APPLICATION OF QUEUEING THEORY IN TACKLING THE PROBLEM OF PORT CONGESTION AT APAPA PORT, LAGOS, NIGERIA

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JULY, 2018

Declaration

I declare that the work in this dissertation titled "APPLICATION OF QUEUING THEORY IN TACKLING THE PROBLEM OF PORT CONGESTION AT APAPA PORT, LAGOS, NIGERIA" has been carried out by me in the Department of Statistics. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this dissertation was previously presented for another degree or diploma at this or any other institution.

________________ ________________

Muhammadu Umaru Date

Certification

This dissertation titled "APPLICATION OF QUEUING THEORY IN TACKLING THE PROBLEM OF PORT CONGESTION AT APAPA PORT, LAGOS, NIGERIA" meets the regulations governing the award of the degree of Master of Science of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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Dedication

To my wife, Fatima.

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Table of

Contents

Contents

List of Tables

Abstract

The Apapa port is part of the ports operated by the Nigerian Ports Authority which was established in 1955 to oversee the activities and operations of all Nigerian sea ports. Over the years the ports have witnessed tremendous increase in volume of trade causing heavy congestion as ships queue up waiting to discharge importers' cargoes. The study demonstrates the applicability of queuing theory models in addressing the problem of congestion in Nigerian Ports, by dealing with the application of multi-queue multi-server queuing model with infinite capacity, First come-First Served model in tackling the congestion problem at the Apapa port, Lagos, Nigeria. The performance indicators of the existing single-queue multi-server model at the general goods cargo terminal of the Apapa port were computed while the results for the performance indicators of the multi-queue multi-server queuing model were also computed and examined. The results obtained were found to be effective in improving port efficiency as it shows that average number of ships in the system (queue and at berth), average queue length, average waiting time of ships in the queue and in the system are reduced in the proposed model by up to 73%, 93%, 78% and 93% respectively. Consequently the study recommended that the multi-queue multi-server queuing model should be the model of choice to solve the ship congestion problem at the Nigerian Ports. This is more so as its implementation is at very minimal cost which is attractive to the port owners.

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Queuing theory deals with the study of waiting for services at service point of any kind (Sztrik, 2012). It is always desirable by all parties to stay on the queue for as short a time as possible. However, reduction of the waiting time usually requires extra and most times huge capital outlays. To decide whether or not to use any port by merchants they always consider the efficiency of the port operations and for port owners (Federal Government of Nigeria) to invest, it is important to know if their investment reduces the waiting time. Queuing models are able to analyze such situations. In this study, attention is paid to application of multiple queues to a multi-server situation to improve on the efficiency of operations in Nigerian Ports. Data collected from Apapa Ports for a period of five months, August to December 2016 were analyzed. Shipping is the cheapest method of all other common modes of transportation. Shipping carries large volume of cargo, which is almost four times more than rail and four hundred times higher than air transportation in total (Martin and Stopford, 2009). Most of goods shipped through Apapa ports are in containers.

Containerization comes with opportunities for safe and secure shipping and handling of cargo. Challenges come into consideration because containerization significantly changes the requirements for terminal facilities (Islam and Olsen, 2011). Important application areas of queuing models are production systems, transportation and stocking systems, communication systems and information processing systems. Queuing models are particularly useful for the design of these systems in terms of layout, capacities and control. In this work, our attention is restricted to a model with multiple queues (i.e. in tackling the problem of port congestion at Apapa Port, Lagos, Nigeria). Queuing theory deals with one of the most unpleasant experiences of life, such as waiting. Erlang (1909) was the first to treat congestion problems in the beginning of the 20^{th} century using the principle of queuing theory. (Sztrik, 2012)

One of the characteristics of the queuing model is the arrival process of ships. Here the ships arrive according to a Poisson stream (i.e. exponential inter arrival times). Furthermore the behavior of arriving ships at the fairway buoy is such that they cannot leave after arrival irrespective of the state of the queue and how long they may have to wait.

The service times are independent and identically distributed, and are independent of the inter-arrival times. The service time were exponentially distributed.

The service discipline was observed to be on a one by one and a first-come-first-served and there are multiple numbers of berths that offer services. Hence the situation was a single queue multiple server type. The fairway buoy where ships first weigh anchor and wait to be served was assumed also to have an infinite capacity.

Kendall introduced the notations used to characterize queuing models. It is a three-part code denoted as M/M/c. The first letter specifies the inter-arrival time distribution and the second one the service time distribution. The letter M is for the exponential distribution and stands for Memory less property. The third and last letter specifies the number of berths. In this case, the M/M/c model was applied. The notation can be extended with an extra letter to cover port waiting area capacity. For example, a system with exponential inter- arrival and service times, c number of berths and having a waiting room only for N customers (including the one in service) is abbreviated by the four letter code M/M/c/N. (Adan and Resing, 2015)

In the case of the Nigerian Ports Authority, the ships arrivals are independent of each other and are suspected to be Poisson distributed. Once a ship arrives at the fair way buoy it cannot leave irrespective of the length or state of the queue. It has to wait until served. Hence the importers have to have a fair idea of the anticipated waiting time at each port to decide on which port will serve as final destination economically. The queue systems at Apapa Port are being served by parallel service berth channels where each berth has an independently and identically distributed exponential service time distribution.

The mechanism of a queue process is such that ships arrive at fairway buoy to join the queue and are called to berth as soon as a service point (berth) is free. The ships are served upon arrival at the berth and then leave the system thereafter. Thus the queuing system may be described as composed of ships arriving for service, waiting for service if all berths are occupied (busy) and leaving the system after being served.

Ships arrive randomly and independently and in accordance to a Poisson process i.e. the number of ships arriving until any specific time has a Poisson distribution. The pattern of arrival in a time period t follows Poisson distribution which assumes that arrival is completely independent of other arrivals i.e. arrivals are completely random.

Obviously if the arrival process follows the Poisson distribution, an associated random variable defined as the time T (inter-arrival time) follows the exponential distribution.

Finally, arrivals of ships are said to be in tandem with Poisson process are random since the probability of arrival of a ship in a small interval of time of length h is proportional to the length h, and is independent of the amount of lapsed time from the arrival period of the last ship.

The service discipline at the Nigerian Ports is first come first served (FCFS), no reneging, balking, jockeying or collusion is allowed or possible. If the service is assumed continuous, then the mean service time μ is also assumed to be constant over long time and is independent of the number of units already serviced, queue length or any other random property of the system.

Therefore, probability of complete services implies that the density function of inter-service time is exponential.

However in the case under study it is not possible for service to be continuous within a long time interval as the possibility of a service berth to be idle for sometimes exists, therefore Poisson distribution cannot be applied to servicing.

The exponential distribution is the only continuous distribution with the important property of forgetfulness or lack of memory suitable for the service process (Cooper, 1972).

1.2 Statement of the Problem

Capacity shortage increases port costs (i.e. higher surcharge or demurrage for port users in a competitive open market). This implies that if costs are kept low then other cheaper ports or less congested ports (e.g. Cotonou and Lome) become more noticeable (Dekker,2005). The ocean transport industry is growing at a faster rate than seaports can build facilities (Pallis and de Langen, 2010) and it takes from two to over ten years from decision to completion of changes in the infrastructure to increase capacity (Henesey, 2006). As many ports are exceeding their capacities and finding it difficult to find the capital outlay, ports worldwide are increasingly faced with the problem of congestion as time goes on. Hence the significance of falling on the use of queuing theory to address the problem becomes pertinent. The case of Nigeria is of great concern to port and economic planners as was re-emphasized in a Patrick Doyle Nigeria Television Authority (NTA) international production for Nigeria Maritime Safety Agency (NIMASA) on August 31, 2017 that maritime trade in Nigeria will quadruple by the year 2035.

Congestion brings delays for port users and increases the cost to many stakeholders; for example, to shipping lines, it brings about increase in cost of charter, while to terminals it brings yard congest and increases cost of re-handling (Mabs, 2009).

The Nigerian Ports Authority (NPA) was established in 1955 to manage all Nigerian Sea and Inland Container Terminals or Ports. Apapa Ports, Lagos is one of such ports. The volume of international trade in and out of Nigeria has been on a steady increase, hence the demand for the services of the various ports have been stretched causing delay and ships have to wait for their turn to discharge / load cargo into/out of the various ports. In a shipping business, a wait of a single day is of tremendous cost and should be avoided as much as possible. The importer/exporter would rather not wait, while the port owners cannot afford to provide a dedicated berth for every ship that calls as the cost will be unbearable, so a balance must be stroke. The importers, on the one hand, would not want to pay heavy demurrage and ship charter on the one hand while the Nigerian Ports would not want to lose business (if importers re-route their cargo through neighboring country's ports which may be cheaper or better managed) as a result of its high charges or having to run the ports at a loss on the other hand. If the number of berths were to be infinite then all the ships will be served instantaneously on arrival and there will be no queue. However,, this is not practically possible. In view of these problems, we intend to examine the existing queuing system in the Nigerian Ports with a view to proposing a better model that would improve overall Ports Operations in Nigeria.

1.3 Aim and Objectives of the Study

The aim of this work is to examine the existing queuing system at Apapa Port, Lagos with a view to applying a model that would substantially improve the port's operations and the objectives are:

i. to establish the arrival and service distributions of ships at Apapa Port by conducting a chi-square goodness-of-fit test respectively
ii. to identify and apply the existing queuing model for the port's operations;
iii. to apply the appropriate queuing model in order to reduce the length of the queues and length of stay by the ships.
iv. to compare the multiple queue model with the existing model

1.4 Significance of the Study

The problem of congestion is a source of concern to the Federal Government in that different solutions proffered were either capital intensive to implement or unattractive to importers. This study is therefore significant in that it seeks to provide another method of solving or reducing the congestion problem to a mutually agreeable level. The study is significant in providing better services with tolerable waiting.

The Nigerian Ports Authority (NPA)'s poor performance restricts national growth, adds to the cost of all goods that transit in and out of the country. Considering that through the years 2000 to 2003, the NPA cost the Federal Government N86.7billion naira (Bureau for Public Enterprise, BPE, 2005) it is therefore of great significance to examine ways of improving the efficiency of NPA at minimum costs.

Furthermore the study is significant in evaluating the efficiency of the port operations, port planning, capacity assessment and building.

1.5 Contribution to Knowledge

The dissertation's contribution to knowledge is in the application of this variation of multiple queue multiple berth model to port ship arrival and services process which have hitherto not been seen in literature. The model applied is a multiple queue multiple berth model which is found to shorten the number of ships in the system and on the queue. Similarly the applied model gave results indicating shorter waiting time in the system and on the queue without having to resort to infrastructural development like building more service berths.

1.6 Limitations of the Study

The study was limited by time and financial resources to go round all the ports and collect real time data from the several seaports and inland container terminals spread in the country. Furthermore, this study did not cover all the different aspects of the port operations which could also cause queuing or congestion problems. For example the efficiency of the port bureaucratic processes, port union activities, Nigerian Customs, clearing agents, stevedores, adequacy of space to store the cargo after offloading, efficiency of other material handling equipments like forklifts, trucks, cranes etc. All other activities that could cause queuing were assumed to be at optimal performance levels. It was not possible to obtain actual cost in monetary value of the different required operating indices like cost of ship wait, berth un-occupancy cost etc. per unit time.

Finally, queuing models that could possibly be better but require increasing the number of berths and hence heavy capital outlay and considerable number of years to put in place were not considered.

1.7 Assumptions of the Study

The assumptions of the study are as follows:

i. Ships arrivals are independent, random and are served on a first-come-first-served basis.

ii. The fairway buoy has infinite capacity to accommodate arriving ships.

iii. The arrival of ships is Poisson distributed with a mean arrival rate of λ and the inter-arrival rate is exponentially distributed.

iv. Balking or jockeying is not allowed as all arriving ships are held until served.

v. The service times are assumed to be independent and identically distributed, and are independent of the inter-arrival times. Service times are exponentially distributed.

vi. The mean service rate μ is the same for each server.

1.8 Definition of Terms

a) Queue: This is a line or sequence of ships waiting to be served.

b) Berths: This refers to the service point where offloading/loading services are being provided to ships calling.

c) M/M/c Model: Is a queuing model with queues which are being served by parallel service channels c, in which each server has an independently and identically distributed exponential service time distribution.

d) Demurrage: Those are charges that a charterer or importer pays to the ship or port owner for extra time or over usage of the vessel or space at the port.

e) Inter-Arrival time: This is the time between successive arrivals and is exponentially distributed.

f) **Steady State**: On the long run, the probability distribution of arrivals, waiting time and service times are independent of time and are hence said to be in steady state since its operating characteristics are independent of time.

g) **Saddle Point or equilibrium point:** This is when the average arrival rate equals average service rate.

h) **Service Discipline:** This refers to the manner, in which those in a queue are chosen for service. The different types of service discipline are First come, first served (FCFS);Last come, first served (LCFS);Service in Random Order (SIRO); and General Service Discipline (GSD)

CHAPTER TWO

LITERATURE REVIEW

2.1 Review of Literature

Queuing models provide the possibilities to evaluate existing port operation and solve realistic problems in the ports by using real data obtained from the ports. These models in most cases, are used for strategic or tactical decisions.

There have been many research of immense importance in the application of queuing theory to address varying problems (theoretical and practical) in the sea ports worldwide; some of which include Lopez *et al*. (2012)who studied non Markovian queuing systems in container terminals of the Port of Valencia, Spain. The results of the study showed that simulation model based on queuing theory was found to be effective in replicating realistic ship traffic operations in the ports as well as in conducting capacity evaluations. Their objective was to investigate to what extent a simulation model could predict the actual container operations with a high order of accuracy. The results helped in visualizing the development of quays and understanding the system behaviors in order to evaluate the berth capacity at the port. Concluding that of very important consideration is the availability of adequate and sufficient berth capacity at the port. The simulated scenarios of each container terminal represented an important management tool for tracking progress against strategic goals and helps the port operators to maximize the number of ships serviced.

Dragovic *et al*. (2005) carried out the performance evaluation of ship-berth link using cost function, computational experiments are reported to evaluate the efficiency of Pusan East Container Terminal in South Korea. Their study came up with the optimum number of berths required and recommended port expansion to solve the queue congestion. The queue discipline considered was of first come first served.

Dragovic *et al*. (2006) on the other hand considered a queue discipline of priority service in carrying out their study of the ship-birth link in the Pusan East Container terminal in South Korea They compared results of the performance indicators obtained from simulation and analytical methodology. The study computed the optimal number of births required to reduce or eliminate queues. This requires an immense investment on the part of the port owners which is not favorable as it is a long term investment whereas a short to medium term solution is needed.

Kuo *et al*. (2006) studied the evolution of ship arrival and service time distribution in public and dedicated container berths in the port of Koohsiung (Taiwan) by comparing the variance in the patterns of 1400 ships in the container terminal and 7,700 ships in the entire port. The study recommended the optimum number of berths required to eliminate queues at the ports given the current traffic volumes.

Islam and Olsen (2011) examined the factors affecting seaports capacity by identifying the consequences of capacity shortage at sea ports and corresponding supply chains, offering conceptual framework to summarize the factors influencing seaport capacity using holistic approach and suggested promising research tracks on factors affecting capacity. Specifically they explored the roles of deterministic simulation and stochastic simulation as future research direction in this rapidly charging and challenging maritime domain. Their research findings were that the factors affecting seaports capacity were the container yard, which is used temporarily for container storage cranes, stressing that the performance of both on and off the ship is an important factor to support the port efficiency labour, which affects the performance of almost all of the processes rigorously from gate operations for truckers to berthing activities for shipping lines. They also considered gates and inland waterways as factors affecting seaport capacity but fell short of examining other factors like the queuing system processes in the ports.

El-Naggar (2010) determined the optimum number of berths at the seaports of Alexandria, Egypt that minimizes the total ports' operating costs and to be able to meet future traffic volumes. The study relied on the Erlang's queuing model with multiple servers (berths) and Poisson input for the analysis and came up with an optimum number of berths required. The study also used chi-square to test the goodness of fit between the observed frequency distribution and the postulated Erlang function. The aim was to avoid inadvertent over and under building. The waiting time of vessels outside the port and in queue was calculated. For economic consideration cost characteristics were also computed. The results of the study showed that the computed optimum berths proposed will be in the best interest of ship operators and port authorities. Concluding that maximum port efficient results when total port cost is minimum i.e. the cost of vacant berths over a substantial period plus the time cost of ships waiting for a berth having the same time period.

Similarly in Novaes *et al*. (2010),a short survey on mathematical approach of queuing models applied to container terminals planning has been presented. In addition, they described some approximate methods for more complex queuing models based on approximate formulae to estimate mean waiting times. The result of their work is quite suitable at the planning stage to determine the number of berths to be constructed based on the estimated expected traffic volume.

Mirano (2007) fit a M/M/I queuing model to the port operations of Bakar, Croatia and determined the optimum levels of the various operations of the port. Characteristically the system had a Poisson arrivals and exponentially distributed service times. Subsequently the study tested the model with data obtained from the bulk cargo ports at Bakar and Rijeka, Croatia. The application of the model made it possible to take a decision on how to optimize the trans-shipment processes in the port with a view to increase its efficiency. The study fixed the bulk terminal in Bakar with M/M/I model which proved useful in modeling of capacity employment and computing of realistic indices of the terminals behaviour. The results were used in interpretation of the terminal's workload which supports decision making and future planning by management. The obtained parameters and the calculated indices point to solid capacity utilization in 2005 and there was a relatively low probability that the berth was unoccupied during the period under review.

Nafees (2007) in her MSc thesis carried out queuing performance indicators computation for a multiple server process as well as for single queue models. The study relied on questionnaire responses and observations over a period of time on two busy days in a supermarket in Kuala Lumpur, Malaysia where the queue behaviour was jockey i.e. arriving customers can leave a long queue after joining to a shorter queue to at will. The result of the study showed that a multiple queueing model gives better results than a single queueing model. The efficiency performance of the supermarket was found to be best under the application of a multiple queueing model.

Dragovic *et al*. (2011) discussed system performance evaluation in the river port on the Dunube River, Smedervo, Serbia utilizing queuing models with batch arrivals. The study applied the M/M/n/M queue model i.e. a queue with finite waiting areas and identical and independent cargo handling capacities. The model was considered with whole and part batch acceptance (or whole or part batch rejections) and the inter-arrival and service times were exponentially distributed, results related to the part batch blocking probability and the blocking of an arbitrary vessel in non-stationary and stationary states were obtained. The performance of the models was evaluated using the obtained basic parameters as in all queuing models.

Mestrovic (2013) established an analytic method for determining the optimal number of berths in a port with finite waiting areas (berths) modeled by $M/M/n_b/K$ queue. In particular the study presented the optimal number of berths under condition that the number of waiting areas is fixed.

The optimal number of berths was computed in order to minimize usage cost and the capacity of servers. They also predicted specific cost ratio in port and ranges of optimum berth capacity. The explicit specific cost ratio in a port for M/M/1/k queue was derived and compared with $M/M/n_b$ (where n_b number of berths, and k is the number of waiting area). The results are useful tools for decision making process in port management and better understanding of the relative merits of new strategy of considering multi-berth situation as against a single berth service point and its applicability.

Zenzerovic and Mrnjavac (2000) demonstrated the application of a single server multi berth queuing model to a hypothetical port and showed the possibility to achieve acceptable port container terminal operational efficiency by determination and using the optimal terminal berth capacity accommodation. They aimed to show that port operations can be monitored with the aid of the queuing theory model by determining the optimal capacity of a port terminal. The results of their study showed that a change in the number of berths impacts on the increased or reduced values of particular container terminal indices by an increase in the number of berths, the number of ships in the queue and at the terminal, as well as waiting time and length of the ship's stay at the terminal are reduced, but the berth un-occupancy rate is increased. Similarly a curtailment in ship service time may affect the quality of service in a negative way thus reducing the number of ship arrivals. Therefore, they concluded that it is best to look at cost indices of the port operations.

Since in practice, a ship's waiting time has to be paid for and the un-occupancy of the berth can also be expressed in terms of monetary value. The study took costs as optimization criteria. Hence they posited that the servicing process solution at the container terminal represented the optimum number of berths for which the total expenses of ship waiting time and expenses of berth un-occupancy is minimum in an observed unit of time. In this regard, the optimal variant will be that one which will reduce to a minimum losses resulting from waiting.

Dragovic and Zrnic (2011) carried their study describing port performance evaluation by the use of queuing models. Their results showed that queuing models had positive influence on port performance evaluation in the areas of faster development, greater flexibility, less data requirement and being easier to understand and interpret the results among others. Their methodology highlighted the development of queuing models for port systems performance evaluation indicators based on various mathematical approaches such as: queuing formulae, analytical formulations and formulae, comparison with simulation models, integration simulation and optimization with queuing network models, cyclic queuing models and Markovian decision process, analytical formulations and numerical solutions of bulk arrival queues, steady state versus time-dependent among others. In a nutshell the paper examines different queuing modeling approaches to port performance evaluation.

Lother (2014) provided how a non-finite queuing theory model was applied to the Port of Durban Container terminal. The study used the Port of Rotterdam's Container terminal in Netherlands as a benchmark to evaluate the Port of Durban. The study concluded that queuing theory provides reliable results in improving port efficiency as it highlighted the indices where the Port of Durban is performing lower than Rotterdam and therefore providing management with information on areas of focus to improve port efficiency.

Raz *et al*. (2005) studied the fairness of multi-queue and multi-server queuing systems by dealing mainly on queue-multiplicity, queue joining policy and queuing jockeying and used a quantitative measure Resource Allocation Queuing Fairness Measure (RAQFM) to evaluate them. Their result yields the relative fairness of the mechanisms as a function of the system configuration and parameters. The result of the study is useful to practitioners to quantitatively account for system fairness and to weigh efficiency aspects versus fairness in designing and controlling queuing systems. The study demonstrated that joining a shortest queue increases fairness, a single queue is more fair than a multi queue system and also that jockeying from the head of a queue is more fair than jockeying from its tail.

Budiono (2016) discussed the development of the queuing system for optimization of harbor facilities. The model applied was that of queue with server service pattern that follows the Erlang distribution which consists of two phases of service and priority queue service discipline. The methodology was of two approaches, mathematical model approach which was to determine queue statistics such as waiting times and length of queues. While the second approach was the use of fuzzy queues which was done to facilitate the optimization process so that the optimization model is simplified. The results obtained from simulation and analysis show that the queuing models can in fact improve the optimization of services provided so that the ship waiting time becomes shorter.

Mallidis (2010) applied queuing theory to model the delivery and receipt operations of the container terminal and improve the service characteristics for specific yard management operations and recommended infrastructural improvement of the terminal. Having examined the current situation in the port, the study calculated the mean service rate, and mean arrival rate at the port presently and applied the existing queuing model M/M/K. The study recommended the need to purchase additional resources and infrastructural improvement at the ports. Both recommendations require high capital outlay.

Oyatoye *et al*., (2011) used the M/M/c queuing model to predict the average arrival rate of ships at Tin Can Island ports and the average service rate per ship in a month and provided the optimum number of berths that will bring about quick turn-around of ships which may lead to making the ports more efficient. The study also recommended that for improvement of quality of services provided by Nigerian Ports in terms of better efficiency and effectiveness; the port should reduce the dwell time by introducing punitive measures to discourage importers from using the ports as storage area for example demurrage charges. They also recommended the physical expansion of port capacity to reduce port congestion thereby making the ports more attractive to users etc. However, due to the high and prohibitive cost of physical expansion and the fact that it will take up to ten years to implement or more, implementation of this aspect of the work has become difficult to implement.

On the other hand the study here looked at the option of proposing a better model which is cost effective as it requires no need to invest in port expansion with its prohibitive cost and the consequent loss of revenue during construction work and hopefully will be more acceptance to all stakeholders in port operations.

CHAPTER THREE

METHODOLOGY

3.1 Chi-Square Goodness-of-fit-tests

To determine if the arrival distribution was Poisson and if the services was exponentially distributed, chi-square criteria goodness of fit test on month by month analysis was employed.

The test involves the statistical comparison between actual observed frequencies and computed expected frequencies of the random variables.

For this test the Null Hypothesis, H_0 =The arrivals of ships were Poisson distributed; While the alternative H_1 = The arrivals were not Poisson distributed. We fail to reject the Null hypothesis if the chi-square statistics calculated is less than the chi-square value obtained at 0.05, and r-1 degrees of freedom, while we reject the null hypothesis otherwise. Similarly, the null hypothesis for service times is H_0 = Services rendered to ships are exponentially distributed while the alternative hypothesis H_1= the services rendered by the Apapa ports to ships is not exponentially distributed. We fail to reject the null hypothesis if the chi-square calculated is less than chi-square from the table.

3.1.1 Inter-arrivals are exponentially distributed

Ships arrive randomly and independently. We have assumed that they arrive according to a Poisson process i.e. the number of ships arriving until any specific time has a Poisson distribution. The pattern of arrival in a times period t follows Poisson distribution which assumes that arrival is completely independent of other arrivals i.e. arrivals are completely random.

Probability of n arrivals $P_n = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$

Where $n =$ number of arrivals per unit time

$t =$unit time

$\lambda =$mean arrival rate

We expect that the probability density function of inter-arrival time is $\lambda e^{-\lambda t}$if exponentially distributed.

Obviously, if the arrival process follows the Poisson distribution $P_n = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$ an associated random variable defined as the time T (inter-arrival time) follows the exponential distribution $f(t) = \lambda e^{-\lambda t}$, since $F(t) = prob(T \leq t)$

Where

T = inter-arrival time

t = any time $t \geq 0$

$$F(t) = 1 - prob(T > t)$$

$$= 1 - P_0(t)$$

$$= 1 - \frac{(\lambda t)^0 e^{-\lambda t}}{0!}$$

$$= 1 - e^{-\lambda t}$$

Obtain the derivatives of both sides with respect to t

$\frac{dF(t)}{dt} = F'(t) = f(t) = \lambda e^{-\lambda t}$, hence the inter-arrival process is exponentially distributed

3.2 The Port Basic Parameters and Utilization Coefficient

The basic parameters of a queuing model for the shipping port are the arrival rate λ and the service rateμ. The arrival rate λ is the average number of ships arriving during an observed time unit (e.g. during a year, month or day). The service rate μ is the average number of ships that are served during an observed time unit.

The parameter μ represents the accommodative capacity of one berth, while the c μ represents the accommodative capacity of the entire port. c is the number of berth.

The port utilization coefficientρ is represented by the quotient of the arrival rate and service rate.

i.e. $$\rho = \frac{\lambda}{c\mu} \qquad 3.1$$

If λ is greater than $c\mu$, a congestion exists since the utilization factor is greater than 1. In this event, the number of berths should be increased, the queuing model should be examined or other factors should be considered until the service system satisfies the stability condition that the utilization coefficient of the system ρ is less than 1.

Adedayo *et al* (2006) stated that the closer the utilization coefficient (traffic intensity) is to zero the more efficient the operations of the port.

3.3 The Single Server, Unlimited Queue Model(M/M/I): (∞/FCFS)

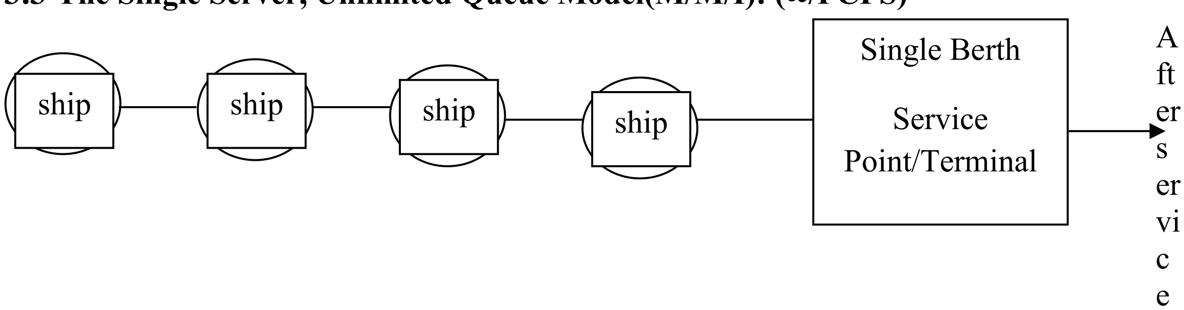

Fig.3.1: Single Queue Single Server Model

The model as depicted in figure 3.1 is of Poisson Arrival, Exponential Inter-arrival and Exponential Service time, with infinite capacity and with a service discipline of the type 'first come, first served". Arriving ships cannot leave the queue. Once at fairway buoy they must wait until served. Port of destination is usually pre-determined. Service discipline is of the type Blocked Customers Delayed (BCD). Both the arrival and service distributions are represented by the letter "M" in the Kendall notation. The letter "M" represents the queue's memory-less nature and is also used to honor the Russian mathematician Andreyevich Markov who was associated with this (Verma, 2014).

Let P_n be the Probability that there are n ships in the system (total number in queue and at berth) at

any time,

λ =mean arrival time

μ = mean service time

t =time

n =no of ships and $n \geq 0$

To obtain P_n we have to consider four mutually exclusive events (these are occasions when only *n* ships are in the system at a particular time *t* in each case. These events are independent of each other) as follows:

i. Events at time t — Probability

   a. If n ships are in the system — $P_n(t)$

   b. During time Δt, if 0 ship arrived — $1 - \lambda(\Delta t)$

   c. During time Δt, 0 ship served — $1 - \mu(\Delta t)$

Total probability, P_1 $= P_n(t)[1-\lambda(\Delta t)][1-\mu(\Delta t)]$

$= P_n(t)[1-\lambda(\Delta t)-\mu(\Delta t)+\lambda\mu(\Delta t)^2]$

$= P_n(t)[1-\lambda(\Delta t)-\mu(\Delta t)]as(\Delta t)^2 \to 0$

$P_1 = P_n(t)[1-(\Delta t)(\lambda+\mu)]$ 3.2

ii. Events at time t — Probability

a. If $(n-1)$ ships are in system — $P_{n-1}(t)$

b. During time Δt, if only one ship arrives — $\lambda(\Delta t)$

c. During time Δt, if no ship is served — $1-\mu(\Delta t)$

Total probability P_2 $= P_{n-1}(t)\lambda(\Delta t)[1-\mu(\Delta t)]$

$= P_{n-1}(t)\lambda[(\Delta t)-\mu(\Delta t)^2]$

$P_2 = P_{n-1}(t)\lambda(\Delta t)$ 3.3

as$(\Delta t)^2 \to 0$

iii. Events at time t — Probability

a. If $(n+1)$ ships are in system — $P_{n+1}(t)$

b. During time Δt, if only one ship is serviced — $\mu(\Delta t)$

c. During time Δt, if no ship arrived — $1-\lambda(\Delta t)$

Total probability P_3 $= P_{n+1}(t)\mu(\Delta t)[1-\lambda(\Delta t)]$

$= P_{n+1}(t)[\mu(\Delta t)-\mu\lambda(\Delta t)^2]$

$P_3 = P_{n+1}(t)\mu(\Delta t)$ 3.4

as$(\Delta t)^2 \to 0$

iv. Events at time t — Probability

a. If n ships are in system — $P_n(t)$

b. If during Δt, one ship arrives — $\lambda(\Delta t)$

c. If during Δt, one ship is serviced $\mu(\Delta t)$

Total probability $= P_n(t)\lambda(\Delta t)\mu(\Delta t)$

$= P_n(t)\lambda\mu(\Delta t)^2 = 0$

$as(\Delta t)^2 \to 0$

To obtain the probability of *n* units in the system at time $(t + \Delta t)$ we must add up the independent compound probabilities in equations 3.2, 3.3 and 3.4

$$P_n(t + \Delta t) = P_n(t)[1 - (\Delta t)(\lambda + \mu)] + P_{n-1}(t)\lambda(\Delta t) + P_{n+1}(t)\mu(\Delta t)$$

$$= P_n(t) - P_n(t)(\Delta t)(\lambda + \mu) + P_{n-1}(t)\lambda(\Delta t) + P_{n+1}(t)\mu(\Delta t)$$

$$= P_n(t) - P_n(t)(\Delta t)(\mu + \lambda) + P_{n-1}(t)\lambda(\Delta t) + P_{n+1}(t)\mu(\Delta t)$$

$$P_n(t + \Delta t) = P_n(t) + (\Delta t)[\lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\mu + \lambda)P_n(t)]$$

$$P_n(t + \Delta t) - P_n(t) = (\Delta t)[\lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\mu + \lambda)P_n(t)]$$

Divide both sides by Δt

$$\frac{P_n(t + \Delta t) - P_n(t)}{(\Delta t)} = \lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\mu + \lambda)P_n(t)$$

Take limits as $\Delta t \to 0$

$$\lim_{\Delta t \to 0}\left[\frac{P_n(t + \Delta t) - P_n(t)}{(\Delta t)}\right] = \lim_{\Delta t \to 0}[\lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\mu + \lambda)P_n(t)]$$

$\therefore \frac{dP_n(t)}{dt} = \lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\mu + \lambda)P_n(t)$ for $n > 0$ 3.5

while for $n = 0$, $\frac{dP_0(t)}{dt} = \mu P_1(t) - \mu P_0(t) - \lambda P_0(t)$, as $\lambda P_{n-1}(t)$ has no meaning as you can not have a negative number of ships .

The term $\mu P_0(t)$ conveys no meaning since for zero ship in service, hence

$\frac{dP_0(t)}{dt} = \mu P_1(t) - \lambda P_0(t)$ 3.6

Since the probability of n ships in queue at any time remains the same with the passage of time and therefore equations 3.5and 3.6 can be equated to zeros.

Therefore,

$$\lambda P_{n-1}(t) + \mu P_{n+1}(t) - (\mu + \lambda)P_n(t) = 0 \quad for\, n > 0 \qquad 3.7$$

and

$$\mu P_1(t) - \lambda P_0(t) = 0 \quad for\, n = 0 \qquad 3.8$$

Equations 3.7 and 3.8 are called steady state difference equations

From equation 3.8

$$P_1(t) = \frac{\lambda}{\mu} P_0 \qquad 3.9$$

while from equation 3.7 for $n = 1,2,3 \ldots n$

$$P_2(t) = \frac{\lambda}{\mu} P_1 = \left(\frac{\lambda}{\mu}\right)^2 P_0$$

and

$$P_3(t) = \left(\frac{\lambda}{\mu}\right)^3 P_0$$

.

.

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 \qquad 3.10$$

We know that $\sum_{n=0}^{\infty} P_n = 1$

$$P_0 + \frac{\lambda}{\mu} P_0 + \left(\frac{\lambda}{\mu}\right)^2 P_0 + \ldots = 1$$

$$P_0 \left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \ldots\right] = 1$$

Sum of infinite geometric progression applies

$$P_0\left[\left(1-\frac{\lambda}{\mu}\right)^{-1}\right]=1$$

$$P_0\left[\frac{1}{1-\frac{\lambda}{\mu}}\right]=1$$

$$\frac{P_0}{1-\frac{\lambda}{\mu}}=1$$

$$\Rightarrow P_0=1-\frac{\lambda}{\mu}=1-\rho \qquad \text{where } \rho=\frac{\lambda}{\mu} \qquad 3.11$$

Substituting equation 3.11 into 3.10

$$P_n=\left(\frac{\lambda}{\mu}\right)^n(1-\rho)$$

$$P_n=\rho^n(1-\rho) \qquad 3.12$$

3.3.1 Expected Number of Ships in the System

This is the sum of the expected number of ships in the queue and those at berth.

Let L_S be the Expected Number of ships in the system

Where *n* is the number of ships, and P_{n} = Probability of *n* ships in the system

Therefore, $L_S=\sum_{\mathrm{n}=1}^{\infty}\mathrm{n}\,P_{\mathrm{n}}$

$=\sum_{\mathrm{n}=1}^{\infty}\mathrm{n}\,[\rho^n(1-\rho)]$ Since $P_{\mathrm{n}}=\rho^n(1-\rho)$

$=(1-\rho)\sum_{\mathrm{n}=1}^{\infty}\mathrm{n}\,\rho^n$

$=(1-\rho)\rho\sum_{\mathrm{n}=1}^{\infty}\mathrm{n}\,\rho^{n-1}$

$=(1-\rho)\rho[1+2\rho+3\rho^2+4\rho^3+---]$

$=(1-\rho)\rho(1-\rho)^{-2}$

$$=\frac{\rho}{1-\rho}$$

Hence, $L_s = \frac{\lambda}{\mu-\lambda}$; $where\ \rho = \frac{\lambda}{\mu}$ 3.13

3.3.2 Expected Number of Ships in the Queue

Let L_q be the Expected Number of ships in the queue

L_q= Expected Number of ships in the system – Expected number of ships being served.

To obtain the expected number of ships being served, we consider the probability that the berth is busy $\rho = \frac{\lambda}{\mu}$ and the probability that the berth is idle is $1 - \rho = 1 - \frac{\lambda}{\mu}$.

But we know that the berth is busy when serving one (1) ship and idle when serving zero (0) ships.

Therefore, the expected number of customers being served = $1 \times \frac{\lambda}{\mu} + 0 \times \left(1 - \frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} = \rho$

For a queue system with c servers, the expected number of customers being served is $c\frac{\lambda}{\mu} + 0 \times \left(1 - \frac{\lambda}{\mu}\right) = \frac{C\lambda}{\mu} = c\rho$

hence,

$$L_q = L_s - \rho$$

$$= \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu}$$

$$= \frac{\mu\lambda-\lambda(\mu-\lambda)}{(\mu-\lambda)\mu}$$

$$= \frac{\mu\lambda-\mu\lambda+\lambda^2}{\mu^2-\mu\lambda}$$

$$= \frac{\lambda^2}{\mu^2-\mu\lambda}$$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$= \frac{\lambda^2/\mu^2}{(\mu-\lambda)/\mu}$$

$$L_q = \frac{\rho^2}{1-\rho} \qquad 3.14$$

3.3.3 Expected Time a Ship Spends in the System

Let W_s be the expected time a ship spends in the system (on queue and being served at berth)

Since λ is the average arrival rate of ships, the inter-arrival rate $= \frac{1}{\lambda}$

Therefore, the waiting time in the system

W_s = Inter-arrival time × no of ships in the system

$$= \frac{1}{\lambda} \times L_s$$

$$= \frac{1}{\lambda} \times \left(\frac{\lambda}{\mu-\lambda}\right)$$

Therefore $$\text{Ws} = \frac{1}{\mu-\lambda} \qquad 3.15$$

3.3.4 Expected Time a Ship Spends in the Queue

Chen-Hsiu and Kuang-Che (2004) posited that port system efficiency may be measured by the average time ship spends on a queue alone. This was therefore used in arriving at the number of queues for the proposed model.

Let W_q = Expected time a ship spends on the queue before being served

= Inter-arrival time × average number of ships in queue

$$= \left(\frac{1}{\lambda}\right) L_q$$

$$= \frac{1}{\lambda}\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right)$$

Hence $$Wq=\frac{\lambda}{\mu(\mu-\lambda)} \qquad 3.16$$

3.4 Little's Law

John D. C. Little and S. Stidham were the first to prove the interrelationship that exist among specific operating characteristics for any queuing system in steady state.

The Little's formula gives the inter-relationship that exist between L_s , L_q, W_s and W_q which generally holds even where the service is not in order of arrival.(Kalavathy,2006)

$L_s = \lambda W_s$ and $L_q = \lambda W_q$ or $W_q = \frac{L_q}{\lambda}$ and $s = \frac{L_s}{\lambda}$ are popularly known as the little's law or formulae of flow equations.

We know that,

$$L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

and $$W_s = \frac{1}{\mu-\lambda}$$

$$\therefore L_s = \lambda W_s \qquad 3.17$$

Similarly $$L_q = \lambda W_q \qquad 3.18$$

We know already that $$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

And $$W_s = \frac{1}{\mu-\lambda}$$

$$\therefore W_s - \frac{1}{\mu} = \frac{1}{\mu-\lambda} - \frac{1}{\mu} = \frac{\mu-(\mu-\lambda)}{\mu(\mu-\lambda)} = \frac{\lambda}{\mu(\mu-\lambda)} = W_q$$

Hence

$$W_q = W_s - \frac{1}{\mu}, \text{ and } \quad W_s = W_q + \frac{1}{\mu}$$

and if we multiply both sides by λ ; $\lambda W_q = \lambda\left(W_s - \frac{1}{\mu}\right)$

$$\lambda W_q = \lambda W_s - \frac{\lambda}{\mu}$$

$$\therefore L_q = L_s - \frac{\lambda}{\mu} \qquad 3.19$$

3.5 Multi-Berth Queuing Model (M/M/c): (∞/FCFS)

The multi-berth queuing model represented by (M/M/c): (∞/FCFS) is the queuing system in existence in the Apapa port. It is a single queue serviced by parallel multipleberths in which each has an independently distributed service time distribution. The arrival process is Poisson and inter-arrival times are exponentially distributed. The performance indicators of the model are to be computed for evaluation and then for comparison with the new model. The model is of infinite capacity and the discipline is on first come first served.

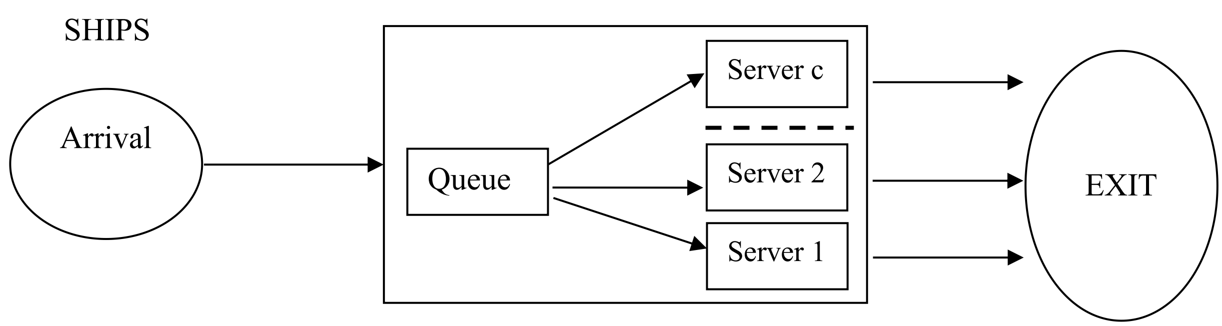

Fig.3.2: Single Queue Multiple Server Model
Source: Sheu and Babbar (1996)

Ships arrived into a single queue as depicted in figure 3.2 then proceed to be served as soon as a berth becomes available out of the *c* berths and exits the queue system thereafter.

3.6 Multiple Queue - Multiple Berth Queuing Model

This is the situation where multiple queues in accordance to the criteria convenient are served by multiple berths (Figure 3.3). Each queue is to be served by fewer berths. The total number of berths in this model is equal to c and the total number of ships n remain the same also as in the existing model.

The value of each respective performance indicator in the new model would be compared with the corresponding performance indicator obtained for the existing model.

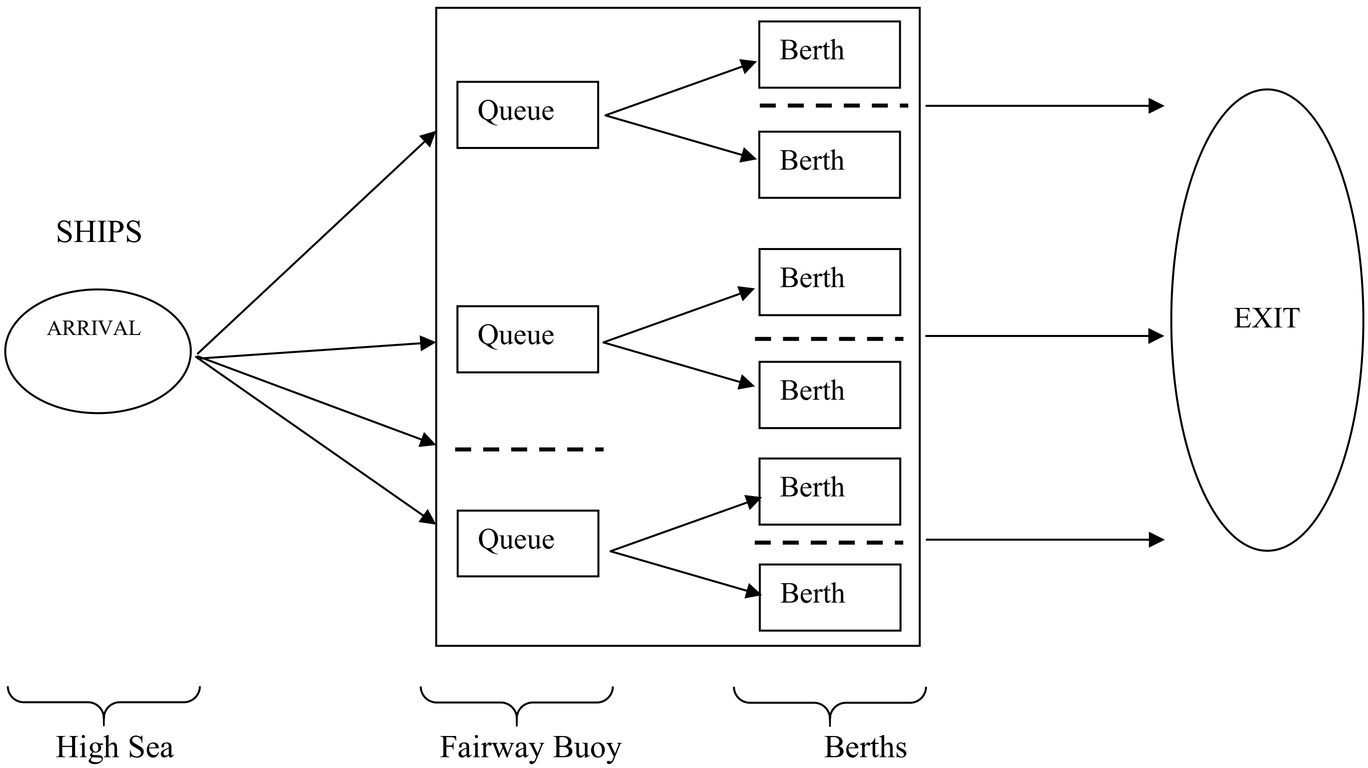

Fig. 3.3 Single Stage Queuing Model with Multiple Queues and Multiple Parallel Berths, Source: Sheu and Babbar (1996)

3.7 Performance Indicators of the Multiple Server Queuing model

If there are n ships in the system, the following two situations exist:

i. If $n \leq c$, where c is number of berths, all the ships will be served simultaneously and there will be no queue. $c - n$ berths will in fact be idle.

   Hence, the mean service rate $\mu_c = n\mu, \;\; 1 \leq n \leq c$

ii. If $n \geq c$, all the c number of berths will be occupied and $n - c$ ships will be waiting to be served on the queue.

In all cases $\lambda_n = \lambda$ for all $n\{n = 0,1,2 \dots\}$

The steady state differential-difference equations of the model are

$$P_n = \frac{\lambda^n}{(n\mu)[(n-1)\mu]\dots(1.\mu)} P_0 \quad for\ 1 \leq n < c$$

$$= \frac{\lambda^n}{n!\mu^n} P_0 \ = \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} P_0$$

$$= \frac{\rho^n}{n!} P_0$$

and for $n \geq c$, $\mathrm{P}_n = \frac{\lambda^n}{(c\mu)(c\mu)\dots(c\mu)\times(c\mu)[(c-1)\mu]\dots(1.\mu)} P_0 = \frac{\lambda^n}{c^{n-c}c!\mu^n} P_0$

$$\mathrm{P_n} = \frac{\rho^n}{c^{n-c}c!} P_0 \qquad 3.20$$

Furthermore, $\sum_{n=0}^{\infty} P_n = 1$

$$= \sum_{n=0}^{c-1} P_n + \sum_{n=c}^{\infty} \rho_n$$

$$= \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \sum_{n=c}^{\infty} \frac{\rho^n}{c^{n-c}c!}\right] P_0 = 1$$

and

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!} \rho^c \sum_{n=c}^{\infty} \left(\frac{\rho}{c}\right)^{n-c}\right]^{-1}$$

if $n = c, n = c + 1, n = c + 2 \dots$ we have

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!} \left(\frac{\lambda}{\mu}\right)^c \left\{1 + \frac{\lambda}{\mu c} \left(\frac{\lambda}{\mu c}\right)^z + \dots\infty\right\}\right]^{-1}$$

since $\rho = \frac{\lambda}{\mu}$ $\quad and\ \frac{\rho}{c} = \frac{\lambda}{c\mu}$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{1}{\left(1-\frac{\lambda}{\mu}\right)}\right]^{-1}$$

Since $s = 1 + r + r^2 + r^3 + \ldots\infty = \frac{1}{1-r}$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{\mu c}{\mu c-\lambda}\right)\right]^{-1} \quad 3.21$$

iii). Let $P(n \geq c)$ = Probability that an arriving ship will have to wait

$$= \sum_{n=c}^{\infty} P_n$$

$$= \sum_{n=c}^{\infty} \frac{\rho^n}{c^{n-c}c!} P_0$$

$$= \frac{\left(\lambda/\mu\right)^c \mu c}{c!(c\mu-\lambda)} P_0 \quad 3.22$$

iv). The probability that an arriving ship will not have to wait

$$P(n<c) = 1 - P(n \geq c)$$

$$P(n<c) = 1 - \frac{\left(\lambda/\mu\right)^c \mu c}{c!(c\mu-\lambda)} P_0 \quad 3.23$$

v). Average queue length

L_q = Average queue length; $n > c$ means $n - c$ ships in queue $= \sum_{n=c}^{\infty}(n-c)P_n$

$$L_q = \frac{1}{(c-1)!}\left(\frac{\lambda}{\mu}\right)^c \left[\frac{\mu\lambda}{(\mu c-\lambda)^2}\right] P_0 \quad 3.24$$

vi). L_s = Average number of ships in the system $= L_q + \frac{\lambda}{\mu}$

$$L_s = \frac{1}{(c-1)!}\left(\frac{\lambda}{\mu}\right)^c \left[\frac{\mu\lambda}{(\mu c-\lambda)^2}\right] P_0 + \frac{\lambda}{\mu} \quad 3.25$$

vii). W_q = Average waiting time of ships in the queue

$$W_q = \frac{L_q}{\lambda} \quad 3.26$$

viii) W_s =Average waiting time a ship spends in the system

$$W_s = W_q + \frac{1}{\mu} \quad 3.27$$

ix) The mean number of waiting ships who actually wait is

$$L(L > 0) = \frac{1}{1-\rho} \quad 3.28$$

x) The mean waiting time in the queue for those who actually wait is

$$\left(\frac{W}{W} > 0\right) = \frac{1}{c\mu-\lambda} \quad 3.29$$

Due to the complexities of computations for large values of *c* and *n* in the above equations, a software WINQSB which employs discrete event Monte Carlo simulation was used for the queuing analysis computations specifically in the computation of L_q.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Goodness of fit test

4.11 Arrivals of ships into the queue at Apapa Port

The null hypothesis for the respective arrivals for all the months H_0: Arrivals are Poisson distributed, while the alternative hypothesis H_1: Arrivals are not Poisson distributed

Table 4.2: Analysis of arrival distribution for August 2016

No of ship arrived per day, x	Observed frequency, O	P(X=x)	Expected , E = N.P(x)	Chi square, $(O-E)^2/E$
0	2	0.1075	3.33	0.5312
1	6	0.2398	7.43	0.2752
2	11	0.2674	8.29	0.8859
3	7	0.1987	6.16	0.1145
≥ 4	5	0.1866	5.78	0.1053
Sum	31			1.9121

$$\lambda = \frac{\Sigma f(x)}{\Sigma f} = \frac{69}{31} = 2.23, \qquad n = 5$$

$$P(X = x) = \frac{\lambda x e^{-\lambda x}}{x!} \qquad for\ x = 0,1,2,3\ and\ 4$$

Chi – square calc = sum of the chi square column = 1.9121

While the table chi-square value at 0.05 and 4 df = 9.488

We therefore fail to reject the null hypothesis and conclude at 95% certainty that the arrival process for the month of August is Poisson distributed.

Table 4.2 Analysis of arrival distribution for September 2016

No of ships arrived per day	Observed frequency (O)	P(x)	Expected frequency (E)	Computed chi-square
0	4	0.1604	4.812	0.1370
1	10	0.2936	8.808	0.1613
2	5	0.2686	8.058	1.1605
3	9	0.1639	4.917	3.3905
≥4	2	0.1135	3.405	0.5797

In table 4.2 the mean =1.83 and the sum of the computed chi square =5.4290 which is less than the table chi square of 9.488 at 0.05 and 4 degrees of freedom. Therefore we fail to reject the null hypothesis.

Table 4.3 Analysis of arrival distribution for October 2016

No of ships arrived per day	Observed frequency O	P(x)	Expected frequency (E)	Computed chi-square
0	7	0.1013	3.140	4.7451
1	9	0.2319	7.189	0.4562
2	8	0.2655	8.231	0.0065
3	6	0.2027	6.284	0.0128
≥4	1	0.1986	1.348	0.0898

In table 4.3 the mean =2.29 and the sum of the computed chi square =5.3104 which is less than the table chi square of 9.488 at 0.05 and 4 degrees of freedom. Therefore we fail to reject the null hypothesis.

Table 4.4 Analysis of arrival distribution for November 2016

No of ships arrived per day	Observed frequency O	P(x)	Expected frequency (E)	Computed chi-square
0	4	0.1604	4.812	0.1370
1	10	0.2936	8.808	0.1613
2	7	0.2686	8.058	0.1389
3	5	0.1639	4.917	0.0014
≥4	4	0.1135	3.405	0.1040

Here the mean =1.83 and the sum of the computed chi square =0.5426 which is less than the table chi square of 9.488 at 0.05 and 4 degrees of freedom. Therefore we fail to reject the null hypothesis.

Table 4.5 Chi-Square Goodness-of-fit test of arrival distribution for December 2016

No of ships arrived per day	Observed frequency (O)	P(x)	Expected frequency (E)	Computed chi-square
0	3	0.1604	4.9724	0.7824
1	9	0.2936	9.1016	0.0011
2	13	0.2686	8.3266	3.4075
3	4	0.1639	5.0809	0.2299
≥4	2	0.1135	3.5185	0.6553

Here the mean =1.83 and the sum of the computed chi square =5.0762 which is less than the table chi square of 9.488 at 0.05 and 4 degrees of freedom. Therefore we fail to reject the null hypothesis.

Consequently from the data available, the arrivals for the five months are said to be Poisson distributed.

4.12 Service distribution

The Null hypothesis for the respective months is H_0: The services are exponentially distributed and the alternative hypothesis H_1: The service times are not exponentially distributed

Table 4.6 Service time goodness-of-fit, August 2016

Service time (days)	Observed ships	P(X=x)	Expected ships	Chi-square
1	31	0.3161	23.08	2.72
2	18	0.2162	15.79	0.31
3	9	0.1479	10.80	0.30
4	6	0.1011	7.38	0.26
≥5	9	0.2187	15.97	3.04
Sums	73			6.63

The Chi-Square Calculated is less than Chi-square from the table which is 7.81 hence we fail to reject the null hypothesis and conclude that the service time at the ports from the available data is Exponentially distributed.

Table 4.7 Service time goodness-of-fit, September 2016

Service time (days)	Observed ships	P(X=x)	Expected ships	Chi-square
1	26	0.3687	20.28	1.61
2	10	0.2328	12.80	0.61
3	8	0.1469	8.08	0.00
4	6	0.0928	5.10	0.16
≥5	5	0.1588	8.73	1.59
Sums	55		3.97	

The Chi-Square Calculated is less than Chi-square from the table which is 7.81 hence we fail to reject the null hypothesis and conclude that the service time at the ports from the available data is exponentially distributed.

Table 4.8 Service time goodness-of-fit, October 2016

Service time (days)	Observed ships	P(X=x)	Expected ships	Chi-square
1	35	0.3995	33.16	0.10
2	21	0.2399	19.91	0.06
3	12	0.1441	11.96	0.00
4	5	0.0865	7.18	0.66
≥5	6	0.1300	10.79	2.13
Sums	**83**			**2.95**

The Chi-Square Calculated is less than Chi-square from the table which is 7.81 hence we fail to reject the null hypothesis and conclude that the service time at the ports from the available data is Exponentially distributed.

Table 4.9 Service time goodness-of-fit, November 2016

Service time (days)	Observed ships	P(X=x)	Expected ships	Chi-square
1	30	0.3935	21.64	3.23
2	10	0.2386	13.12	0.74
3	5	0.1448	7.96	1.10
4	5	0.0878	4.83	0.01
≥5	5	0.1353	7.44	0.80
Sums	**55**			**5.88**

The Chi-Square Calculated is less than Chi-square from the table which is 7.81 hence we fail to reject the null hypothesis and conclude that the service time at the ports from the available data is Exponentially distributed.

Table 4.10 Service time goodness-of-fit, December 2016

Service time (days)	Observed ships	P(X=x)	Expected ships	Chi-square
1	17	0.3363	19.17	0.25
2	20	0.2233	12.73	4.15
3	6	0.1481	8.44	0.71
4	5	0.0983	5.60	0.06
≥5	9	0.1940	11.06	0.38
Sums	**57**			**5.55**

The Chi-Square Calculated is less than Chi-square from the table which is 7.81 hence we fail to reject the null hypothesis and conclude that the service time at the ports from the available data is Exponentially distributed.

4.2 The Port Basic Parameters

The data obtained for Apapa port was for five months August to December 2016.

Only data on the container laden ships who arrived the port for service were extracted and considered for this work.

The existing operating system at the port have twenty (20) berths available for their operations (hence c=20) being fed by a single queue.

Table 4.11: Computation of mean Ship Arrivals/Serviced per hour

Month	Total Arrivals (a)	Container ships Arrived (b)	Total Serviced (c)	Container ships Serviced (d)	Mean arrival Per/hr (λ=b/Q)	Mean service per/hr/berth (μ=d/Q*20)
August	129	87	109	73	0.1169	0.0049
September	106	66	88	55	0.0917	0.0038
October	113	99	87	83	0.1331	0.0056
November	101	60	94	55	0.0833	0.0038
December	111	69	84	57	0.0927	0.0038

Q= number of days in the month multiplied by the number of hours in a day (24 since the port is operational round the clock). While 20 used in computing the mean service per hour is the number of berths available for the port container operations.

The number of container ships that arrived in each month are as shown in Table 4.1 above and the number of ships serviced in each month. These figures were then converted to arrivals and service means per hour, (λ) and(μ) respectively. The rest of the cargo not considered were bulk and of no interest to this study because they are mainly petroleum products that have priority status and are offloaded at dedicated berths reserved for such purposes.

The mean arrival rate was obtained by dividing the respective monthly arrivals by the number of days in the month and the number of hours in a day. On the other hand the mean service rates were obtained by dividing the number of ships serviced by the number of days in the month and the number of hours in a day and the number of berths.

4.3 Existing Single Queue – Multiple Server Model

Table 4.12 Single Queue Multiple Server Analysis

Month	Mean Arrival Per hour (λ)	Mean served Per hour per berth (μ)	Average no of ships in the system *Ls*	Average no of ships in the queue *Lq*	Average time spent in the system in hrs *Ws*	Average time spent in the queue in hrs *Wq*
August	0.1169	0.0049	30.11	6.25	257.54	53.46
September	0.0917	0.0038	30.99	6.86	337.97	74.81
October	0.1331	0.0056	32.80	9.03	246.41	67.84
November	0.0833	0.0038	25.90	3.98	310.94	47.78
December	0.0927	0.0038	30.87	6.48	333.06	69.90

To obtain the average number of ships in the queue, *Lq* from equation 3.24 was made cumbersome by the need to first compute *Po* from equation 3.21 before substituting same into equation 3.24 hence the software was used to obtain *Lq* while the relevant equations 3.25, 3.27 and 3.26 were applied respectively to compute *Ls, Ws,* and *Wq*.

4.4 Multi-Queue Multi-Berth Model

It is proposed that the berths should be split into five per queue. This implies that we would have four queues, each being served by five berths. Here c =5.

All the number of ships in the system was reduced as shown in Table 4.13 for each month. The number of ships in the queue and all the times spent in the queue and in the system were equally reduced in comparison to what was obtainable in Table 4.12.

Table 4.13 Multi-Queue Multi-Berth model analysis

Month	Mean Arrival Per hour *(λ)*	Mean served Per hour per berth *(μ')*	Average no of ships in the system *(Ls')*	Average no of ships in the queue *(Lq')*	Average time spent in the system Hrs.*(Ws')*	Average time spent in the queue Hrs.*(Wq')*
August	0.1169	0.0196	6.37	0.41	54.53	3.51
September	0.0917	0.0152	7.74	1.71	84.44	18.65
October	0.1331	0.0224	9.14	3.20	68.68	24.04
November	0.0833	0.0152	6.18	0.70	74.19	8.40
December	0.0927	0.0152	7.66	1.56	82.62	16.83

The calculation for *Lq'* from equation 3.24 is complicated by the need to first compute *Po* from equation 3.21 before substituting same into equation 3.24 hence the software was used to obtain *Lq'* while the relevant equations 3.25, 3.27 and 3.26 were applied respectively to compute *Ls', Ws',* and *Wq'*.

4.5 Evaluation of the Multiple Queue Model against the existing Single queue model

An evaluation of the results from the multiple queue model against the existing single queue model yielded the following outcome in terms of percentage improvements.

Table 4.14: Percentage difference between the numbers of ships in the system

Months	Average no of ships in the system		% difference
	Existing model	New model	
August	30.11	6.37	78.84
September	30.99	7.74	75.02
October	32.80	9.14	72.13
November	25.90	6.18	76.14
December	30.87	7.66	75.19

Overall there was a marked decrease in all the performance indicators with up to a decrease in the average number of ships in the system dropping by up to 72.7% while average number of ships on the queue dropped up to a maximum of 83.3% . Interestingly the proposed model impacted positively on the times spent on the queue with an observed drop in time spent on the system and on the queue proper dropping by up to a maximum of 66.1% and 86.5% respectively.

Hence the data strongly indicate that a multiple queue model is more acceptable to all stakeholders in a multiple berth situation.

Table 4.15: Percentage difference between the Average number of ships in the queue

Months	Average no of ships in the queue		% difference
	Existing model	New Model	
August	6.25	0.41	93.44
September	6.86	1.71	75.07
October	9.03	3.20	64.56
November	3.98	0.70	82.41
December	6.48	1.56	75.93

Table 4.16: Percentage difference between the Average times spent in the system

Months	Average time spent in the system in hours		% difference
	Existing Model	New Model	
August	257.54	54.53	78.83
September	337.97	84.44	75.02
October	246.41	68.68	72.13
November	310.94	74.19	76.14
December	333.06	82.62	75.19

Table 4.17: Percentage difference between average times spent on the queue

Months	Average time spent in the queue in hours		% difference
	Existing Model	New Model	
August	53.46	3.51	93.43
September	74.81	18.65	75.07
October	67.84	24.04	64.56
November	47.78	8.40	82.42
December	69.90	16.83	75.92

All the performance indicators clearly shows that the multiple queue – multiple berth model is more efficient and would make the Apapa port services more attractive to stake holders. This is obvious from the respective reductions in the number of ships in the system, the number of ships in the queue, the mean time spent in the system Ws and on the queue, Wq.

4.6 Selecting the number of queues needed for the multiple queue multiple server model

The assertion by Chen-Hsiu and Kuong-Che (2004) that port system efficiency may be measured by the average time ship spends on a queue alone was relied upon in arriving at the number of queues for the multiple queue model.

Table 4.18: Analysis of times spent on the queue for different number of berths in August 2016

Number of berths per queue , c (Max 20)	Number of queues	Mean service rates in hours per berth = (73/31*24*c)	Number of ships in the queue L_q'	Average time a ship spends on the queue in hours, Wq'
1	20	0.0980	0.22	1.88
2	10	0.0490	0.74	6.33
4	5	0.0245	3.26	27.89
5	4	0.0196	0.51	3.51
10	2	0.0098	1.01	8.64
20	1	0.0049	6.2504	53.47

The mean rate of arrival remains 0.1169 for all c.

The maximum numbers of berths in use at the ports are 20 hence only a combination of berth to number of queues whose multiplicand yields 20 are possible and considered as can be seen in the first two columns in the table 4.15 above.

The mean service rate column is obtained by taking the quotient of the total container vessels serviced in the month of August which is 73 and the number of days in the month by the number of hours worked by the number of berths. The berths work 24 hours daily hence 24 hours was used.

The direct computation of the average time a ship spends on the queue, *Wq* in the fourth column could have been easy as can be seen in equation 3.26 but for the complex formula of equation 3.21 in obtaining *Po* which was needed in equation 3.24 to compute *Lq'*. First$L_q^{'}$ was obtained from software for the entire port then divided by the respective number of queues which in turn was required to obtain *Wq'*.

As can be seen from the last column of the Table 4.18 the shortest time (at c=1) is when you have maximum number of queues which is however not feasible. This is because it doesn't make sense to have a dedicated queue for every berth, hence the next shortest time which is at c=5 is selected.

Hence the new model is a multiple (5) queue multiple (4) berths queueing model with Poisson arrival, exponential service with infinite capacity and infinite source and a first in first out type is the model of choice since it gives the shortest stay in the queue by a calling ship.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMENDATIONS

5.1 Summary

The study examined the application of a multi-queue multi-berth model to the arrival and departure of ship operations at Apapa Port of the Nigeria Ports Authority in Lagos.

The existing single queue multi-berth operations of the port was also reviewed.

Since it is not possible to obtain solutions for multi-server models in closed form or by solving a set of equations the study therefore relied on simulation using the WINQSB software for the computation of the number of ships on the queue L_q.

The data obtained was for five months and the results clearly showed that the queue length for the system and in the queue was shorter for a multiple queue than for the single queue presently in operation.

Similarly it was observed that the time a ship will spend on the queue and in the system is much shorter for a multiple queue than for a single queue.

5.2 Conclusion

The application of queuing theory modeling to analyze various aspects of port operations was shown to be very useful for understanding and meeting challenges encountered in port activities.

In this study we showed from results obtained that for a multi queue multi berth very important variables like the number of ships on queue and in the system were fewer than in a single queue multi berth model.

Likewise other variables supported the proposed model over the existing model since the mean time ships spent on the queue and in the system were relatively shorter in the proposed multi queue model than in the single queue model.

Therefore the proposed model should be of great interest to the port owners and the port users alike since the port would become attractive with the reduced turnaround time and which will make the port busier and generate more revenue for the port owner while the shorter time means less expenditure by the port user. The implementation of the multiple queue model is the more attractive as it involves no additional cost by the port owners in contrast to the previous works of Oyatoye *et al.*(2011).

5.3 Recommendations

Recommendations arising from the study includes that research students could explore unequal berth numbers and other queuing models' application to various port operations.

A student of queuing theory could also examine the inter relationships between various port operations like dockyard activities, materials handling, warehousing, gate operations.

The result of this study could also generate interest in looking at various modes of arrivals and even distribution-less arrivals and services as in the deterministic queuing models.

It is also recommended that other queuing disciplines and behaviours different from the blocked customer first in first out could be another area of research interest.

Finally it is strongly recommended that the multiple queue model should be adopted for the containerized general goods operations of the port.

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APPENDICES

System Performance Summary for Single Queue Multi Berth August 2016

	Performance Measure	Result
1	System: M/M/20	From Simulation
2	Customer arrival rate (lambda) per hour =	0.1169
3	Service rate per server (mu) per hour =	0.0049
4	Overall system effective arrival rate per hour =	0.1179
5	Overall system effective service rate per hour =	0.0829
6	Overall system utilization =	78.6755 %
7	Average number of customers in the system (L) =	21.9855
8	Average number of customers in the queue (Lq) =	6.2504
9	Average number of customers in the queue for a busy system (Lb) =	10.7900
10	Average time customer spends in the system (W) =	176.5195 hours
11	Average time customer spends in the queue (Wq) =	36.8026 hours
12	Average time customer spends in the queue for a busy system (Wb) =	63.5321 hours
13	The probability that all servers are idle (Po) =	0.5585%
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	57.9276 %
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour =	1000.0000
24	Starting data collection time in hour =	0
25	Number of observations collected =	83
26	Maximum number of customers in the queue =	17
27	Total simulation CPU time in second =	0.0470

System Performance Summary for Single Queue Multi Berth September 2016

	Performance Measure	Result
1	System: M/M/20	From Simulation
	Customer arrival rate (lambda) per hour =	0.0917
3	Service rate per server (mu) per hour =	0.0038
4	Overall system effective arrival rate per hour =	0.1007
5	Overall system effective service rate per hour =	0.0608
6	Overall system utilization =	72.9928 %
7	Average number of customers in the system (L) =	21.4551
8	Average number of customers in the queue (Lq) =	6.8566
9	Average number of customers in the queue for a busy system (Lb) =	14.7605
10	Average time customer spends in the system (W) =	206.2871 hours
11	Average time customer spends in the queue (Wq) =	30.0900 hours
12	Average time customer spends in the queue for a busy system (Wb) =	64.7760 hours
13	The probability that all servers are idle (Po) =	0.7108%
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	46.4524 %
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour =	1000.0000
24	Starting data collection time in hour =	0
25	Number of observations collected =	61
26	Maximum number of customers in the queue =	21
27	Total simulation CPU time in second =	0.0470

System Performance Summary for Single Queue Multi Berth October 2016

	Performance Measure	Result
1	System: M/M/20	From Simulation
2	Customer arrival rate (lambda) per hour =	0.1131
3	Service rate per server (mu) per hour =	0.0056
4	Overall system effective arrival rate per hour =	0.1365
5	Overall system effective service rate per hour =	0.1027
6	Overall system utilization =	81.2867 %
7	Average number of customers in the system (L) =	25.2861
8	Average number of customers in the queue (Lq) =	9.0288
9	Average number of customers in the queue for a busy system (Lb) =	14.3000
10	Average time customer spends in the system (W) =	193.4924 hours
11	Average time customer spends in the queue (Wq) =	59.2682hours
12	Average time customer spends in the queue for a busy system (Wb) =	93.8702 hours
13	The probability that all servers are idle (Po) =	0.4893%
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	63.1384 %
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour =	1000.0000
24	Starting data collection time in hour =	0
25	Number of observations collected =	103
26	Maximum number of customers in the queue =	21
27	Total simulation CPU time in second =	0.0620

System Performance Summary for Single Queue Multi Berth November 2016

	Performance Measure	Result
1	System: M/M/20	From Simulation
	Customer arrival rate (lambda) per hour =	0.0833
3	Service rate per server (mu) per hour =	0.0038
4	Overall system effective arrival rate per hour =	0.0871
5	Overall system effective service rate per hour =	0.0614
6	Overall system utilization =	69.3104%
7	Average number of customers in the system (L) =	17.8415
8	Average number of customers in the queue (Lq) =	3.9794
9	Average number of customers in the queue for a busy system (Lb) =	9.5932
10	Average time customer spends in the system (W) =	191.5402 hours
11	Average time customer spends in the queue (Wq) =	23.5157hours
12	Average time customer spends in the queue for a busy system (Wb) =	56.6902 hours
13	The probability that all servers are idle (Po) =	0.7768%
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	41.4811%
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour=	1000
24	Starting data collection time in hour=	0
25	Number of observation collected=	62
26	Maximum number of customers in the queue=	14
27	Total simulation CPU time in second=	0.0470

System Performance Summary for Single Queue Multi Berth December 2016

	Performance Measure	Result
1	System: M/M/20	From Simulation
	Customer arrival rate (lambda) per hour =	0.0927
3	Service rate per server (mu) per hour =	0.0038
4	Overall system effective arrival rate per hour =	0.0941
5	Overall system effective service rate per hour =	0.0604
6	Overall system utilization =	73.6076%
7	Average number of customers in the system (L) =	21.2053
8	Average number of customers in the queue (Lq) =	6.4838
9	Average number of customers in the queue for a busy system (Lb) =	13.6725
10	Average time customer spends in the system (W) =	211.9439 hours
11	Average time customer spends in the queue (Wq) =	27.7661hurs
12	Average time customer spends in the queue for a busy system (Wb) =	58.5509 hours
13	The probability that all servers are idle (Po) =	0.6979%
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	47.4222%
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour=	1000.0000
24	Starting data collection time in hour=	0
25	Number of observation collected=	61
26	Maximum number of customers in the queue=	19
27	Total simulation CPU time in second=	0.0620

System Performance Summary for Multi Queue Multi Berth August 2016

	Performance Measure	Result
1	System: M/M/5	From Simulation
2	Customer arrival rate (lambda) per hour =	0.1169
3	Service rate per server (mu) per hour =	0.0196
4	Overall system effective arrival rate per hour =	0.1102
5	Overall system effective service rate per hour =	0.1022
6	Overall system utilization =	86.0721 %
7	Average number of customers in the system (L) =	6.3346
8	Average number of customers in the queue (Lq) =	2.0310
9	Average number of customers in the queue for a busy system (Lb) =	2.8475
10	Average time customer spends in the system (W) =	59.4507 hours
11	Average time customer spends in the queue (Wq) =	19.0486 hours
12	Average time customer spends in the queue for a busy system (Wb) =	26.7058 hours
13	The probability that all servers are idle (Po) =	1.6950 %
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	71.3275 %
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour =	1000.0000
24	Starting data collection time in hour =	0
25	Number of observations collected =	103
26	Maximum number of customers in the queue =	8
27	Total simulation CPU time in second =	0.0630

System Performance Summary for multiple Queue Multi Berth September 2016

	Performance Measure	Result
1	System: M/M/5	From Simulation
2	Customer arrival rate (lambda) per hour =	0.0917
3	Service rate per server (mu) per hour =	0.0152
4	Overall system effective arrival rate per hour =	0.0967
5	Overall system effective service rate per hour =	0.0708
6	Overall system utilization =	86.0684 %
7	Average number of customers in the system (L) =	12.8399
8	Average number of customers in the queue (Lq) =	8.5365
9	Average number of customers in the queue for a busy system (Lb) =	11.1745
10	Average time customer spends in the system (W) =	119.3030 hours
11	Average time customer spends in the queue (Wq) =	65.0255 hours
12	Average time customer spends in the queue for a busy system (Wb) =	85.1203 hours
13	The probability that all servers are idle (Po) =	2.1513 %
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	76.3925 %
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour =	1000.0000
24	Starting data collection time in hour =	0
25	Number of observations collected =	71
26	Maximum number of customers in the queue =	23
27	Total simulation CPU time in second =	0.0620

System Performance Summary for multiple Queue Multi Berth October 2016

	Performance Measure	Result
1	System: M/M/5	From Simulation
2	Customer arrival rate (lambda) per hour =	0.1331
3	Service rate per server (mu) per hour =	0.0224
4	Overall system effective arrival rate per hour =	0.1445
5	Overall system effective service rate per hour =	0.1036
6	Overall system utilization =	91.0642 %
7	Average number of customers in the system (L) =	20.5687
8	Average number of customers in the queue (Lq) =	16.0155
9	Average number of customers in the queue for a busy system (Lb) =	18.5651
10	Average time customer spends in the system (W) =	135.7944 hours
11	Average time customer spends in the queue (Wq) =	93.5335hrours
12	Average time customer spends in the queue for a busy system (Wb) =	108.4236 hours
13	The probability that all servers are idle (Po) =	1.4992 %
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	86.2667 %
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour =	1000.0000
24	Starting data collection time in hour =	0
25	Number of observations collected =	104
26	Maximum number of customers in the queue =	41
27	Total simulation CPU time in second =	0.0780

System Performance Summary for multiple Queue Multi Berth November 2016

	Performance Measure	Result
1	System: M/M/5	From Simulation
2	Customer arrival rate (lambda) per hour =	0.0833
3	Service rate per server (mu) per hour =	0.0152
4	Overall system effective arrival rate per hour =	0.0888
5	Overall system effective service rate per hour =	0.0728
6	Overall system utilization =	83.0708 %
7	Average number of customers in the system (L) =	7.6585
8	Average number of customers in the queue (Lq) =	3.5050
9	Average number of customers in the queue for a busy system (Lb) =	4.8649
10	Average time customer spends in the system (W) =	87.6749 hours
11	Average time customer spends in the queue (Wq) =	36.2828 hours
12	Average time customer spends in the queue for a busy system (Wb) =	50.3603hours
13	The probability that all servers are idle (Po) =	3.0919%
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	72.0465 %
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour =	1000.0000
24	Starting data collection time in hour =	0
25	Number of observations collected =	73
26	Maximum number of customers in the queue =	12
27	Total simulation CPU time in second =	0.0620

System Performance Summary for multiple Queue Multi Berth December 2016

	Performance Measure	Result
1	System: M/M/5	From Simulation
2	Customer arrival rate (lambda) per hour =	0.0927
3	Service rate per server (mu) per hour =	0.0152
4	Overall system effective arrival rate per hour =	0.0982
5	Overall system effective service rate per hour =	0.0704
6	Overall system utilization =	86.5015 %
7	Average number of customers in the system (L) =	12.1298
8	Average number of customers in the queue (Lq) =	7.8047
9	Average number of customers in the queue for a busy system (Lb) =	10.1037
10	Average time customer spends in the system (W) =	104.2865 hours
11	Average time customer spends in the queue (Wq) =	51.9766hours
12	Average time customer spends in the queue for a busy system (Wb) =	67.2868 hours
13	The probability that all servers are idle (Po) =	2.0991%
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	77.2463 %
15	Average number of customers being balked per hour =	0
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour =	1000.0000
24	Starting data collection time in hour =	0
25	Number of observations collected =	71
26	Maximum number of customers in the queue =	23
27	Total simulation CPU time in second =	0.0470

Notations Used

n =number of ships in the system

λ =average number of ships arriving per unit of time

μ =average number of ships being served per unit of time

ρ =berth utilization factor or traffic intensity ($\rho=\lambda/\mu$)

c =number of berths

L_s =expected number of ships in the system

L_q =expected number of ships in the queue excluding those at berth

W_s =expected waiting time per ship in the system

W_q =expected waiting time per ship in the queue excluding service time or time at berth

$P_n(t)$ =transient state probability of exactly "n" ships in the system at

time t assuming the port started operations at time zero 0

P_n =steady state probability of having "n" ships in the system