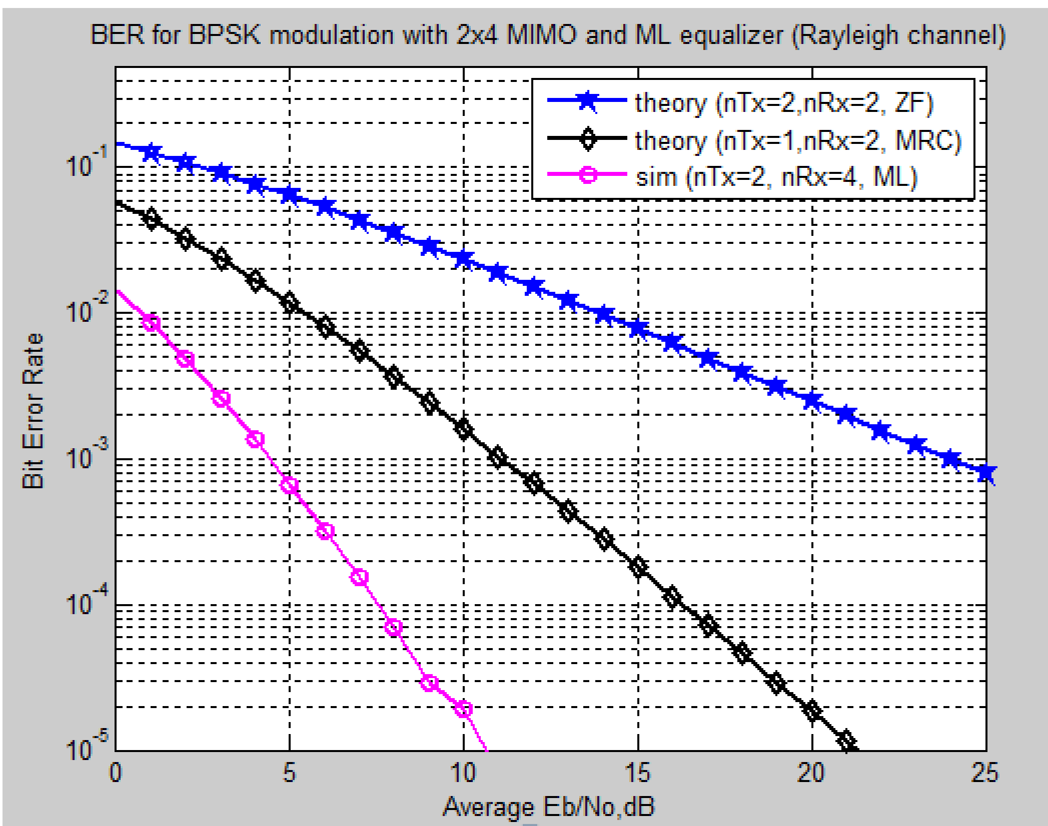


Figure 4.15 Plot of BER for MIMO 2 × 4 Maximum Likelihood equalization

Figure 4.15 shows the BER againstE_b/N_o, the receiving antenna has increase from 2-to-4.

At BER point of 10^{-3}; the E_b/N_o, of ML is $4.5dB$, this increase the quality of the output signal.At random point of $5dB$ using multiple antennas, that is transmit antenna= 2, receive antenna=4, BER for ZF= 0.7, BER for ML = 0.0007,

$$\% \; of \; BER \; improvement = \frac{ZF(BER) \quad ML(BER) \times 100}{ZF(BER)} \tag{4.16}$$

$$\% \; of \; BER \; improvement = \frac{(0.7 \quad 0.0007) \times 100}{0.7} = 99.9\%$$

As a result of varying the receiving antenna from 3-4 the ML MIMO system improvement increase by3.65% of BER, which increases from 96.25% to 99.9%

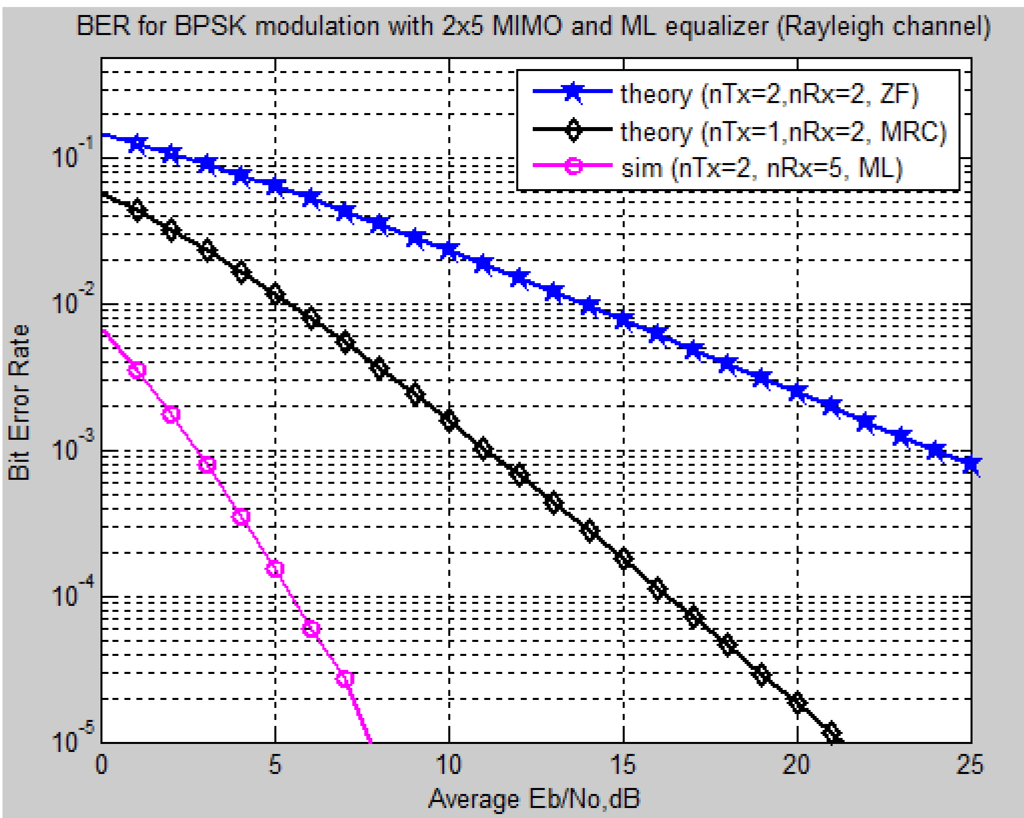


Figure 4.16 Plot of BER for MIMO 2 × 5 Maximum Likelihood equalization

Figure 4.16 shows the BER against E_b/N_o, the receiving antenna has increase from 2-to-5. At BER point of 10^{-3}; the E_b/N_o,of ML is $3.0dB$, this increase the quality of the output signal.

Atrandom point of$5dB$ using multiple antennas, that is transmit antenna = 2, receive antenna =5, BER for ZF= 0.7, BER for ML = 0.0001,

$$\% \, of \, BER \, improvement = \frac{ZF(BER) \quad ML(BER) \times 100}{ZF(BER)} \tag{4.17}$$

$$\% \, of \, BER \, improvement = \frac{(0.7 \quad 0.0001) \times 100}{0.7} = 99.987\%$$

As a result of varying the receiving antenna from 4-5 the ML MIMO system improvement increase by3.737% of BER, which increases from 96.25% to 99.9857%

4.6 Comparison of BER of MIMO2×2(SIC) ZF,MMSE and ML

This section presents the comparative analysis of the performance of the three different equalization schemes of 2×2 MIMO system. The ZF has poor performance, as well as the MMSE while the ML has the best performance.

4.7Comparison of BER of MIMO 2×3 (SIC) ZF,MMSE and ML

This section presents the comparative analysis of the performance of the three different equalization schemes of 2×3 MIMO system. SIC-ZF has low performance, as well as SIC-MMSE while ML is the best among the three algorithms since is an optimal algorithms. As the number of the receiving antenna increases the BER and E_b/N_o reduces and the performance increases.

4.8 Comparison of BER of MIMO 2×4(SIC) ZF,MMSE and ML

This section presents the comparative analysis of the performance of the three different equalization schemes of 2×4 MIMO system.The SIC-ZF has the least performance,then the SIC-MMSE while the ML channel estimation equalizer is the best among the three algorithms. As the number of the receiving antenna increases the BER and E_b/N_oreduces and there is an improvement in the quality of signal at the receiver's side.

The comparative analysis of the performance of the three different equalization schemes of MIMO system. The ZF has least performance, SIC-ZF, while the MRC channel estimation has the best performance, but as the number of the receiving antenna increases the BER and E_b/N_o reduces and there is an improvement in the quality of signal at the receiver's side. At lower data rate the graph is not a perfect straight line which does not favors voice application system.Therefore, based on the results obtained from the simulation of the various channel estimation equalizers algorithms for ZF, MMSE, SIC ZF, SIC MMSE and ML, it is obvious that the SIC ZF and SIC MMSE is an improvement to ordinary ZF and MMSE algorithms while ML) out-performed both ZF, MMSE and SIC ZF, SIC MMSE. This implies that with ML equation, the probability of error occurring is minimal at lowest possible energy per bit to noise density as compared to ZF, MMSE and SIC ZF, SIC MMSE which is very important to wireless operators in wireless communication system.

4.9 Conclusion

In this thesis, MIMO system has been implemented using various model from the simulation carried out in chapter three. Using modulation technique of BPSK, the results obtained were compared with each other based on BER against E_b/N_o,. The simulation study on the performance analysis of MIMO 2×2 based on channel estimation algorithms like ZF, SIC ZF, MMSE, SIC MMSE and the ML for MIMO wireless channel. From the simulation it can be conclude that the SIC ZF gives a better performance than the ZF, also SIC MMSE gives a better performance than the MMSE, but in all cases the MLout-performed all the various channel estimation algorithms.

CHAPTER FIVE

SUMMARY AND RECOMMENDATION FOR FURTHER WORK

This thesis is aimed at analyzing the performance of MIMO channel estimation equalizers used for wireless communication system. A MIMO system increases the spectral efficiency which provides increase in data throughput, link reliability and high quality of service delivery without additional bandwidth or transmit power. The MIMO system achieves this by transmission of more bits per second per hertz of bandwidth and reduces multipath fading. This technology is currently being used in conjunction with other techniques such as OFDM for the next generation mobile systems.

Channel estimation equalizers are of great importance in wireless communication system design due to their ability to combat the effect of ISI in multipath fading environment and noise reduction from the transmitted symbol. In MIMO systems, channel estimation equalizers are much more complex due to the fact that the channel must be estimated and equalized over both space and time. This poses a challenge to wireless communication receiver designers. One major problem with linear channel estimation equalizers is the fact that they are not optimal in terms of minimizing the average symbol error probability. This is because the effect of a symbol is spread to other symbols. This has been addressed in optimal receiver like the ML channel estimation equalizers where the entire segment of the received signal is observed not only the desired symbol.

5.1CONTRIBUTION TO THESIS

- The research presents an improvement to error reduction in a wireless MIMO system and also provides a means of optimizing the linear algorithm (ZF and MMSE) system as a result of addition of more receiving antennas, which are model base on BER in terms of energy per bit per noise density, an improvement that suggest the value of error to be expected at particular data transmitted. The percentage of improvement was also calculated based on the values obtained from each of the various simulation results obtained from the graph. Through:

- Bit Error rate reduction in a wireless MIMO system by varying the receiving antennas for ZF, SIC ZF, MMSE, SIC MMSE, and ML
- Adoption of channel estimation equalizer equation of (ZF, MMSE, ML)
- Improvement on ZF to achieve SIC ZF and also MMSE to achieve SIC MMSE

5.2 Summary

In this thesis, the performance of MIMO Channel estimation equalizers used for reducing the effect of Inter-symbol interference was studied and presented. For MIMO system, the effect of varying antenna configuration was investigated and results were compared for all the various channel estimation equalizers under study. In all the cases, a better performance was achieved when the number of receiving antennas exceeded that of transmitting antennas.

Comparative analysis of the results based on bit error rate shows that SIC is an improvement ZF and MMSE. The ML out-performed the (MMSE SIC, ZF SIC) because is a non-linear channel estimation algorithms which reduces the error probability, this indicates that there is much reduction in the transmitter power requirement when using maximum likelihood channel estimation as compared to others. This is important due to energy conservation and reduction in cost of wireless facilities. The successive interference cancellation is an improvement to the two linear channel estimation ZF, MMSE. As a result of reduction in the amplified noise in ZF system, ZF SIC was formed, there is an improvement in the system performance. Also as a result of reducing the noise content of MMSE channel estimation system, MMSE SIC was formed.

5.3 Recommendations and Further Work

Though the aim and objectives of this thesis have been achieved to a certain level, further improvements can be done to make it better. Suggestions, for future work on this thesis are as follows:

(i). Forward Error Correction, parity checksum and low density parity checksum, techniques can be investigated and incorporated into a single model to ascertain and analyze the overall system performances.

(ii). Application of channel coding such as the turbo coding, reed Solomon coding, Viterbi Coding to detect error in the transmitted signal and to correct it to avoid the signal from corrupting before getting to the receiver.

(iii). Research into other forms of equalization techniques such as Adaptive Blind and Decision Feedback can also be carried out for better understanding of the concept

(iv). Laboratory set up for the practical research and a comparative analysis between the simulated BER and that of the laboratory practical.

(v) Research on the Evaluation of Turbo Coding equalizer in channel estimation.

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APPENDICES

Appendix A1:

MATLAB code for analyzing the BER performance of MIMO system with BPSK modulation in Rayleigh channel using ZF channel estimation equalizer

clear

N = 10^6; % number of bits or symbols,Eb_N0_dB = [0:25]; % multiple Eb/N0 values, nTx = 2; nRx = 2;for ii = 1:length(Eb_N0_dB)% Transmitter, ip = rand(1,N)>0.5; % generating 0,1 with equal probability,s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 0, sMod = kron(s,ones(nRx,1)); % sMod = reshape(sMod,[nRx,nTx,N/nTx]); % grouping in [nRx,nTx,N/NTx] matrix

h = 1/sqrt(2)*[randn(nRx,nTx,N/nTx) + j*randn(nRx,nTx,N/nTx)]; % Rayleigh channel

n = 1/sqrt(2)*[randn(nRx,N/nTx) + j*randn(nRx,N/nTx)]; % white gaussian noise, 0dB variance,% Channel and noise Noise addition, y = squeeze(sum(h.*sMod,2)) + 10^(-Eb_N0_dB(ii)/20)*n;% Receiver % Forming the Zero Forcing equalization matrix W = inv(H^H*H)*H^H,% H^H*H is of dimension [nTx x nTx]. In this case [2 x 2] % Inverse of a [2x2] matrix [a b; c d] = 1/(ad-bc)[d -b;-c a],hCof = zeros(2,2,N/nTx); hCof(1,1,:) = sum(h(:,2,:).*conj(h(:,2,:)),1);% d term, hCof(2,2,:) = sum(h(:,1,:).*conj(h(:,1,:)),1); % a term hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1);% c term,hCof(1,2,:) =sum(h(:,1,:).*conj(h(:,2,:)),1); % b term, hDen = ((hCof(1,1,:).*hCof(2,2,:)) - (hCof(1,2,:).*hCof(2,1,:))); % ad-bc term

hDen = reshape(kron(reshape(hDen,1,N/nTx),ones(2,2)),2,2,N/nTx); % formatting for division

hInv = hCof./hDen; % inv(H^H*H),hMod = reshape(conj(h),nRx,N); % H^H operation

yMod = kron(y,ones(1,2)); % formatting the received symbol for equalization, yMod = sum(hMod.*yMod,1); % H^H * y , yMod = kron(reshape(yMod,2,N/nTx),ones(1,2)); %formatting, yHat = sum(reshape(hInv,2,N).*yMod,1); % inv(H^H*H)*H^H*y

% receiver - hard decision decoding, ipHat = real(yHat)>0;% counting the errors,nErr(ii) = size(find([ip- ipHat]),2);end,simBer = nErr/N; % simulated ber,EbN0Lin = 10.^(Eb_N0_dB/10);

```
theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5)); p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);
theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p)); close all, figure
semilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);hold on
semilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);
semilogy(Eb_N0_dB,simBer,'mo-','LineWidth',2);axis([0 25 10^-5 0.5]),grid on
legend('theory (nTx=2,nRx=2)', 'theory (nTx=2,nRx=2, MRC)', 'sim (nTx=2, nRx=2, ZF)');
xlabel('Average Eb/No,dB');ylabel('Bit Error Rate');
title('BER for BPSK modulation with 2x2 MIMO and ZF equalizer (Rayleigh channel)');
```

Appendix A2

Improve Mat-lab code for analyzing the performance of BER for MIMO system using BPSK modulation in a Rayleigh fading channel with NTx, MRx MIMO channel

```
% Minimum Mean Square Error Equalization with Successive Interference
% Cancellation (ZF-SIC) with optimal ordering
clear
N = 10^6; % number of bits or symbols,Eb_N0_dB = [0:25]; % multiple Eb/N0 values,nTx = N;
nRx = M;for ii = 1:length(Eb_N0_dB), % Transmitter
ip = rand(1,N)>0.5; % generating 0,1 with equal probability
s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 0   sMod = kron(s,ones(nRx,1)); %
sMod = reshape(sMod,[nRx,nTx,N/nTx]); % grouping in [nRx,nTx,N/NTx ] matrix
   h = 1/sqrt(2)*[randn(nRx,nTx,N/nTx) + j*randn(nRx,nTx,N/nTx)]; % Rayleigh channel
   n = 1/sqrt(2)*[randn(nRx,N/nTx) + j*randn(nRx,N/nTx)]; % white gaussian noise, 0dB
variance
   % Channel and noise Noise addition
   y = squeeze(sum(h.*sMod,2)) + 10^(-Eb_N0_dB(ii)/20)*n;
   % Receiver
   % Forming the MMSE equalization matrix W = inv(H^H*H + sigma^2*I)*H^H
   % H^H*H is of dimension [nTx x nTx]. In this case [2 x 2]
   % Inverse of a [2x2] matrix [a b; c d] = 1/(ad-bc)[d -b;-c a]
hCof = zeros(2,2,N/nTx)  ;
hCof(1,1,:) =  sum(h(:,2,:).*conj(h(:,2,:)),1) + 0*10^(-Eb_N0_dB(ii)/10);  % d term
hCof(2,2,:) =  sum(h(:,1,:).*conj(h(:,1,:)),1) + 0*10^(-Eb_N0_dB(ii)/10);  % a term
```

```
hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c termhCof(1,2,:) =
sum(h(:,1,:).*conj(h(:,2,:)),1); % b term  for kk = 1:2, if kk == 1
sortIdx = [];  hCof(1,1,:) =  sum(h(:,2,:).*conj(h(:,2,:)),1) + 10^(-Eb_N0_dB(ii)/10);  % d term
hCof(2,2,:) =  sum(h(:,1,:).*conj(h(:,1,:)),1) + 10^(-Eb_N0_dB(ii)/10);  % a term
hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c term
hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:)),1); % b term,elseifkk == 2
      % Sorting the equalization matrix based on the channel power on each dimension
      % since the second spatial dimension is equalized first, the channel
      % with higher power assigned to second dimension
      normSS1 = squeeze(hCof(2,2,:));normSS2 = squeeze(hCof(1,1,:));
         sortIdx = find(normSS2 < normSS1);end% sorting the H^H*H  + sigma^2*I matrix
hCofSort = hCof;  if ~isempty(sortIdx)
hCofSort(2,2,sortIdx) = hCof(1,1,sortIdx) + 10^(-Eb_N0_dB(ii)/10);;
         hCofSort(1,1,sortIdx) = hCof(2,2,sortIdx) + 10^(-Eb_N0_dB(ii)/10);;
         hCofSort(1,2,sortIdx) = hCof(2,1,sortIdx);  hCofSort(2,1,sortIdx) = hCof(1,2,sortIdx)
end
hDen = ((hCofSort(1,1,:).*hCofSort(2,2,:)) - (hCofSort(1,2,:).*hCofSort(2,1,:))); % ad-bc term
hDen = reshape(kron(reshape(hDen,1,N/nTx),ones(2,2)),2,2,N/nTx);  % formatting for
divisionhInvSort = hCofSort./hDen; % inv(H^H*H)% sorting the H matrixhSort = h;
     if ~isempty(sortIdx)  hSort(:,2,sortIdx) = h(:,1,sortId hSort(:,1,sortIdx) = h(:,2,sortIdx);
end % Equalization - Zero forcing
hModSort =  reshape(conj(hSort),nRx,N); % H^H operation
yModSort = kron(y,ones(1,2)); % formatting the received symbol for equalization
```

```
yModSort = sum(hModSort.*yModSort,1); % H^H * y
yModSort =  kron(reshape(yModSort,2,N/nTx),ones(1,2)); % formatting
yHatSort = sum(reshape(hInvSort,2,N).*yModSort,1); % inv(H^H*H)*H^H*y
    % receiver - hard decision decoding on second spatial dimension
    ipHat2SS = real(yHatSort(2:2:end))>0;ipHatMod2SS = 2*ipHat2SS-1;
    ipHatMod2SS = kron(ipHatMod2SS,ones(nRx,1));
    ipHatMod2SS = reshape(ipHatMod2SS,[nRx,1,N/nTx]);
    % new received symbol - removing the effect from second spatial dimension
    h2SS = hSort(:,2,:); % channel in the second spatial dimension
    r = y - squeeze(h2SS.*ipHatMod2SS);
    % maximal ratio combining - for symbol in the first spatial dimension
    h1SS = squeeze(hSort(:,1,:));yHat1SS = sum(conj(h1SS).*r,1)./sum(h1SS.*conj(h1SS),1);
yHatSort(1:2:end) = yHat1SS; yHatSort = reshape(yHatSort,2,N/2) ;
    if ~isempty(sortIdx),yHatSort(:,sortIdx) = flipud(yHatSort(:,sortIdx));
end  yHat = reshape(yHatSort,1,N);% receiver - hard decision decoding
ipHat = real(yHat)>0; % counting the errors, nErr(kk,ii) = size(find([ip- ipHat]),2);
end,end,simBer = nErr/N; % simulated ber,EbN0Lin = 10.^(Eb_N0_dB/10);
theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5)); p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);
theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p)); close all
semilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);hold on
semilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);
semilogy(Eb_N0_dB,simBer(1,:),'mo-','LineWidth',2);
semilogy(Eb_N0_dB,simBer(2,:),'gp-','LineWidth',2);
```

```
axis([0 25 10^-5 0.5])grid on
legend('sim (nTx=2,nRx=2, ZF)', 'sim (nTx=2,nRx=2, MRC)', 'sim (nTx=2, nRx=2, MMSE-SIC)','sim (nTx=2, nRx=2, MMSE-SIC-Sort)');xlabel('Average Eb/No,dB');
ylabel('Bit Error Rate');title('BER for BPSK modulation with 2x2 MIMO and MMSE-SIC equalizer (Rayleigh channel)');
```

APPENDIX A3

Improve Mat-lab code for computing the BER for BPSK modulation in arayleigh fading channel with 2Tx, 2Rx MIMO channel Minimum Mean Square Error Equalization with Successive Interference Cancellation (ZF-SIC) with optimal ordering.

```
clear
N =  10^6; % number of bits or symbols,Eb_N0_dB = [0:25]; % multiple Eb/N0 values
nTx = 2;nRx = 2;for ii = 1:length(Eb_N0_dB) % Transmitter
ip = rand(1,N)>0.5; % generating 0,1 with equal probability  s = 2*ip-1; % BPSK modulation 0 -
> -1; 1 -> 0sMod = kron(s,ones(nRx,1)); %sMod = reshape(sMod,[nRx,nTx,N/nTx]); %
grouping in [nRx,nTx,N/NTx ] matrix h = 1/sqrt(2)*[randn(nRx,nTx,N/nTx) +
j*randn(nRx,nTx,N/nTx)]; % Rayleigh channel
 n = 1/sqrt(2)*[randn(nRx,N/nTx) + j*randn(nRx,N/nTx)]; % white gaussian noise, 0dB variance
   % Channel and noise Noiseaddition  y = squeeze(sum(h.*sMod,2)) +
10^(Eb_N0_dB(ii)/20)*n; % Receiver
   % Forming the MMSE equalization matrix W = inv(H^H*H + sigma^2*I)*H^H
   % H^H*H is of dimension [nTx x nTx]. In this case [2 x 2]
   % Inverse of a [2x2] matrix [a b; c d] = 1/(ad-bc)[d -b;-c a]hCof = zeros(2,2,N/nTx)  ;
hCof(1,1,:) =  sum(h(:,2,:).*conj(h(:,2,:)),1) + 0*10^(-Eb_N0_dB(ii)/10);  % d term
hCof(2,2,:) =  sum(h(:,1,:).*conj(h(:,1,:)),1) + 0*10^(-Eb_N0_dB(ii)/10);  % a term
hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c term
hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:)),1); % b termforkk = 1:2
  if kk == 1 sortIdx = []; hCof(1,1,:) =  sum(h(:,2,:).*conj(h(:,2,:)),1) + 10^(-Eb_N0_dB(ii)/10);
% d termhCof(2,2,:) =  sum(h(:,1,:).*conj(h(:,1,:)),1) + 10^(-Eb_N0_dB(ii)/10);  % a term
```

hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:)),1); % c term,hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:)),1); % b termelseifkk == 2 % Sorting the equalization matrix based on the channel power on each dimension % since the second spatial dimension is equalized first, the channel % with higher power assigned to second dimension normSS1 = squeeze(hCof(2,2,:));
normSS2 = squeeze(hCof(1,1,:)); sortIdx = find(normSS2 < normSS1);end
% sorting the H^H*H + sigma^2*I matrix hCofSort = hCof;
if ~isempty(sortIdx) hCofSort(2,2,sortIdx) = hCof(1,1,sortIdx) + 10^(-Eb_N0_dB(ii)/10);;
hCofSort(1,1,sortIdx) = hCof(2,2,sortIdx) + 10^(-Eb_N0_dB(ii)/10);;
hCofSort(1,2,sortIdx) = hCof(2,1,sortIdx);hCofSort(2,1,sortIdx) = hCof(1,2,sortIdx); end hDen = ((hCofSort(1,1,:).*hCofSort(2,2,:)) - (hCofSort(1,2,:).*hCofSort(2,1,:))); % ad-bc term
hDen = reshape(kron(reshape(hDen,1,N/nTx),ones(2,2)),2,2,N/nTx); % formatting for divisionhInvSort = hCofSort./hDen; % inv(H^H*H) % sorting the H matrix
hSort = h;if ~isempty(sortIdx)hSort(:,2,sortIdx) = h(:,1,sortIdx); hSort(:,1,sortIdx) = h(:,2,sortIdx); end % Equalization - Zero forcing hModSort = reshape(conj(hSort),nRx,N); % H^H operation yModSort = kron(y,ones(1,2)); % formatting the received symbol for equalization ModSort = sum(hModSort.*yModSort,1); % H^H * y
yModSort = kron(reshape(yModSort,2,N/nTx),ones(1,2)); % formattinyHatSort = sum(reshape(hInvSort,2,N).*yModSort,1); % inv(H^H*H)*H^H*y
% receiver - hard decision decoding on second spatial dimension ipHat2SS = real(yHatSort(2:2:end))>0;ipHatMod2SS = 2*ipHat2SS-1ipHatMod2SS = kron(ipHatMod2SS,ones(nRx,1));ipHatMod2SS = reshape(ipHatMod2SS,[nRx,1,N/nTx]);% new received symbol - removing the effect from second spatial dimensionh2SS = hSort(:,2,:); %

channel in the second spatial dimension r = y - squeeze(h2SS.*ipHatMod2SS);% maximal ratio combining - for symbol in the first spatial dimensionh1SS = squeeze(hSort(:,1,:));

yHat1SS = sum(conj(h1SS).*r,1)./sum(h1SS.*conj(h1SS),1);yHatSort(1:2:end) = yHat1SS;

yHatSort = reshape(yHatSort,2,N/2) ;if ~isempty(sortIdx)yHatSort(:,sortIdx) = flipud(yHatSort(:,sortIdx)); endyHat = reshape(yHatSort,1,N); % receiver - hard decision decodingipHat = real(yHat)>0;% counting the errorsnErr(kk,ii) = size(find([ip- ipHat]),2); endendsimBer = nErr/N; % simulated berEbN0Lin = 10.^(Eb_N0_dB/10);

theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5)); p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);

theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p)); close allsemilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);hold onsemilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);

semilogy(Eb_N0_dB,simBer(1,:),'mo-

','LineWidth',2);semilogy(Eb_N0_dB,simBer(2,:),'gp','LineWidth',2);axis([0 25 10^-5 0.5])grid onlegend('theory (nTx=2,nRx=, ZF)', 'theory (nTx=1,nRx=2, MRC)', 'sim (nTx=2, nRx=2, MMSE-SIC)','sim (nTx=2, nRx=2, MMSE-SIC-Sort)');xlabel('Average Eb/No,dB');ylabel('Bit Error Rate');

title('BER for BPSK modulation with 2x2 MIMO and MMSE-SIC equalizer (Rayleigh channel)');

APPENDIX A4

Matlab code for computing the BER for BPSK modulation in a rayleigh fading channel with NTx, MRx MIMO channel for Maximum Likelihood channel estimation Equalization scheme

```
clear
N =10^6; % number of bits or symbolsEb_N0_dB = [0:25]; % multiple Eb/N0 values
nTx = 2;nRx = 2;for ii = 1:length(Eb_N0_dB)  % Transmitterip = rand(1,N)>0.5; % generating
0,1 with equal probability   s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 0
sMod = kron(s,ones(nRx,1)); % sMod = reshape(sMod,[nRx,nTx,N/nTx]); % grouping in
[nRx,nTx,N/NTx ] matrixh = 1/sqrt(2)*[randn(nRx,nTx,N/nTx) + j*randn(nRx,nTx,N/nTx)]; %
Rayleigh channel n = 1/sqrt(2)*[randn(nRx,N/nTx) + j*randn(nRx,N/nTx)]; % white gaussian
noise, 0dB variance % Channel and noise Noise addition
   y = squeeze(sum(h.*sMod,2)) + 10^(-Eb_N0_dB(ii)/20)*n; % Maximum Likelihood Receiver
   % if [s1 s2 ] = [+1,+1 ]  sHat1 = [1 1];  sHat1 = repmat(sHat1,[1 ,N/2]);
   sHat1Mod = kron(sHat1,ones(nRx,1)); sHat1Mod = reshape(sHat1Mod,[nRx,nTx,N/nTx]);
   zHat1 = squeeze(sum(h.*sHat1Mod,2));J11 = sum(abs(y - zHat1),1);
   % if [s1 s2 ] = [+1,-1 ] sHat2 = [1 -1]; sHat2 = repmat(sHat2,[1 ,N/2]);
   sHat2Mod = kron(sHat2,ones(nRx,1));sHat2Mod = reshape(sHat2Mod,[nRx,nTx,N/nTx]);
   zHat2 = squeeze(sum(h.*sHat2Mod,2)); J10 = sum(abs(y - zHat2),1); % if [s1 s2 ] = [-1,+1 ]
   sHat3 = [-1 1];    sHat3 = repmat(sHat3,[1 ,N/2]);sHat3Mod = kron(sHat3,ones(nRx,1));
   sHat3Mod = reshape(sHat3Mod,[nRx,nTx,N/nTx]); zHat3 = squeeze(sum(h.*sHat3Mod,2)) ;
   J01 = sum(abs(y - zHat3),1);  % if [s1 s2 ] = [-1,-1 ]  sHat4 = [-1 -1]; sHat4 = repmat(sHat4,[1
,N/2]);   sHat4Mod = kron(sHat4,ones(nRx,1));sHat4Mod
```

```
reshape(sHat4Mod,[nRx,nTx,N/nTx]); zHat4 = squeeze(sum(h.*sHat4Mod,2)) ; J00 =
sum(abs(y - zHat4),1);% finding the minimum from the four alphabet combinations
rVec = [J11;J10;J01;J00]; [jjdd] = min(rVec,[],1); % mapping the minima to bits
ref = [1 1; 1 0; 0 1; 0 0 ]; ipHat = zeros(1,N);ipHat(1:2:end) = ref(dd,1);
ipHat(2:2:end) = ref(dd,2); % counting the errorsnErr(ii) = size(find([ip- ipHat]),2);
endsimBer = nErr/N; % simulated berEbN0Lin = 10.^(Eb_N0_dB/10);
theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5)); p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);
theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p)); close allfigure
semilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);
holdonsemilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);
semilogy(Eb_N0_dB,simBer,'mo-','LineWidth',2);axis([0 25 10^-5 0.5])grid on
legend('theory (nTx=1,nRx=1)', 'theory (nTx=1,nRx=2, MRC)', 'sim (nTx=2, nRx=2, ML)');
xlabel('Average Eb/No,dB');ylabel('Bit Error Rate');
title('BER for BPSK modulation with 2x2 MIMO and ML equalizer (Rayleigh channel)');
```