RELIABILITY BASED CALIBRATION OF PARTIAL SAFETY FACTOR ON CHARACTERISTICS STRENGTH OF STEEL REINFORCEMENT BARS

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DECLARATION

I hereby declare that the work in this Dissertation entitled reliability based calibration of partial safety factor on characteristics strength of steel reinforcement bars has been performed by me in the Department of Civil Engineering. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this Dissertation was previously presented for another degree or diploma at this or any other Institution.

______________________________ ________________

Mohammed AbdulmuminNda **Date**

CERTIFICATION

This dissertationentitled RELIABILITY BASED CALIBRATION OF PARTIAL SAFETY FACTOR ON CHARACTERISTICS STRENGTH OFSTEEL REINFORCEMENT BARS by AbdulmuminNdaMOHAMMED meets the regulations governing the award of Master of Science (MSc) Degree of the Post Graduate School of Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

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ABSTRACT

In the calibration of partial safety factor of steel reinforcement bars conducted in this study, necessary parameters (i.e. steel diameter, area, yield strength, ultimate strength, and elongation) needed for the calibration were obtained from the steel testing laboratory, Department of Civil Engineering, Ahmadu Bello University Zaria. The test results were analyzed in accordance with Eurocode, JSCC and ISO standards with the aid of Microsoft Excel, SPSS, EasyFit and COMREL software packages. The steel reinforcement parameters were checked for outliers and the PDF of the steel reinforcement parameters were analyzed using the Kolmogorov-Smirnov goodness of fit test. The result from the analysis showed that the probability density function of the steel reinforcement bars diameter follows a Log-Normal distribution, the yield strength and ultimate yield strength fits a Gumbel(Min) and Gumbel(Max) distribution respectively. Also, the calibrated design values of the steel reinforcement bar diameter, yield and ultimate strength are 2.167 (8.7mm), 398.33N/mm^2, 842.7N/mm^2 respectively with a reliability index value of 3.805 and probability of failure of 7.08E-5 which yielded a partial safety factor 1.07(0.93) with a characteristics value of approximately 426N/mm^2. Hence, it was recommended that the characteristic strength of the steel reinforcement bars should be set to 426N/mm^2 with a safety factor of 0.93(1.07).

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LIST OF ABBREVIATIONS

AD	Anderson-Darling
AFOSM	Advanced First-Order Second-Moment
ASTM	American Society for Testing and Materials
BS	British Standard
C	Carbon
CDF	Cumulative Distribution Function
CoV	Coefficient of Variation
EC/EN	EuroCode
FLIM	Limit State Function
FORM	First Order Reliability Method
FOSM	First-Order Second-Moment
ISO	International Standard Organization
JCSS	Joint Commission on Structural Safety
KS	Kolmogorov-Smirnov
Mn	Manganese
P	Phosporus
PDF	Probability Density Function
PSF	Partial Safety Factor
S	Sulphur
Si	Silicon
SORM	Second Order Reliability Method

LIST OF NOTATIONS

β_{FORM}	Reliability Index Computed by FORM
β_{HL}	Hasofer-Lind Reliability Index
d	Diameter of Steel reinforcement bars
F_d	Design Strength of Steel reinforcement bars
F_y	Characteristic Strength of Steel reinforcement bars
L	Ultimate Strength of Steel reinforcement bars
N	Number of Samples
P_f	Probability of Failure
s_p	Sensitivity
t	T-value for One-sample t-test
X_m	Model Uncertainty
Y_L	Log of High or Low Outlier Limit
α	Sensitivity Factor
β	Reliability Index
μ	Mean
σ	Standard Deviation
Φ	Standard Normal Cumulative Distribution Function

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Buildings are designed to support certain loads without deforming excessively. The loads are made up principally of dead loads and imposed loads. The process of designing a building starts with the selection of materials based on their properties and the type of stresses to be supported. A structural material is a material that carries its self-weight and tributary load transferred from other structural members. The material to be selected must be capable to withstand the stresses generated by the design loads.Today, the most common type of structures is a reinforced concrete structures and the two commonly used structural materials for a reinforced concrete structures are concrete and steel(Ede*et al.*, 2015).

Steel is produced from two key components: iron which is one of earth's most abundant elements and recycled steel. Its combination of strength, recyclability, availability, versatility and affordability makes steel unique. Steel is manufactured under carefully controlled conditions in specialized plants. The properties of every type of steel are determined in a laboratory and described in a manufacturer's certificate. Thus, the designer only specifies the steel complying with a relevant standard, and the builder ensures that the correct steel is used appropriately (World Steel Association, 2012).

The quality of reinforcing steel in structural engineering is usually investigated in terms of its mechanical properties such as, yield strength, ultimate strength and elongation. The characteristic resistance of a material (such as Concrete or Steel) is defined as that value of resistance below which not more than a prescribed percentage of test results may be expected to fall,(for example the characteristic yield stress of steel is usually defined as that value of yield stress below which not more than 5% of the test values may be expected to fall). In other words, this strength is expected to be exceeded by 95% of the cases.However, material design values are obtained by dividing the characteristics values by an appropriate partial safety factor. This partial safety factor takes care of difference between specified strength and actual strength, inaccuracies in the assessment of resistance sections, etc. (Akinsola *et al,* 2016).

Using a wrong safety factor can either make the structure unsafe due to under design or make it costly due to over design. Selection of the right factor of safety is very essential. Standards organizations based on scientific studies, experience and engineering judgments give safety factors in product standards which can be taken as acceptable factors. However, the process of determining a suitable partial safety factor involves purely stochastic/statistical method, code calibration method, reliability method etc. (EN, 1990).

Reliability engineering is a well-established field of study. In general, it refers to the ability of a system to maintain its function in both routine and unexpected circumstances, evaluated in a probabilistic manner. The subject has grown rapidly in the last decade. It has evolved from a research topic to become a set of procedures and methodologies with a wide range of practical applications, and it has been used in code developments worldwide. Providing adequate safety is the main aim in any probabilistic based design code. Limit state design method, which is used worldwide by many design codes, is the latest probabilistic based design method. In order to have a reliable design, the structure should satisfy all limiting states beyond which the structure no longer satisfies the chosen performance criteria. Strength, serviceability, fatigue and stability are the most important limit states in the design of any structure (Baji, 2014).

Code calibration refers to that particular activity that is exercised when some superior method is applied to assign values to the variables of a code format such that a specific design code is formulated. For a code format of the partial safety factor type, the variables are characteristic values, partial coefficients, and load reduction factors (Friis-Hansen and Sorensen, 2002). A code can be calibrated on different levels of superior methods. (Friis-Hansen and Sorensen, 2002).

Structural design codes have been developed around the globe over the past few decades and these will continue to evolve as engineers adapt their design methods to the improving understanding of structural behaviour, material strength and nature/values of imposed loads (Bulleit, 2008).

In recent times, it is observed that the quality of steel bars used in Nigeria has diminished due to various reasons. Therefore, the aim of this research is to evaluate the quality of steel reinforcing bars and to propose a satisfactory material safety factor to be adopted for locally produced reinforcements in Nigeria in order toeffectively and ultimately prevent premature collapse of buildings, preventing loss of life and property in the country.

1.2 Statement of the Problem

Over the years and up-to this moment, Nigeria as a nation have been adopting the British code as a basis for structural design even if these codes were designed on the basis of the various factors/challenges in Britain. These British codes over the years have not been stable as it is constantly being reviewed and updated. A good example is in 1994 when a proposal was made to reduce the partial safety factor for reinforcement from 1.15 to 1.05 which was accepted and incorporated in BS8110-1997. Also, a change was subsequently made to the specified characteristic strength of reinforcement; changing it from 460 N/mm^2 to 500 N/mm^2 as a temporary measure until data was available on the characteristics of the new specification

(Beeby and Jackson, 2016). However, the partial safety factor was changed back to 1.15 upon several cases of building collapse all over the world including Nigeria as a result of adopting the change in the partial safety factor of reinforcement without due consideration.

Also, the characteristics value of our locally produced reinforcement as reported by so many literatures is not up to standard compared to the characteristics value of the Britain's whose code we have adopted. These characteristics value is one of the major basis on which the material partial safety factor is being derived. However, adopting this code without similarities in process of manufacture of these reinforcements nor similarities in their characteristics strength might be a great risk/gamble. Hence, there is need to derive a value for our reinforcement factor of safety to be adopted in Nigeria based on our own engineering practice.

1.3 Justification of the Study

In principle numerical values for partial factors can be determined either on the basis of calibration to a long experience of building tradition (Deterministic approach), or on the basis of statistical evaluation of experimental data and field observations (Probabilistic based reliability approach) (EN, 1990). For most of the partial factors proposed in currently available codes, the deterministic approach is the leading principle adopted based on the fact that when they were developed, there were so many challenges ranging from insufficient data, slow computers for analysis, etc. Hence, there is need to develop a partial safety factor shifting away from the deterministic to reliability based approach.

Also, the Eurocode and BS code recognizes that there may be cases where due to particular nature of loading of the materials, other safety factors would be more appropriate for design (BS8110 Part 2 – 1985, EN1992 Part 1). This suggests that design codes in different regions and different engineering professions are based on diverse criteria and cannot ensure structural members the same safety margin or reliability. Hence, the same reinforcement may be given different strength values under different specifications and condition and can also be given a different factor of safety.

More also, using a wrong safety factor can either make the structure unsafe due to under design or make it costly due to over design. However, it is for these reasons that the researcher deemed it fit to come up with a satisfactory factor of safety with respect to our engineering practice, material properties, etc.

1.4 Aim and Objectives

1.4.1 Aim of Research

The aim of this research is to calibrate partial safety factor for characteristics strength of steel reinforcement barswhile the objectives are;

1.4.2 Objective of Study

i. To determine the statistical parameters of the steel reinforcement bars.

ii. To determine the distribution (Probability Density Function) of the steel reinforcement bars.

iii. To perform sensitivity analysis on the steel reinforcement bars.

iv. To calibrate the design values and establish the reliability index and probability of failure.

v. To determine the partial safety factor and characteristic strength.

CHAPTER TWO
LITERATURE REVIEW

2.1 Overview of Reinforcement for Concrete

Steel has a large field of application ranging from small appliances to big construction industries. Most steels have a crystalline structure and consist of a basic iron-carbon system. Relatively small changes in the carbon content and/or other alloys result in significant changes in the mechanical behavior of the resultant steel. The mechanical properties of steel that are of interest to the design engineer are the stress-strain curve; the yield strength; the amount of strain at yield, the percentage elongation at failure, or ductility; the amount and rate of strain hardening; and the ultimate tensile strength.While the mechanical behavior of a particular steel is significantly influenced by its carbon content, other factors that influence its properties are the chemical composition and the method used to shape the ingot into the final form as steel bar(Chatterjee, 1995). The mechanical properties of steel are mainly affected by the following parameters:

(i) Chemical Composition

• Carbon content

• Presence of alloying elements such as manganese, silicon, chromium, vanadium and copper

(ii) Physical Condition

• Slow cooling from the molten state or quenching

• Annealing

• Hardening characteristics

• Shaping operation (e.g. cold working)

• Weldability

Carbon is the most important element that governs the mechanical properties of steel and most heat treatments of steel are based primarily on controlling the distribution of carbon. Low and medium carbon steels are used extensively for construction of buildings, in most cases as reinforcing bars in concrete(Chatterjee, 1995). Previous research have been conducted to provide detailed information on the strength and ductility properties of reinforcing bars that are manufactured from scrap metals in developing countries. Typical examples are the cases of Ghana and Nigeria (Kankam and Asamoah, 2012) and (Balogun *et al.,* 2009). From the statistical analysis, these steel bars exhibited significant variability in yield strength. From the same study it was observed that the chemical composition in these steel bars could not meet the standard requirement for the limit of carbon content and other associated elements such as silicon, sulphur, phosphorus and manganese present in the steel bars. The final product was a bar with high tensile strength with low % elongation.

The grades of the reinforcing bars are set by the well-known standards such as ISO, ASTM, and BS. Low alloy steel of grade 60 or 500MPa (ASTM A706) is useful for application of reinforcing steel bars that involve both welding and bending. The carbon content of these steels is approximately 0.25%.

The grades are designated by the specified minimum yield strength. For example Grade 460 denotes the minimum yield strength of 460N/mm^2 as per BS4449:1997, Grade 500 denotes the minimum yield strength of 500N/mm^2 as per ASTM A706 or BS4449:2005 for both twisted and ribbed bars. ISO 6935-2:2007 covers ten steel grades not intended for welding and eleven steel grades intended for welding.

Steel derives its mechanical properties from a combination of chemical composition, heat treatment and manufacturing processes (Ullah *et al.,* 2008). In the normalized condition, steel exhibits maximum toughness but a lower strength as compared to oil quenched conditions. The strength of steel may be improved by oil quenching as well as water quenching followed by tempering at 300^oC and 400^oC with some compromise on toughness (Ndaliman, 2006).

There is a considerable effect of the processes of steel making on the quality of concrete reinforcement (Singh and Kaushik, 2002). It is important that quality norms are exercised in the case of reinforcing bars which should invariably have been rolled from billets of known composition. Reinforcing steel bars have remarkable benefits in the concrete because besides the increased strength, the bars can reduce or control crack width of the concrete and help maintain aggregate interlock. The change in strength is such that even the smallest cross-sectional area of steel wire will increase the value by 16% or more (Roy, 1996). Reinforcing steel bars also contribute considerably to earthquake resistance. Under the action of loads, they act together as a frame transferring forces from one to another. With the use of longitudinal bar (large diameter), and vertical stirrup (smaller diameter bar) a beam can withstand the seismic damage.

However reinforced steel bars of high tensile strength and high ductility are required. Research has shown that for the same manufacturer of reinforcing steels the Coefficient of Variability (CoV), which is the ratio of standard deviation of the tensile strength to the mean value of the same for a number of samples, could be noticed from the same batch and a slight variability along the same bar. Constructional bars of a given nominal type may display variation in strength from piece to piece even when made by a controlled standardized process. Investigation by Clifton (1969) on structural material showed that noticeable variation of mechanical properties not only occurs between one batch and another but also within the same batch. Later Mirza (1979) found out that there is variation in yield strength for reinforcing steel bar of Grades 40 and 60 with CoV of 10.7% and 9.3% respectively.

2.1.1 Chemical Composition, Grade, and Type of Steel Reinforcement Bars

According to chemical composition, steel bars can be categorized as carbon steel bars and ordinary low-alloy steel bars. Carbon steel contains ferrum and other trace elements, such as carbon, silicon, manganese, sulfur, and phosphorus. Experimental results show that the strength of a steel bar increases with carbon content, but at the cost of plasticity and weldability (Gu *et al.*, 2016). Generally, carbon steels with a carbon content of less than 0.25 %, 0.25–0.6 %, and 0.6–1.4 % are called low-carbon steel, medium-carbon steel, and high-carbon steel, respectively. Low- and medium-carbon steels are mild steels, while high-carbon steel is hard steel (Gu *et al.*, 2016).

Ordinary low-alloy steel contains added carbon steel alloying elements such as silicon, manganese, vanadium, titanium, and chromium to efficiently increase strength and improve steel properties (Gu *et al.*, 2016).

According to the processing method, steel bars can be categorized as hot-rolled steel bars, heat treatment steel bars and cold working steel bars. Steel wires include carbon steel wires, indented steel wires, steel strands, and cold stretched low-carbon steel wires (Gu *et al.*, 2016).

Cold stretched steel bars are made by mechanically tensioning hot-rolled steel bars in normal temperature. Heat treatment steel bars are ordinary low-alloy steel bars that have undergone the process of heating, quenching, and tempering. High-carbon-killed steel becomes carbon steel wires after several times of cold stretching, stress relieving, and tempering. Rolling indentations into steel wires make indented steel wires, which enables a more secure bond with surrounding concrete. Steel strands are fabricated by twisting several steel wires of the same diameter. Mechanically, tensioning low-carbon steel wires at room temperature makes cold stretched low-carbon steel wires (Gu *et al.*, 2016).

2.1.2 Strength of Steel Reinforcement Bar

The two important characteristics which determine the character of reinforcement bar are the yield point and the modulus of elasticity of the reinforcing steel bars. Generally, the modulus of elasticity of the steel is taken as equal to 200kN/mm^2. In addition, the shape of the stress-strain curve of tensile test of steel has significant influence on the performance of reinforced concrete members.Reinforcement undergoes large plastic strain after yielding. The resulting excessive deformation and crack widths of structural members violate the serviceability requirement. So when the capacity of a reinforced concrete member is calculated, yield point (or conditional yield point) is taken as the upper bound. Reinforcement bar strength is obtained by tests. But the strengths of different specimens, even when of the same classification or type, are generally different due to the inherent variability of reinforcement materials. Statistical analysis shows that experimental data of reinforcement strength obeys Gaussian distribution as shown in Figure 2.1 (Gu *et al.*, 2016).

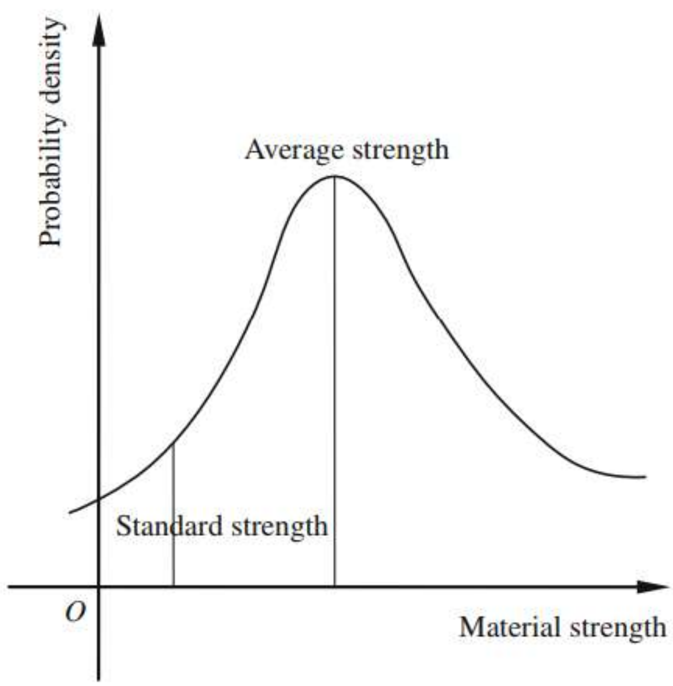


Figure 2.1: Distribution of Reinforcement Material Strength

2.1.3 Creep and Relaxation of Reinforcement Bars

The strain on the reinforcement bar will increase with time if continuously subjected to high stress. This phenomenon is called creep. Relaxation will occur if the length of a steel bar is kept constant, and the stress of the steel bar will decrease with time. Creep and relaxation have the same physical nature. Creep and relaxation increase with time and depend on initial stress, steel material, and temperature. Generally, high initial stress will cause large creep or great relaxation-induced stress loss. The creep and relaxation of cold stretched hot-rolled steel bars are lower than those of cold stretched low-carbon steel wires, carbon steel wires, and steel strands. If the temperature increases, the creep and relaxation will also increase (Gu *et al.,* 2016).

2.1.4 Ductility of Reinforcing Steel Bar

Ductility can be determined by a tensile or bending test; the higher the percentage of elongation the more the material becomes ductile. Research by Allen(1972) has shown that the service life, strength and ductility of concrete structures depend to a large extent on certain properties of reinforcing bars such as Modulus of Elasticity, Yield Stress, Ultimate Stress and Elongation. The values are controlled in practice by the international standards specifications such as American Society of Testing Materials (ASTM). Most reinforcing steel bars will require bending before being installed into a concrete structure. Because they are relatively high strength steels, and because the ribs on the bar surface act as stress concentrators, reinforcing steels bars may fracture on bending if the radius of bend is too tight. The presence of crack in bending test reveals that the material is brittle hence the ductility is low. The bending test predicts the ductility of the reinforcing steel bar. The specimen is subjected to the prescribed sequence of operations and should not show any sign of crack or fracture.The ISO 6935-2:2007(E) and BS4449:1997, and BS4449:2005 have specified the standards requirement for yield strength for most of the reinforcing bars. The minimum yield strength is 460MPa (BS4449:1997), 500MPa

(BS4449:2005) for high tensile deformed bars and 250MPa (BS4449:1997) for mild steel round bars. The tensile strength should exceed the yield strength by 10 to 15% and the minimum percentage elongation should be 14% for Grade 460-500 and 22% for Grade 250.

2.2 Steel Reinforcing Bar from Scrap

The process consists of collecting scrap metals, sorting them, melting in a furnace, mixing with ingredients (additives), cast the molten metal into moulds for making the ingots. After the ingots solidify, the moulds are stripped. Before rolling of each stock, the ingot is placed in a soaking pit for heating to ensure that the entire cross section is uniform. The rolled bar is normally cooled in air. To increase the strength of the bar, cold working by twisting is performed and finally the bar is inspected and stored.

The production of reinforcing steel bars from scrap metals has raised interest of researchers (Buro *et al.*, 2006) who tried to classify the scrap metal commonly used in three categories (home scrap, process scrap and obsolete scrap) according to the place of generation, chemical composition or physical properties. Obsolete scrap causes the biggest trouble for steel maker, because its recovery is difficult, and this type of scrap is often mixed or coated with other materials such as copper, brass, plastic, zinc, tin etc. The chemical composition of obsolete scrap fluctuates widely depending on its origin and can affect the mechanical properties of the reinforcing bars. A complete understanding and knowledge of the real behaviour of construction materials is of prime importance for the proper behaviour of engineered structures. The physical properties of structural materials are expected to meet the demand of the fundamental assumptions underlying structural codes of practice on which designs are based. Locally manufactured reinforcing steel bars from scrap metal are typical examples. In developing countries where imported steel is very expensive, milling companies have taken up the challenge to re-cycle obsolete vehicle and machine metal parts for the production of structural and reinforcing steel bars. The study in Ghana by Kankam and Asamoah (2002) on Strength and ductility characteristics of reinforcing steel bars milled from scrap metals, showed that reinforcing bars did not meet the BS4449 maximum limit of 0.25% for carbon requirements for mild steel. The phosphorus and sulphur impurities in the steel bars from three companies exceeded the preferred limit of 0.05% for phosphorus and 0.01% for sulphur. These excess carbon, sulphur and phosphorus contents increase the strength and hardness of the steels and decreases their ductility, making them brittle (Kankam and Asamoah, 2002).

The British Standard BS 4449: 2005 defines the characteristic strength of steel reinforcement as that value of the yield stress below which not more than 5% of the test material should fall. The presence of variation in the strength of bars is as a result of such factors as variation in the chemical composition and heat treatment. With regard to quality control of chemical properties, steel manufacturers must give the results of analysis for carbon (C), manganese (Mn), silicon (Si), sulphur (S) and phosphorus (P) for all steels. The KS 573 and BS 4449:2005 give maximum percentage composition of mild steel as S (0.06%), P (0.06%), C (0.25%), Mn (0.65%) and Si (0.25%). The different elements have varying effects on the behaviour of mild steel. The carbon level affects the strength and hardening properties of steel. Higher carbon contents increase strength but reduce ductility. Excessive levels of phosphorus and sulphur, which are non-metallic impurities reduce fracture toughness (Munyazikwiye, 2013). For modern steel-making practice, sulphur and phosphorus are preferably maintained at less than 0.01%. Steel grades with a high level of dissolved gases, particularly oxygen and nitrogen, can behave in a brittle manner, if not controlled by addition of small elements with a particular affinity for them to float out in the liquid steel at high temperature. Manganese, chromium, molybdenum, nickel and copper also affect the strength to a lesser extent than carbon, although their sole effect is on the microstructure of the steel (Munyazikwiye, 2013). Research by Shunichi and Morifumi(2006) showed that addition of alloying elements such as Niobium and Vanadium were effective to increase strength of reinforcing steel bars.

2.3 Types of Uncertainty

2.3.1 Alerotic and Epistemic Model Uncertainties

A rigorous structural reliability assessment involves modeling all of the sources of uncertainty that may affect failure of the component or system. This clearly involves modeling all of the fundamental quantities entering the problem, and also the uncertainties that arise from lack of knowledge and idealized modeling. These terms are referred to as basic variables. Basic variables representing common engineering quantities include: diameter, wall thickness, material and contents density, yield stress, maximum operating pressure, maximum operating temperature, corrosion rate, etc.

The sources of uncertainty that are relevant to structural reliability analysis can be primarily classified into two categories: those that are a function of the physical uncertainty or randomness (aleatoric uncertainties), and those that are a function of understanding or knowledge of the uncertainty (epistemic uncertainties) (BOMEL, 2001).

2.3.1.1 Aleatoric Uncertainties

Aleatoric uncertainty (originating from the Latin aleator or aleatorius meaning dice thrower) refers to the natural randomness associated with an uncertain quantity, and is often termed Type I uncertainty in reliability analysis. It can be further subdivided into three categories:

Inherent or intrinsic uncertainty in time:- In reliability analysis variables that change with time are often referred to as processes, stochastic processes or stochastic variables. Examples include the fluctuations of pressure or temperature in an operating pressure system, wind velocity, corrosion rate, etc. (BOMEL, 2001).

The inherent, intrinsic or physical uncertainty of an object or property in space:- examples include the natural variations of the strength parameters from one specimen to another, the spatial variation in soil strength, and the incidence of corrosion in a pipeline(BOMEL, 2001).

The inherent uncertainty associated with the measuring device:- With modern techniques, and properly calibrated instruments, this source of uncertainty should be small, but it can never be fully eliminated. Aleatoric uncertainty is quantified through the collection and analysis of data. The observed data may be fitted by theoretical distributions, and the probabilistic modelling may be interpreted in the relative frequency sense (BOMEL, 2001).

2.3.1.2 Epistemic Uncertainties

Epistemic uncertainty (originating from the Greek episteme, meaning knowledge) reflects a lack of knowledge or information about a quantity, and is often termed Type II uncertainty in reliability analysis. Epistemic uncertainty can also be further subdivided into:

i. Model uncertainty:- This is due to the simplifications and idealizations necessary to model the behaviour in a reliability analysis, or to an inadequate understanding of the physical causes and effects.
ii. Statistical uncertainty:- This is solely due to a shortage of information, and originates from a lack of sufficiently large samples of input data.

Model uncertainty arises because many of the engineering models that are used to describe natural phenomena, to analyze stresses and to predict failure of components are imperfect. Models are often based on idealized assumptions, they may be based on empirical fits to test results or observed behaviour, and variables of lesser importance may be omitted for reasons of efficiency (or ignorance). In many components and structures, model uncertainties have a large effect on structural reliability and should not be ignored.

It is very difficult to quantify model uncertainty adequately. The errors in the model may be known relative to more elaborate models, but at any level of modelling there are errors relative to the unknown reality. Model uncertainty is often assessed on the basis of experimental test results, but this too has a number of limitations. Tests themselves are idealizations or simplifications of reality, and are affected by scale, boundary effects, load rates, measurement errors, etc. Tests are expensive, and the data need to be carefully screened to ensure that they are consistent. Ideally, the test data should cover uniformly the full range of the applicability of the model.

Model uncertainty may be expressed in terms of the probability distribution of a variable X_m

$$\text{Where } X_m = \frac{\text{Actual or modeled strength (response)}}{\text{Predicted strength (response) using model}}$$

Statistical uncertainty can be considered to arise from:

- Parameter uncertainty:- This occurs when the parameters of a distribution are determined from a limited set of data. The smaller the data set the larger the parameter uncertainty.
- Distribution type uncertainty:- This uncertainty arises from the choice of a theoretical distribution fitted to empirical data. It is a particular problem when deriving extreme value distributions.

Often it may not be possible to differentiate between the two types of statistical uncertainty in practice, since with limited data both the parameters and distribution type may be uncertain.

Statistical uncertainty can be divided further into 'statistical uncertainty due to variations in time' and 'statistical uncertainty due to variations in space'. Statistical uncertainty can be modelled by a variety of techniques, including classical statistical techniques, Bayesian methods, and Bootstrapping (Efron, 1979).

Because epistemic uncertainty is associated with a lack of knowledge and/or information it follows that it can be reduced through an increase in knowledge. In general there are three ways to increase knowledge:

- Gathering data
- Research
- Expert judgment.

Statistical uncertainty associated with variations of variables in time can, in principle, be reduced by observing the phenomena for a longer period. Statistical uncertainty associated with the variability of properties in space can be reduced by taking more measurements or carrying out further tests. Model uncertainty can be reduced by further research and testing of the phenomena, and by improved modelling.

The modelling of a basic variable can be updated with the help of expert judgment. This may be informal, on the basis of one or two specialist's opinions, or a number of techniques exist to solicit, collate and analyze the views of a circle of experts – the so-called Delphi technique. Expert opinion can be incorporated into the reliability analysis using Bayesian methods.

Epistemic uncertainties influence the confidence in the evaluated failure probability that is they add to the 'uncertainty' in the probability of failure. A problem with low epistemic uncertainty leads to a failure probability with a high degree of confidence that tends towards the 'true' failure probability (BOMEL, 2001).

It is important to distinguish between uncertainty and ignorance. Ignorance reflects a lack of awareness of factors influencing the problem or issue, and is not, and by its very nature cannot, be included in a reliability analysis. Ignorance is a well-recognized weakness in QRA, where it is manifested by an incomplete identification of hazards. Ignorance of this sort is not often recognized in reliability analysis. In reliability analysis failure events are formulated mathematically; in the early development of the reliability analysis methods applications were theoretical and great care was taken over formal definitions. With the wider use of probabilistic analysis the formal restrictions are not rigorously applied, and a lack of awareness or ignorance of all the factors influencing failure of a system can become more significant. This is particularly important when the results of reliability analyses are used to assess risk, and particularly when they are combined with risks assessed from historical failure rate data and other sources.

2.3.2 Physical, Statistical and Model Uncertainty

The overall aim of structural reliability analysis is to quantify the reliability of structures under consideration of the uncertainties associated with the resistances and loads. The structural performance is assessed by means of models based on physical understanding and empirical data. Due to idealizations, inherent physical uncertainties and inadequate or insufficient data the models themselves and the parameters entering the models such as material parameters and load characteristics are uncertain. Structural reliability theory takes basis in the probabilistic modelling of these uncertainties and provides methods for the quantification of the probability that the structures do not fulfil the performance criteria.

The uncertainties in modeling and calibration, which must be considered, are the physical uncertainty, the statistical uncertainty and the model uncertainty.

The physical uncertainties are typically uncertainties associated with the loading environment, the geometry of the structure and the material properties. The statistical uncertainties arise due to incomplete statistical information e.g. due to a small number of materials tests. Finally, the model uncertainties must be considered to take into account the uncertainty associated with the idealized mathematical descriptions used to approximate the actual physical behavior of the structure (Faber and Sorensen, 2001).

The probabilistic modeling of uncertainties highly rests on a Bayesian statistical interpretation of uncertainties implying that the uncertainty modeling utilizes and facilitates both the incorporation of statistical evidence about uncertain parameters and subjectively assessed uncertainties. Modern methods of reliability and risk analysis allow for a very general representation of these uncertainties ranging from non-stationary stochastic processes and fields to time-invariant random variables. In most cases it is sufficient to model the uncertain quantities by random variables with given distribution functions and distribution parameters estimated on basis of statistical and/or subjective information. In the probabilistic model code by (JCSS, 2001), an almost complete set of probabilistic models are given covering most situations encountered in practical engineering problems.

2.3.3 Uncertainty Due to Human Error

Experience in many areas, including building structures Matousek(1977) and offshore structures shows that gross error is the dominant cause of structural failure. Understanding of the human contribution to failures has grown substantially through studies of major accidents. Matousek's work for instance, based on investigation of 800 cases of major damage to building structures, showed that human errors and gross errors contributed to 75-90% of accidents; the contribution of failures that can be attributed to causes normally covered by rigorous reliability analyses was only 10-25%. A gross error may be defined as a major or fundamental mistake in some aspect of the processes of planning, design, analysis, construction, use or maintenance of a structure that has the potential for causing failure (Thoft-Christensen and Baker, 1982). Human errors can be individual acts, and may be:

• Deliberate acts - sharp practice, fraud, theft, sabotage, etc.

• Non-deliberate acts - (Obvious' (inexperience, negligence) or 'Subtle' – (new material, new structural type, new construction procedure etc.)

Human errors can also be influenced by the company management or culture, which may lead to:

• stress and overwork

• Bad practice, poor communication, etc.

2.3.3.1 Control of Errors in Deterministic Design

In principle, it should be possible to account for human errors or gross errors in design by increasing the safety factors. It has been widely shown that small adjustments to safety factors are ineffectual in mitigating the effects of human error (Thoft-Christensen and Baker, 1982). The interaction between safety factor and probability of failure is illustrated by (Beeby, 1999), which was originally based on work by Lewecki for Eurocode 2 in 1994. It shows that where the safety factors are greater than some level X, the probability of failure is largely independent of the safety factor because the overall probability of failure is dominated by unforeseen events and gross errors occurring. It is generally accepted that current practice lies to the right of X, as there is little evidence that the very different levels of safety in different countries lead to different rates of failure.

Since increasing safety factors is ineffective in reducing the effects of human errors, reliance must be placed on control measures to reduce the risks to an acceptable level. It is often assumed that the frequency of undetected errors is reduced to an acceptable level by inspection, quality control and quality assurance measures. However, the effectiveness of existing control measures is itself highly variable and depends to a large extent on the type and severity of error made, and who is performing the checking. Given the significance of human errors and unforeseen events, there is a strong reason for designing the structure to be damage tolerant or robust.

Robustness is the ability of a structure to absorb energy, and is often defined as the ability of a structure to withstand accidents and unforeseen events without suffering damage disproportionate to the cause. In addition, some accidental scenarios develop over time and their consequences can be mitigated to some extent by Event Control measures, e.g. evacuation procedures, water curtains or sprinklers, etc. (BOMEL, 2001).

2.3.3.2 Treatment of Errors in Probabilistic Assessment

In principle, it should be possible to account for human errors or gross errors in probabilistic assessments by accounting for the uncertainty in the basic variables, or by modifying the evaluated probability to account for the probability of such errors. Gross errors and human-based errors are, by their very nature, difficult to deal with. Work has been undertaken by a number of authors to estimate the probability of a human error in the design phase; as may be expected because of the wide-ranging causes summarized above, the results are variable. However until now, human errors or gross errors are rarely taken into account in reliability analysis calculations. It is clear that a rational solution to structural safety problems cannot be achieved without due consideration of human error. It is also widely accepted that structural reliability analysis is not a suitable tool for addressing human errors (BOMEL, 2001).

2.3.3.4 Probabilistic Design and Assessment Procedures

Probabilistic analysis, based on structural reliability analysis methods, is an extension of deterministic analysis since deterministic quantities can be interpreted as random variables of a particularly trivial nature in which their density functions are contracted to spikes and in which their standard deviations tend to zero (BOMEL, 2001).

Variations in the values of the basic engineering parameters occur because of the natural physical variability, because of poor information, and because of accidental events involving human error. In the past, emphasis has been focused on the former categories, but the last is equally if not more important(BOMEL, 2001).

In addition to the uncertainties associated with the individual load and strength parameters (basic variables) which are mentioned above, it is well known that both the methods of global analysis and the equations used for assessing the strength of individual components are not exact. In the case of global structural analysis, the true properties of the materials and components often deviate from the idealizations on which the methods are based. Without exception, all practical structural systems exhibit behaviour that (to a certain extent) is nonlinear and dynamic, and have properties that are time-dependent, strain-rate dependent and spatially non-uniform. Furthermore, most practical structures are statically indeterminate and contain high levels of residual forces (and hence stresses) resulting from the particular fabrication and installation sequence adopted; in addition they often contain so-called non-structural components which are normally ignored in the analysis, but which often contribute in a significant way, particularly to stiffness. These differences between real and predicted behaviour can be termed global analysis model uncertainty. In general, this is extremely difficult to quantify. Estimates of the magnitude of this type of model uncertainty can be obtained by comparisons using more refined analysis tools and sensitivity studies, or by full-scale physical testing(BOMEL, 2001).

As far as individual components are concerned, the design equations given in Codes of Practice are generally chosen to be conservative, but there are often large variations in the ratio of real to predicted behaviour, even when the individual parameters in the equations are known precisely (e.g. Poisson's ratio).

The variability in load and strength parameters (including model uncertainty) arising from physical variability and inadequacies in modelling are allowed for in deterministic design and assessment procedures by an appropriate choice of safety factors and by an appropriate degree of bias in the Code design equations. In probabilistic methods the variability in the basic design variables, including model uncertainty, is taken into account directly in the probabilistic modeling of the quantities (BOMEL, 2001).

2.3.3.5 Error Due to Outliers

An outlier is a measured value that, according to a statistical test, is unlikely to have been drawn from the same population as the remainder of the sample data.Outliers are caused by either mistakes in data entry or an unusual or unique situation. A mistake in data entry is easier to deal with: You discover and correct the mistake and then redo the analysis. If there is no mistake, you have a bigger problem. In that case you have to study the outlier and decide whether it really belongs with the other data values.

Outliers have been a problematic concern since the inception of statistics. One of the first known efforts to address issues concerning outliers was made by Boscovich in 1755. In an attempt to determine the average ellipticity of the earth using polar degrees over the equatorial, Boscovich (1975) collected ten measures. When he determined that two of the ten measures exceeded the normal range, Boscovich removed the two extraneous values and calculated the mean of the eight remaining values. As one of the earliest attempts to address the presence of outliers, Boscovich set an early precedent for their removal. As attempts to analyze data sets grew popular in several fields such as science, psychology and education, the question of what to do with outliers began to pervade many statistical studies (Farrell-Singleton, 2010).

Since then, identification and treatment of outliers is crucial to statistical research because if left unchecked, outliers can increase error variance, reduce the power of statistical tests, decrease normality (if non-randomly distributed), violate assumptions of sphericity and multivariate normality (in multivariate analyses), as well as significantly bias or influence estimates that may be of considerable interest (Osborne &Overbay, 2004). Removal of outliers has been linked to problems such as increased sampling error, particularly when the underlying distribution is unknown or contaminated, as well as the increased likelihood of violating underlying assumptions. These concerns can have serious effects on the validity of statistical studies and can negatively impact statistical results when making inferences about data.

2.4 Considerations When Modeling Steel Reinforcement

2.4.1 Uncertainty in Steel Reinforcement Bars

The uncertainties in the determination of steel strength are due to the variation in the strength of the material, variation in cross section of the bar, effect of rate of loading, effect of bar diameter on properties of bar and effect of strain at which it is defined (Mirza, 1979). Effort must be made to ensure that distributions determined from test data are properly transformed to represent the in-situ conditions and the type of test performed. Different tests may sometimes be performed to measure the same property. For example, often there are two quoted steel strengths, the mill test strength and the static strength. The mill strength tests are done at a rapid rate of loading and use actual areas. The static strengths are determined based on nominal area and use a strain rate that is similar to what is expected in a structure.

2.4.2 Yield and Ultimate Strength Steel Reinforcement Bars

The yield strength of steel reinforcing bars is taken as the stress at a corresponding strain. This strain normally corresponds to the initial plastic deformation of the reinforcement. A model for the yield strength of reinforcing steel is proposed in JCSS (2001), taking into consideration the variations in global mean of different mills, the variations in a mill from batch (melt) to bath and the variations within the melt. Normal or beta distributions can be used to represent yield strength (JCSS, 2001; Mirza, 1979).

Strength fluctuations along bars are negligible (JCSS, 2001). The yield force of a bundle of bars under static loading is the sum of the yield forces of each contributing bar. In general, it can be assumed that all reinforcing steel used at a job originates from a single mill. The correlation coefficient between yield forces of individual bars of the same diameter can be taken as 0.9 (Rackwitz, 1996). The correlation coefficient between yield forces of bars of different diameter and between the yield forces in different cross sections in different beams in a structure can be taken as 0.4 (JCSS, 2001). The ultimate strength is often represented by normal or beta distributions (Mirza, 1979; JCSS, 2001).

2.4.3 Variations in Area of Bar Cross Section

The actual areas of reinforcing bars tend to deviate from the nominal areas due to the rolling process. In general this value has been found to be less than 1 and that it can be represented by a normal distribution (Mirza, 1979; JCSS, 2001; Allen, 1972;and Wiss, 1973).

2.5 Reliability Analysis Methods

This section of the chapter presented the First Order Reliability Method (FORM) and the Second Order Reliability Method (SORM). The First Order Reliability Method (FORM) makes use of the first and second moments of the random variables. This method includes two approaches. These are First-Order Second-Moment (FOSM) and Advanced First-Order Second-Moment (AFOSM) approaches. In FOSM, the information on the distribution of random variables is ignored; however, in AFOSM (called also Hasofer-Lind approach), the distributional information is appropriately used (Emilo& Abdel-Hamid, 2015).

2.5.1 First Order Reliability Method (FORM)

The First Order Reliability Method (FORM) makes use of the first and second moments of the random variables which includes two approaches such as the First-Order Second-Moment (FOSM) and Advanced First-Order Second-Moment (AFOSM) approaches. In FOSM, the information on the distribution of random variables is ignored; however, in AFOSM, the distributional information is appropriately used.

2.5.2 First Order Second Moment (FOSM)

The First Order Second Moment (FOSM) method makes use of only second moment statistics (i.e. mean and standard deviation) of the random variables and it requires a linearized form of the performance function at the mean values of the random variables. A first-order Taylor series approximation is used to linearize the performance function at the mean values of the random variables. Assuming a Gaussian distribution, the failure probability of the FOSM (P_f) is related to the failure event $R < S$, and is computed using equation (2.1).

$$P_f = P(R < S) = P(R - S < 0) \qquad (2.1)$$

Where Pf is the probability of failure

R is the resistance and

S is the load action

However, a new random variable Z (called performance function) can be introduced using equation (2.2).

$$Z = R - S \qquad (2.2)$$

The performance function Z is characterized by a mean $\mu_Z = \mu_R - \mu_S$ and a standard deviation $\sigma_Z^2 = \sigma_R^2 + \sigma_S^2$. Since R and S are Gaussian, it can be demonstrated that Z also follows a Gaussian distribution and the failure probability is related to the event P(Z < 0). Consequently, P_f could be estimated directly from equation (2.3).

$$P_f = \Phi[(0 - \mu_Z)/\sigma_Z] = \Phi[-\mu_Z/\Phi_Z] = \Phi[-\beta] \qquad (2.3)$$

Where Φ[.] represents the standard normal cumulative distribution function (CDF) and $\beta = \mu_Z/\sigma_Z$ is the 'reliability index' that is also used to quantify risks of failure.

In the general case where the performance function Z is a function of a vector of n random variables, the Taylor series expansion about the mean value is given by equation (2.4).

$$Z = g(\mu_X) + \sum_{i=1}^{n} \frac{\partial_g}{\partial_{Xi}} (X_i - \mu_{Xi}) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 g}{\partial X_i \partial X_j} (X_i - \mu_{Xi})\left(X_j - \mu_{Xj}\right) + \ \ldots. \quad (2.4)$$

Where the derivatives are evaluated at the mean values of the random variables (X_1, X_2,…,X_n), and μ_X is the mean value of X_i.

2.5.3 Advanced First Order Second Moment (AFOSM)

AFOSM is also called 'Hasofer-Lind' method. In this method, the assessment of the reliability index is mainly based on the transformation/reduction of the problem to a standardized coordinate system. Thus, a random variable Xi is reduced and expressed by equation (2.5).

$$X'_i = (X - \mu_{Xi}) / \sigma_{Xi} \quad (i = 1,2,\ldots.n) \qquad (2.5)$$

Where X_i' is a random variable with zero mean and unit standard deviation. Thus, Eq. (2.5) is used to transform the original limit state surface $g(\mathbf{X}) = 0$ into a reduced limit state surface $g(\mathbf{X'})$ = 0. Consequently, **X** denotes 'original coordinate system' and **X'** describes the 'transformed or reduced coordinate system'. In the standardized coordinate system, the Hasofer-Lind reliability index β_{HL}corresponds to the minimum distance from the origin of the axes (in the reduced coordinates system) to the limit state surface shown in equation (2.6).

$$\beta_{HL} = \sqrt{(X'^{*})^T (X'^{*})} \qquad (2.6)$$

The minimum distance point on the limit state surface is called the 'design point'. It is denoted by vector **x*** in the original coordinate system and by vector **x'*** in the reduced coordinate system. These vectors represent the values of all the random variables, *i.e.* X_1, X_2, …,$X_n$at the design point corresponding to the coordinate system being used.

According to eq. (2.5), R and S can be reduced as shown in equation 2(.6).

$$R' = (R - \mu_R) / \sigma_R \text{ and } S' = (S - \mu_S) / \sigma_S \qquad (2.7)$$

The substitution of R' and S' into the limit state surface (Z=0) gives the new limit state surface in the reduced coordinate system given by equation (2.8).

$$Z = \sigma_R R' - \sigma_S S' + \mu_R - \mu_S = 0 \qquad (2.8)$$

The reliability of the problem is estimated by using eq. (2.6) since it corresponds to the minimum distance between the limit state surface and the origin (in the reduced coordinates system). By using simple trigonometry, this distance (reliability index) can be estimated using equation (2.9).

$$\beta_{HL} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (2.9)$$

It should be noted that in the present case of a linear limit state surface, the Hasofer-Lind reliability index corresponds exactly to the reliability index computed from FOSM if both R and S are normal variables. However, this is not the case for other limit state surfaces or random variable distributions. It is apparent that if the limit state line is closer to the origin in the reduced coordinate system, the failure region is larger, and if it is farther away from the origin, the failure region is smaller. Thus, the position of the limit state surface relative to the origin in the reduced coordinate system is a measure of the reliability of the system. Notice that the Hasofer-Lind reliability index can be used to calculate the failure probability as $P_f = \Phi(-\beta_{HL})$. This is the integral of the standard normal density function along the ray joining the origin and **x'***. It is obvious that the nearer **x'*** is to the origin, the larger is also the most probable failure point. The point of minimum distance from the origin to the limit state surface, **x'***, represents the worst combination of the stochastic variables and is appropriately named the 'design point' or the 'most probable point MPP' of failure.

Finally, it should be noted that the Hasofer-Lind reliability index is invariant, because regardless of the form in which the limit state equation is written [e.g., ($R - S$=0) or (R/S=1)], its geometric shape and the distance from the origin remain constant.

For the general case of a non-linear limit state surface, the assessment of the minimum distance can be written as an optimization problem and represented by equation (2.10).

$$\text{Minimize } D = \sqrt{X'^T X'} \qquad \text{subject to } g(x') = 0 \qquad (2.10)$$

By using Lagrange's multipliers, the minimum distance (for n random variables) couldbe estimated using equation (2.11).

$$\beta_{HL} = -\frac{\sum_{i=1}^{n} x_i^{j*}\left(\frac{\partial g}{\partial X'_i}\right)}{\sqrt{\sum_{i=1}^{n}\cdot\left(\frac{\partial g}{\partial X'_i}\right)^{2*}}} \qquad (2.11)$$

Where $(\partial g/\partial X'_i)^*$ is the i^{th} partial derivative evaluated at the design point ($x_1'^*, x_2'^*, \ldots, x_n'^*$). The design point in the reduced coordinates shown in equation (2.12).

$$X_i' = -\alpha_i \beta_{HL} \qquad (I = 1,2,\ldots.n) \qquad (2.12)$$

Where $\alpha_i$are the direction cosines along the coordinate axes X'_i. They are given by equation (2.13).

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial X'_i}\right)^*}{\sqrt{\sum_{i=1}^{n}\cdot\left(\frac{\partial g}{\partial X'_i}\right)^{2*}}} \tag{2.13}$$

An algorithm was formulated by Rackwitz(1976) to compute $\beta_{HL}$and xi'^* as follows:

Step 1: Define the appropriate performance function $g(\mathbf{X})$.

Step 2: Assume initial values for the design point $\mathbf{x}^*_i$. The initial design point is usually assumed to be at the mean value of the n –1 random variables.For the last random variable, its value is obtained from the performancefunction to ensure that the design point is located on the limit state surface $g(\mathbf{X}) = 0$.

Step 3: Obtain the design point in the reduced space $\mathbf{x'}^*= [x1'^*, x2'^*,\ldots,xn'^*]$ as;

$$x_i'^* = (x_i^* - \mu_{xi})/\sigma_{xi} \qquad (i = 1, 2, \ldots.. n)$$

Step 4: Estimate the partial derivatives of the performance function $(\partial g/\partial X'_i)^*$ with respect to the variables in the reduced space and evaluate them at x_i^*. These derivatives can be estimated from the performance function in the original space by using the chain rule of differentiation

$$\frac{\partial g}{\partial X_i'} = \frac{\partial g \partial X_i}{\partial X_i \partial X_i'} = \frac{\partial g}{\partial X_i}\sigma_{x_i}$$

Define the column vector **A** such that $A_i = \left(\frac{\partial g}{\partial X_i'}\right)^*$

Step 5: Compute the reliability index as $\beta_{HL} = -\frac{A^T x^{f*}}{\sqrt{A^T A}}$

Step 6: Determine a vector of directional cosines as $\alpha = \frac{A}{\sqrt{A^T A}}$

Step 7: Obtain the new design point xi'^* for the n –1 random variables.

Step 8: Determine the coordinates of the new design point in the original space for the *n* –1 random variables considered in Step 7 as

$$x_i'^* = \mu_{xi} + x_i^{f*}\sigma_{xi} \qquad (i = 1, 2,\ldots.. n)$$

Step 9: Determine the value of the last random variable in the original space such that the estimated point belongs to the limit state surface $g(\mathbf{X}) = 0$.

Step 10: Repeat Steps 3 to 9 until convergence of β_{HL}.

2.5.4 Ellipsoid Approach

The Hasofer-Lind reliability index can be formulated in a matrix form given by equation (2.14).

$$\beta_{HL} = \min_{g(X)=0} \sqrt{(X-\mu)^T C^{-1} (X-\mu)} \qquad (2.14)$$

Where **X** is a vector containing the *n* random variables, **μ** is a vector containing their mean values, and **C** is the covariance matrix.

When there are only two uncorrelated non-normal random variables $X_1$and X_2, these variables span a two-dimensional random space, with an equivalent one-sigma dispersion ellipse (corresponding to β_{HL}=1 in Eq. (2.14) without the min.), centered at the equivalent mean values μ_1^N and μ_2^N and whose axes are parallel to the coordinate axes of the original space. For correlated variables, a tilted ellipse is obtained.

However, the Hasofer-Lind reliability index β_{HL}may be regarded as the co-directional axis ratio of the smallest ellipse (which is either an expansion or a contraction of the $1-\sigma$ ellipse) that just touches the limit state surface to the $1-\sigma$ dispersion ellipse. They also stated that finding the smallest ellipse that is tangent to the limit state surface is equivalent to finding the most probable failure point.

2.5.5 Response Surface Method (RSM)

In case of analytically-unknown system response (such as the responses computed using a finite element/finite difference method), several approaches based on the Response Surface Method (RSM) can be found in the literature with the aim of calculating the reliability index and the corresponding design point. The basic idea of this method is to approximate the system response $Y(x)$ by an explicit function of random variables, and to improve the approximation *via* iterations. The system response can be approximated (in the original space of random variables) given by equation (2.15).

$$Y(x) = a_0 + \sum_{i=1}^{n} a_1 x_1 + \sum_{i=1}^{n} b_1 x_i^2 \qquad (2.15)$$

Where x_i are the random variables (μ_i and σ_i being their mean and standard deviation values, respectively); n is the number of random variables; and (a_i, b_i) are coefficients to be determined. The RSM algorithm is summarized as follows:

Step 1: Evaluate the system response $Y(x)$ at the mean value point μ and at the $2n$ points each at $\mu \pm k\sigma$ where k can be arbitrarily chosen (k = 1).

Step 2: The above $2n+1$ values of $Y(x)$ are used to solve eq. (2.15) and find the coefficients (a_i, b_i). Then, the performance function g(x) can be constructed to give a tentative response surface function.

Step 3: Solve Eq. (2.14) to obtain a tentative design point and a tentative B_{hl} subjected to the constraint that the tentative performance function of step 2 be equal to zero.

Step 4: Repeat Steps 1 to 3 until convergence of β_{HL}. Each time, Step 1 is repeated, the $2n + 1$ sampled points are centered at the new tentative design point of Step 3.

2.5.6 Second Order Reliability Method (SORM)

Reliability assessment is relatively simple if the limit state function is linear. However, most of the limit state functions are nonlinear. The nonlinearity is due to nonlinear relationship between random variables, to the consideration of non-normal random variables, and/or to the transformation from correlated to uncorrelated random variables. Indeed, a linear limit state in the original space becomes non-linear when transformed to the standard normal space if any of the variables is non-normal. Also, the transformation from correlated to uncorrelated variables might induce nonlinearity.

Both linear and non-linear limit state functions have the same minimum distance point β, but the failure regions are different in both cases. The failure probability of the nonlinear limit state should be less than that of the linear limit state. The FORM approach approximates the limit state function with a linear function and will therefore provide the same assessment of the probability of failure for both cases. This approximation introduces errors in the assessment of the probability of failure. Consequently, it is preferable to use a higher order approximation for the failure probability computation.

The SORM method improves the assessment given by FORM by including information about the curvature (which is related to the second-order derivatives of the limit state function with respect to the basic variables). The Taylor series expansion of a general nonlinear function $g(X_1, X_2,\ldots,X_n)$ at the design point $(x^*_1, x^*_2,\ldots, x^*_n)$ is given by equation (2.16).

$$g(X_1, X_2, X_n) = g(x_1^*, x_2^*, x_n^*) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i}(x_i - x_i^*) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=}^{n} \frac{\partial^2 g}{\partial X_i \partial X_j}(x_i - x_i^*)(x_j - x_j^*) + \ldots \tag{2.16}$$

Where the derivatives are evaluated at the design point

For reliability analysis, the space of standard normal variables is more convenient for a second order approximation of $g()$. In the following, $X_i$and $Y_i$will refer to random variables in the original and equivalent uncorrelated standard normal spaces, respectively. If all the variables are uncorrelated, then use equation (2.17).

$$Y_i=(X_i - \mu^e{}_{Xi}) \,/\, \sigma^e{}_{Xi} \qquad (2.17)$$

Where $\mu^e{}_{Xi}$ and $\sigma^e{}_{Xi}$ are the equivalent normal mean and standard deviation of X_i at the design point x_i.

In the Taylor series approximation given by eq. (2.16), FORM ignores the terms beyondthe first order term, and SORM ignores the terms beyond the second-orderterm (involving second-order derivatives). A simple closed-form solution for the probability computation using the theory of asymptotic approximation is given by equation (2.18).

$$P_f \approx \Phi\left(-\beta_{\mathrm{FORM}}\right) \prod_{i=1}^{n-1} \left(1 + \beta_{\mathrm{FORM}} \; \mathrm{K_I}\right)^{-1/2} \qquad (2.18)$$

Where κ_i represents the principal curvatures of the limit state function at the minimum distance point, and β_{FORM} is the reliability index computed by the FORM method. The assessment of P_f requires the computation of κ_i. Towards this aim, the random variables Y_i(in the **Y** reduced space) are rotated to another set of variables Y_i', such that the last Y_i' variable coincides with the vector $\boldsymbol{\alpha}$ where $\boldsymbol{\alpha}$ is the unit gradient vector of the limit state at the minimum distance point.

2.5.7 Monte-Carlo Reliability Method

The interest in simulation methods started in the early 1940s for the purpose of developing inexpensive techniques for testing engineering systems by imitating their real-world behavior. These methods are commonly called Monte Carlo simulation techniques (Novák, *et al.*, 2014). The principle behind the methods is to develop an analytical model, which is usually computer based, that predicts the behavior of a system. Then parameters of the model are calibrated using data measured from a system. The model can then be used to predict the response of the system for a variety of conditions. Next, the analytical model is modified by incorporating stochastic components into the structure. Each input parameter is assumed to follow a probability function and the computed output depends on the value of the respective probability distribution. As a result, an array of predictions of the behavior are obtained. Then statistical methods are used to evaluate the moments and distribution types for the system's behavior (Novák, *et al.,* 2014). The analytical and computational steps of a Monte Carlo simulation follow the following steps

1. Define the system using a model.

2. Calibrate the model.

3. Modify the model to allow for random variation and the generation of random numbers to quantify the values of random variables.

4. Run a statistical analysis of the resulting model output.

5. Perform a study of the simulation efficiency and convergence.

6. Use the model in decision making.

The definition of the system should include its boundaries, input parameters, output (or behavior) measures, architecture, and models that specify the relationships of input and output parameters. The accuracy of the results of simulation are highly dependent on an accurate definition for the system. All critical parameters and variables should be included in the model. If an important variable is omitted from the model, then the calibration accuracy will be less than potentially possible, which will compromise the accuracy of the results. The definition of the input parameters should include their statistical or probabilistic characteristics, that is, knowledge of their moments and distribution types. It is common to assume in Monte Carlo simulation that the architecture of the system is deterministic, that is, nonrandom. However, model uncertainty is easily incorporated into the analysis by including bias factors and measures of sampling variation of the random variables. The results of these generations are values for the input parameters. These values should then be substituted into the model to obtain an output measure. By repeating the procedure N times (for N simulation cycles), N response measures are obtained. Statistical methods can now be used to obtain, for example, the mean value, variance, or distribution type for each of the output variables. The accuracy of the resulting measures for the behavior are expected to increase by increasing the number of simulation cycles. The convergence of the simulation methods can be investigated by studying their limiting behavior as N is increased (Richard, 2003).

The Monte Carlo techniques involve sampling values of the basic variables in order to conduct many simulated experiments. The input variables for the limit state function are randomly (or rather pseudo randomly) collected from their probabilistic distributions. Every realization of the random variables is checked against the limit state and the probability of failure can be calculated using equation (2.19)

$$P_f \approx \frac{n(G(x_i)\leq 0}{N} \qquad (2.19)$$

Where $(() \leq 0)$ is the number of virtual experiments (or trials) where the limit state is violated and N is the total number of trials. The number of virtual experiments conducted governs the accuracy of the estimation of $P_f$and is logically dependent on the evaluated function. It has been suggested that around 10,000 – 20,000 simulations are needed to reach a 95% confidence limit, Melchers (1999).

2.6 Sensitivity Analysis

The essential objective of sensitivity analysis of any system is to establish a measure of the way of response quantity varies with the change of input parameters that define the system. According to Castillo *et al*., (2008), sensitivity analysis is very useful to:

- a designer who can know which input parameter is most influential to a structural output

- ❖ a builder who can know how changes in component prices influence the total cost of the project being undertaken; and
- ❖ A code maker, who can know safety implications associated with changes in a design format.

In uncertainty quantification, sensitivity analysis is the study of how the uncertainty of an output can be apportioned to different sources of uncertainties in input variables. Methods for the probabilistic sensitivity analysis are usually classified into the following two categories:

- ❖ Local sensitivity analysis concentrates on the sensitivity of distribution parameters of an input random variable on a model output. It employs the gradient of response with respect to each parameter around a nominal value. Moment sensitivity factor and the reliability-based sensitivity factor are of two popular indices of the local sensitivity analysis.
- ❖ Global sensitivity analysis focuses on the output uncertainty over the whole definition domain of input variables. It takes into account the entire variation of input variables and aims to apportion the output uncertainty to each input factor. Therefore, global sensitivity analysis helps analyst identify the key parameters whose uncertainty affect most of the model output, which in turn can be used to establish experimental research priorities, eventually leading to a better definition of the model function (Sudret, 2008).

In local sensitive analysis, the contribution of each parameter (e.g., mean value, standard deviation, etc.) of a random variable with respect to output moment or failure probability is of interest. With the local sensitivity index, critical parameters can be identified. Therefore, it provides the gradient information needed by an optimization procedure to update the distribution parameter in a risk-based design framework (Zhang, 2013).

However, the local sensitivity index is only validated in the vicinity of a nominal value of distribution parameter. The first-order derivative, hence, is only useful to assess the sensitivity information near a predefined reference point, instead of the entire definition domain of input variable.

Reliability sensitivity analysis is used to find the rate of change in the probability of failure (or reliability) due to the changes in the parameters (usually mean and standard deviation) of distributions. For a distribution parameter p of random variable X_i, the sensitivity is defined by equation (2.20).

$$s_p = \frac{\partial P_f}{\partial P} \tag{2.20}$$

$s_p$can be derived as shown in equation (2.21).

$$s_p = \frac{\partial P_f}{\partial P} = \frac{\partial \Phi(-\beta)}{\partial P} = \frac{\partial \Phi(-\beta)\partial\beta}{\partial\beta}\frac{\partial\beta}{\partial P} = -\phi(-\beta)\frac{\partial\beta}{\partial P} \tag{2.21}$$

The derivative of the reliability index with respect to the distribution parameter is given by equation (2.22).

$$\frac{\partial\beta}{\partial p} = \frac{\partial\beta}{\partial u_i^*}\frac{\partial u_i^*}{\partial p} \tag{2.22}$$

Where equation (2.23) expresses the value of:

$$\frac{\partial\beta}{\partial u_i^*} = \frac{\partial\sqrt{\sum_{j=1}^{n}\left(u_j^*\right)^2}}{\partial u_i^*} = \frac{u_i^*}{\sqrt{\sum_{j=1}^{n}\left(u_j^*\right)^2}} = \frac{u_i^*}{\beta} \tag{2.23}$$

and from Eq. 2.17, we obtain equation (2.24).

$$u_i^* = \Phi^{-1}\left[F_{X_i}(x_i^*)\right] = w(p) \tag{2.24}$$

Where w(p) = $\Phi^{-1}\left[F_{X_i}(x_i^*)\right]$ is a function of the distribution parameter *p*.

Therefore, the equation is expressed in equation (2.25).

$$\frac{\partial\beta}{\partial p} = \frac{u_i^*}{\beta}\frac{\partial w}{\partial p} \qquad (2.25)$$

And, similarly, equation (2.26) expresses Sp

$$s_p = \frac{\partial P_f}{\partial P} = -\phi(-\beta)\frac{u_i^*}{\beta}\frac{\partial w}{\partial P} \qquad (2.26)$$

Using Eq. 2.25, the sensitivity of the mean and standard deviation of random variables Xi can be calculated using equation (2.26).

$$s_{?i} = \frac{\partial P_f}{\partial \mu_i} = -\phi(-\beta)\frac{u_i^*}{\beta}\frac{\partial w}{\partial \mu_i} \qquad (2.27)$$

And also equation (2.28).

$$s_{\sigma i} = \frac{\partial P_f}{\partial \sigma_i} = -\phi(-\beta)\frac{u_i^*}{\beta}\frac{\partial w}{\partial \sigma_i} \qquad (2.28)$$

2.6.1 Sensitivity Analysis Algorithm

In this section, a step-by-step algorithm is presented to facilitate a sensitivity analysis process. To begin with, it is required to identify input variables and determine their statistical characteristics, including means and standard deviations. In addition, a performance function should be determined as a function of involved variables. This part of modeling should be completed before the main calculation. Furthermore, following the steps listed below, sensitivity derivatives are estimated according to Shadab& Huang(2019) as:

i. Step 1. Derivatives of performance function with respect to involved variables are evaluated at the mean values.

ii. Step 2. Performance function is evaluated at the mean value. Additionally, the standard deviation of performance function, σ_G, is estimated

iii. Step 3. Safety index, β, and probability of failure, P_f, are calculated

iv. Step 4. Derivatives of safety index with respect to the mean of the involved variables, $\partial\beta/\partial\mu_x$, are calculated

v. Step 5. Derivatives of failure probability with respect to the mean of the involved variables, $\partial P_f/\partial\mu_x$, are estimated

vi. Step 6. Derivatives of safety index with respect to the standard deviation of the involved variables, $\partial\beta/\partial\sigma_x$, are calculated

vii. Step 7. Derivatives of failure probability with respect to the standard deviation of the involved variables, $\partial P_f/\partial\sigma_x$, are approximated

CHAPTER THREE
MATERIALS AND METHODS

3.1 Preamble

In the calibration of partial safety factor for steel reinforcement bars, laboratory results were obtained from the department of Civil Engineering, Ahmadu Bello University Zaria. The laboratory result contained the necessary parameters needed for the research i.e. the yield strength of the reinforcements, ultimate strength, the diameter, and elongation.

3.2 Method of Statistical Data Analysis

The data's were analyzed statistically using Microsoft Excel, EasyFit, and SPSS (Statistical Package for Social Sciences) version 23.

3.2.1 Normality Test and Outliers

The data were analysed to test for normality and identify outlierswhile the Log-Pearson Type III method of outlier detection was adopted for this research because this method can be used to detect both low and high outliers. The high and low outliers are determined with the following equations, respectively:

$$Y_L = \acute{Y} + K_N S_Y \quad (3.1)$$

$$Y_L = \acute{Y} - K_N S_Y \quad (3.2)$$

Where $Y_L$is the log of high or low outlier limit,$\acute{Y}$ is the mean of the log of the sample flows, S_Y is the standard deviation of the logs of the sample flows, and K_N is the critical deviate of the data set.

3.2.2 Kolmogorov–Smirnov One-Sample Test (Goodness of Fit)

The data were subjected to Kolmogorov–Smirnov Test to determine the Probability Density and Cumulative Distribution Function of the data before been used for the reliability analysis, and the data's were fitted using EastFit software.

The Kolmogorov Smirnov (K-S) test was adopted in this study compared to the Anderson Darling and the Chi-square tests because;

1. Kolmogorov Smirnov test do not depend on the specific distribution being tested.
2. Kolmogorov Smirnov test is an exact test. This has overcome the disadvantage of Chi-square test, in which sufficient sample size is required for the Chi-square approximation to be valid.

The Probability Density and Cumulative Distribution Function for a Log-normal distribution are given by equation (3.3) and (3.4) respectively.

$$f_{\sigma F}(y) = \frac{1}{\sqrt{2\pi y \varepsilon}} exp\left[-0.5\left(\frac{\ln y - \gamma}{\varepsilon}\right)^2\right] \qquad 0 \leq y < \infty \qquad (3.3)$$

$$F_{\sigma F}(y) = \int_{-\infty}^{y} f_{\sigma f}(u)du = \Phi\left(\frac{\ln x - \gamma}{\varepsilon}\right) \qquad (3.4)$$

The Probability Density and Cumulative Function for a Gumbel distribution are given by equation (3.5) and (3.6) respectively.

$$f_Q(x) = a\exp[-a(x-b) - e^{-a(x-b)}] \qquad (3.5)$$

$$F_Q(x) = \exp\{ - e^{-a(x-b)}\} \qquad (3.6)$$

The hypothesis is that "The data follow a specified distribution". The hypothesis regarding the distributional form is rejected if the test statistics is greater than the critical value and accepted if it's lower than the critical value.

3.2.3 Coefficient of variability (CoV)

The coefficient of variation was determined by dividing the standard deviation of the various material properties by its mean given by the following equation;

$$\mathbf{CoV} = \frac{\sigma}{\square} \qquad (3.7)$$

3.2.4 One Sample T-test

The One Sample *t* Test determines whether the sample mean is statistically different from a known or hypothesized population mean. However, for this research, the one sample T-test was used to determine if the mean diameter and elongation of the steel bars deviates significantly from standard. The one sample T-test equation is given by equation (3.8).

$$t = \frac{\bar{x} - \mu}{\sqrt{s^2/n}} \qquad (3.8)$$

Where $\bar{x}$ is the sample mean

s^2 is the sample variance

n is the sample size

μ is the population mean and t is a Student t-quartile with n-1 degrees of freedom

3.3 Method of Reliability Analysis

The reliability analysis was done using COMREL (Component Reliability Software 10.0). In the reliability analysis, the reliability index of the First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and Monte-Carlo simulation method was performed and compared. After computing the reliability index, the probability of failure as well as the partial safety factor was also derived.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Statistical Parameters of Reinforcement

This section of the chapter discusses about the statistical parameters of steel reinforcement bars based on the data obtained from the Department of Civil Engineering, ABU Zaria. The data collected was from 2009-2018 and it contains information about the mean diameter, area, yield strength, ultimate strength and elongation of steel reinforcement bars.

Table 4.1: Statistical Parameters of Steel reinforcement bars (2009 - 2018)

Bar size	Diameter (mm)	Area (mm^2)	Yield Strength (N/mm^2)	Ultimate Strength (N/mm^2)	Elongation (%)
10	9.68	73.57	504.40	623.66	11.89
12	11.40	102.26	483.40	624.11	12.92
16	15.35	185.33	483.55	620.73	12.57
20	19.18	289.42	465.38	603.41	14.60
25	24.28	463.61	497.95	645.27	14.48
Total	1150	1150	1150	1150	1150

(Source: Civil Engineering Dept., ABU Zaria, 2009– 2018)

The result of the analysis from Table 4.1 shows that the mean diameter of size 10mm reinforcement is 9.68mm with a mean area of 73.57mm^2, mean yield strength of 504.40N/mm^2, mean ultimate strength of 623.66N/mm^2 and mean elongation of 11.89%. However, the T-test analysis in (Appendix A1 and A2) shows that the mean diameter of 9.68mm is significantly lower than the standard bar size of 10mm (i.e. Sig-value < 0.05) with a mean difference of 0.013and the mean elongation of bar size 10mm which is 11.89 is significantly lower than 14% standard recommended by ISO 6935-2:2007 and BS4449:1997-2005.

The result also shows that the mean diameter for diameter 12mm bars is 11.40mm with a mean area of 102.26mm^{2}and a mean yield strength of 483.40N/mm^2, mean ultimate strength of 624.11N/mm^2 and mean elongation of 12.92%. However, the T-test analysis in (Appendix A1 and A2) shows that the mean diameter of 11.40mm is significantly lower than the standard bar size of 12mm (i.e. Sig-value < 0.05) with a mean difference of 0.602 and the mean elongation of bar size 12mm which is 12.92 is significantly lower than 14% standard recommended by ISO 6935-2:2007 and BS4449:1997-2005.

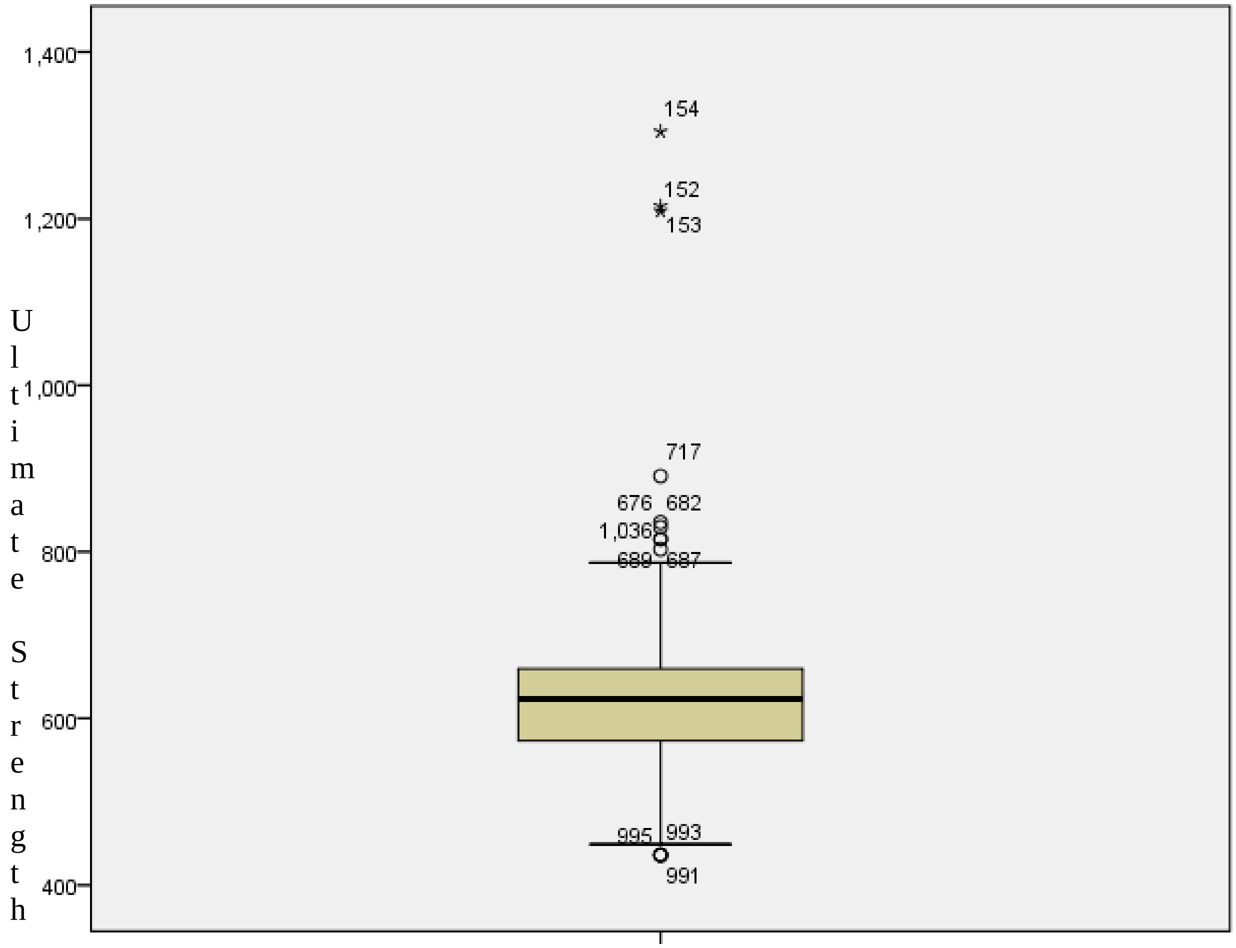
In terms of diameter 16mm steel bars, the result of the analysis shows that the mean diameter is 15.35mm with a mean area of 185.33mm^2 and mean yield strength of 483.55N/mm^2, mean ultimate strength of 620.73N/mm^2 and mean elongation of 12.57%. However, the T-test analysis in (Appendix A1 and A2) shows that the mean diameter of 15.35mm is significantly lower than the standard bar size of 16mm (i.e. Sig-value < 0.05) with a mean difference of 0.652 and the mean elongation of bar size 16mm which is 12.57 is significantly lower than 14% standard recommended by ISO 6935-2:2007 and BS4449:1997-2005.

Also, the mean diameter of size 20mm steel reinforcement bar is 19.18mm with a mean area of 289.42mm^{2}and a mean yield strength of 465.38N/mm^2, mean ultimate strength of 603.41N/mm^2 and mean elongation of 14.60%. However, the T-test analysis in (Appendix A1 and A2) shows that the mean diameter of 19.18mm is significantly lower than the standard bar size of 20mm (i.e. Sig-value < 0.05) with a mean difference of 0.820 and the mean elongation of bar size 20mm which is 14.60 is significantly higher than 14% standard recommended by ISO 6935-2:2007 and BS4449:1997-2005.

Finally from Table 4.1, the result of the analysis shows that the mean diameter for size 25mm steel bar is 24.28mm with a mean area of 463.61mm^2 and mean yield strength of 497.95N/mm^2, mean ultimate strength of 645.27N/mm^2 and mean elongation of 14.48%. However, the T-test analysis in (Appendix A1 and A2) shows that the mean diameter of 24.28mm is significantly lower than the standard bar size of 25mm (i.e. Sig-value < 0.05) with a mean difference of 0.717 and the mean elongation of bar size 25mm which is 14.48 is significantly higher than 14% standard recommended by ISO 6935-2:2007 and BS4449:1997-2005.

4.2 Check for Outliers

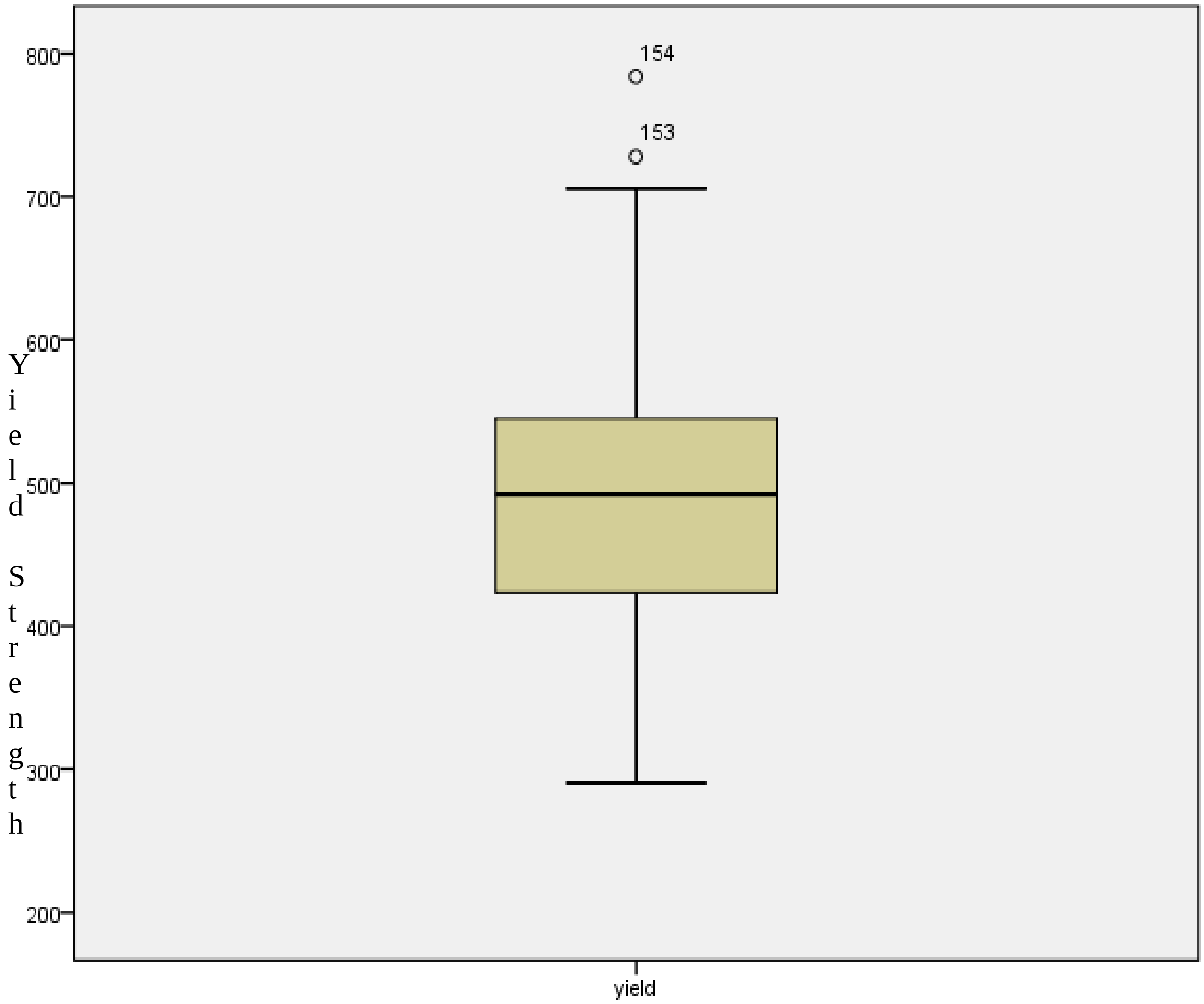
This section of the chapter checks for extreme lower and higher values (outliers) within the statistical parameters of the steel reinforcement bars i.e. steel diameter, steel yield strength and steel ultimate strength. The presence of these outliers can be as a result of typographical error and other factors which can ultimately affect the mean, standard deviation, Probability Density Function (PDF) and other significant parameters in the course of the analysis.



Outliers Box-Plot

Figure 4.1: Outliers in Ultimate Strength of the Reinforcement

The result of the analysis from Figure 4.1 shows that there are 14 cases of extreme lower and high values (outliers) in the ultimate strength of reinforcement having values ranging from 436.0N/mm^2 to 1304.6N/mm^2 (Appendix B).



Outliers Box-Plot

Figure 4.2: Outliers in the Yield Strength of Steel reinforcement bars

The result of the analysis from Figure 4.2 shows that there are 2 cases of extreme higher outliers present in the reinforcement yield strength with values of783.90N/mm^2 and 727.90N/mm^2. See Appendix B for tabulated results.

However, after identifying outliers in the data set, the sample size was reduced from 1150 to 1138, i.e. 1150-16 cases of outliers (Appendix B has full list of Outliers).

4.3 Probability Distribution Function (PDF)

In determining the Probability Density Function (PDF) of the parameters, the following hypotheses are used at a P-value of 0.05. The null and the alternative hypotheses are:

- ❖ H_0: the data follow the specified distribution;
- ❖ H_A: the data does not follow the specified distribution

Also, based on the recommendations from the EuroCode and JCSS, the resistance parameters were modeled with the Log-normal, Gumbel, and Weibull distributions while load parameters were modeled with the Log-normal, Normal and Gumbel distributions.

Furthermore, the Kolmogorov-Smirnov (KS) goodness of fit test was preferred to the Anderson-Darling (AD) and Chi-Square test due to its wide applicability and acceptance.

4.3.1 Steel Reinforcement Bars Diameter PDF

The distribution of the steel reinforcement bars diameter was modeled with the Log-normal, Gumbel, and Weibull PDF’s, and from the result of the analysis, the lognormal distribution significantly fits with the steel reinforcement bars diameter with a P-value greater than 0.05 (i.e. P-value = 0.790) compared to the Gumbel and Weibull distributions. Hence the null hypothesis is retained, and the statistics value indicates that the steel reinforcement bars diameter fits the lognormal distribution at 0.05 – 0.01 significant levels since the statistics value is 0.0197 (Appendix D).

The diagram and parameters of the log-normal distribution with respect to the steel reinforcement bars diameter are shown in Figure 4.3.

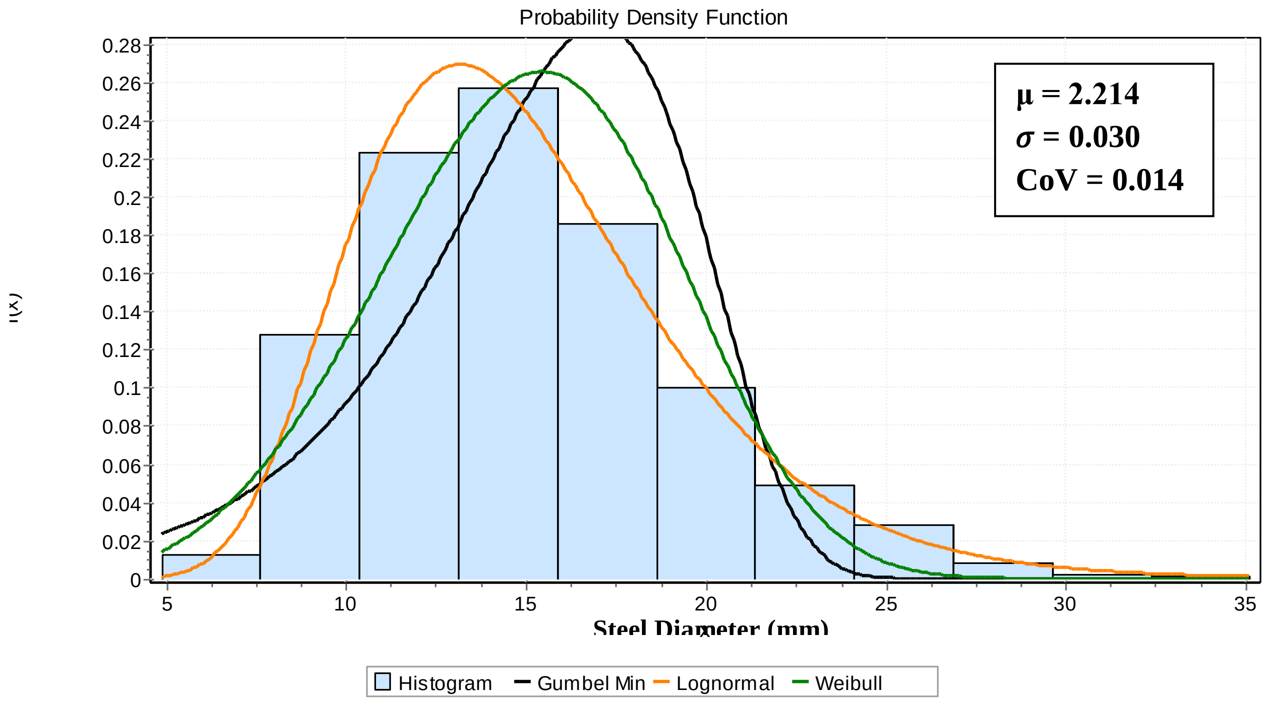


Figure 4.3: Diameter of Steel Reinforcement BarsPDF

From Figure 4.3, it can be seen that the Log-normal PDF satisfactorily fits with the histogram which is the distribution of steel reinforcement bars diameter with a mean value of 2.214, standard deviation of 0.030 and coefficient of variation of 0.014.

4.3.2 Steel Reinforcement BarsYield Strength PDF

The distribution of reinforcement yield strength was also modeled with the Log-normal, Gumbel, and Weibull PDF's, and from the result of the analysis, the Gumbel distribution significantly fits with the reinforcement yield strength with a P-value greater than 0.05 (i.e. P-value = 0.918) compared to the Log-normal and Weibull distributions. Hence the null hypothesis is retained, and the statistics value indicates that the steel reinforcement barsyield strength fits the Gumbel distribution at 0.05 to 0.01 significant levels since the statistics value is 0.017 (Appendix D).

The diagram and parameters of the Gumbel distribution with respect to the steel reinforcement bars yield strength are shown in Figure 4.4

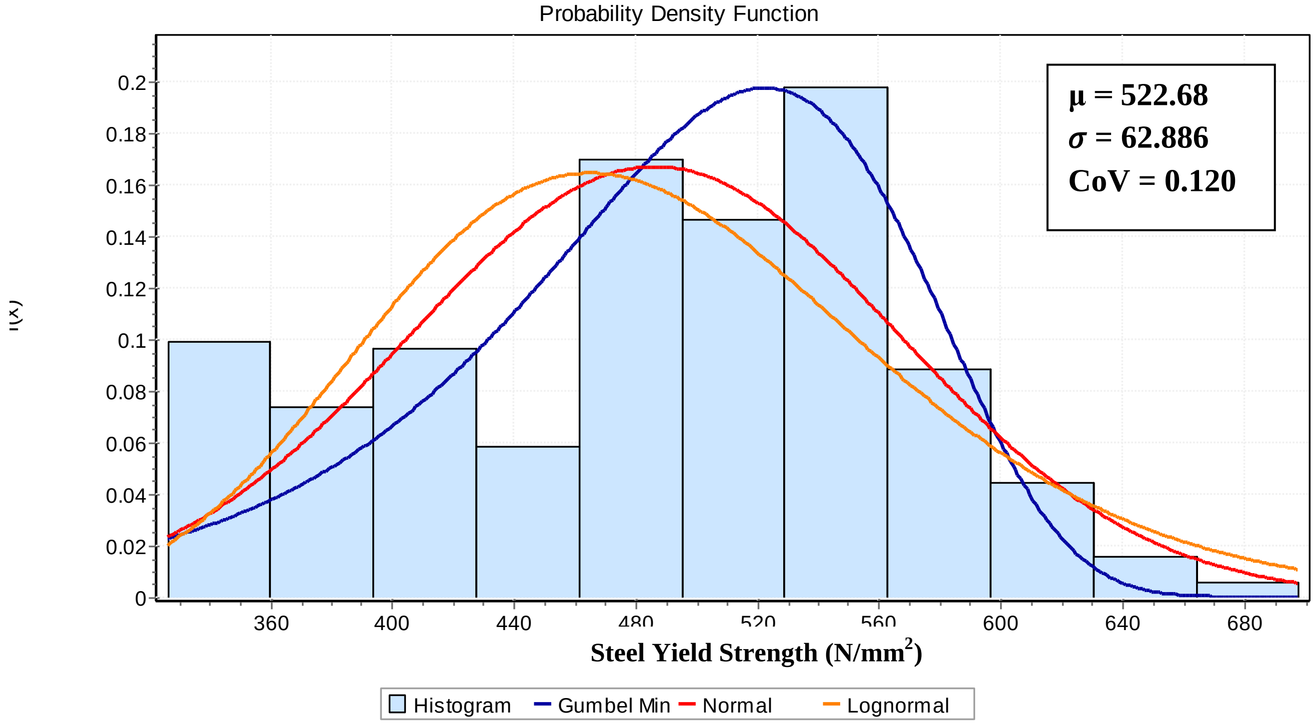


Figure 4.4: Yield Strength of Steel reinforcement bars PDF

From Figure 4.4, it can be seen that the Gumbel(Min) PDF significantly fits with the histogram which is the distribution of steel reinforcement bars yield strength with a mean value of 522.68N/mm^2, standard deviation of 62.886N/mm^2, and coefficient of variation of 0.120.

4.3.3 Steel Reinforcement BarsUltimate Strength PDF

Finally, the distribution of reinforcement ultimate strength was modeled with the Normal, Gumbel, and Weibull PDF's, and from the result of the analysis, the Gumbel distribution significantly fits with the reinforcement yield strength with a P-value greater than 0.05 (i.e. P-value = 0.709) compared to the Normal and Weibull distributions. Hence the null hypothesis is retained, and the statistics value indicates that the steel reinforcement barsultimate strength fits the Gumbel distribution at 0.05 to 0.02 significant levels since the statistics value is 0.02 (Appendix D).

The diagram and parameters of the Gumbel distribution with respect to the steel reinforcement bars ultimate strength are shown in Figure 4.5.

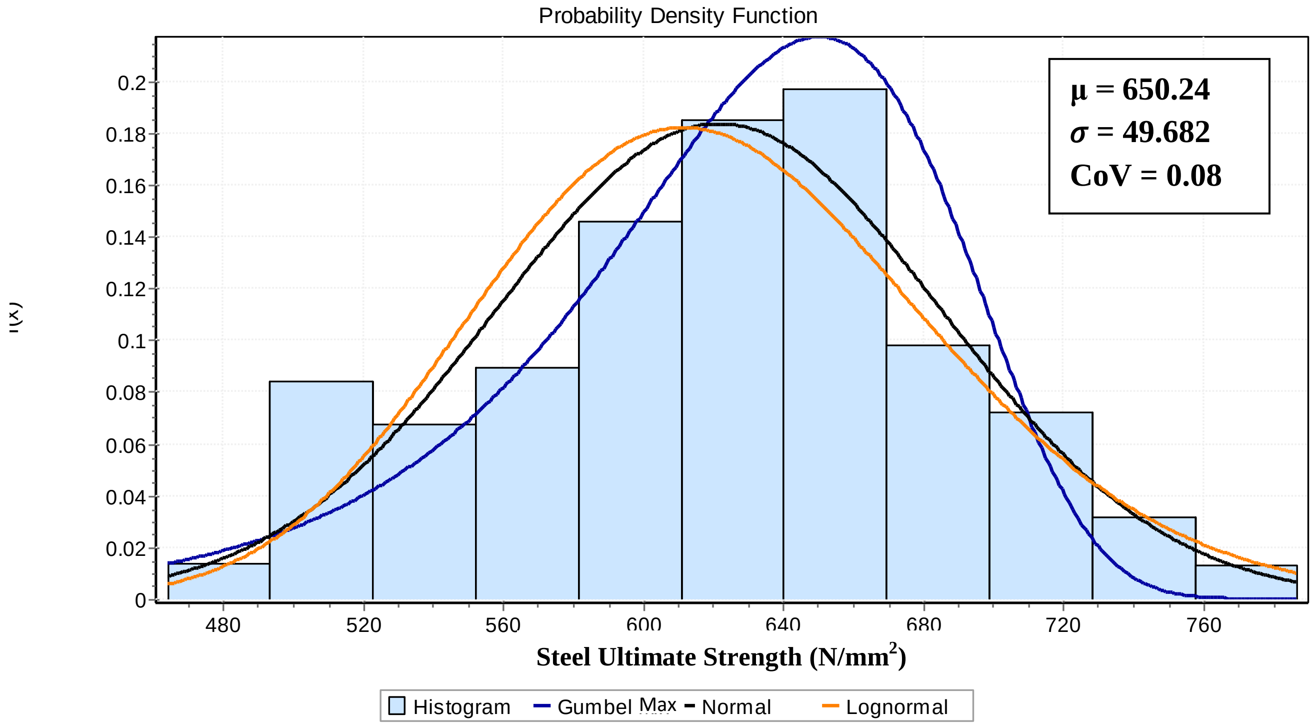


Figure 4.5: Ultimate Strength of Steel Reinforcement Bars PDF

From Figure 4.5, it can be seen that the Gumbel (max) PDF fits significantly with the histogram which is the distribution of steel reinforcement bars ultimate strength with a mean value of 650.24N/mm^2, standard deviation of 49.682N/mm^2, and coefficient of variation of 0.08.

4.4 Statistical Parameters for Reliability Analysis

In the reliability analysis of a component system i.e. steel reinforcement bars, the resistance and the load action are reduced and increased to the 5th and 95th percentile respectively according to the procedures described in (EN, 1990). Hence, the steel reinforcement bars diameter and reinforcement yield strength were reduced to the 5th percentile and the ultimate strength is increased to the 95th percentile.

Table 4.2: 5th and 95th Percentile of Resistance and Load Action

Parameters	Mean	Standard Deviation	Percentile 05	Percentile 95
Diameter	2.214	0.03	2.18	-
Yield Strength	522.68	62.886	411.42	-
Ultimate Strength	650.24	49.682	-	734.79

The result of the analysis from Table 4.2 shows that the 5th percentile of the resistance parameters i.e. diameter and yield strength are 2.18 (8.9mm) and 411.42N/mm^2 respectively and the 95th percentile of the load action i.e. ultimate strength is 734.79N/mm^2. From the result, since the log-normal distribution undergoes a transformation, the real value must be transformed back to a normal value that can be used or accessed in real life situation. Hence, the log-normal transformation of value 2.18mm is 8.9mm of the steel reinforcement bars diameter. However, the value of the Gumbel distribution remains the same since it undergoes no transformation.

Table 4.3: Parameters for Reliability Analysis

Parameters	Design Values	COV	Std. Deviation	PDF
Diameter	2.18	0.014	0.03	Log-Normal
Yield Strength	411.42	0.12	62.89	Gumbel (Min)
Ultimate Strength	735.0	0.08	49.68	Gumbel (Max)

The result of the analysis from Table 4.3 shows that the resistance design values to be used for the reliability analysis with respect to the steel diameter and yield strength are 2.18mm and 411.2N/mm^2 with standard deviation and coefficient of variation of 0.03mm, 62.89N/mm^2 and 0.014, 0.12 respectively, while the steel reinforcement bars diameter follows a Log-Normal PDF, the reinforcement yield Strength follows a Gumbel (Min) PDF. Also, the value to be used for the load action i.e. ultimate yield strength is 735N/mm^{2}with standard deviation of 49.68N/mm^2 and coefficient of variation of 0.08 having a Gumbel (Max) distribution.

4.5 Performance Function for Reliability Analysis

The reliability index and probability of failure were obtained using COMREL (Component Reliability Analysis) software version 10.0. In the reliability analysis, the following limit state function was used;

$$\text{FLIM} = \frac{\pi d^2}{4} . f_y - L \qquad (4.1)$$

Where FLIM is the Limit state function

d is the diameter of the steel reinforcement bars

f_y is the reinforcement yield strength and

L is the reinforcement ultimate yield strength

The reliability analysis was done for the First Order Reliability Method (FORM) and Second Order Reliability Method (SORM) as tabulated in Table 4.4.

Table 4.4: Reliability Analysis

Method	Reliability Index	Probability of Failure
FORM	6.370	9.53E-11
SORM	6.389	8.38E-11

The result from Table 4.4 shows that the reliability index for the FORMand SORM reliability analysis are 6.370 and 6.389 with a probability of failure of 9.53E-11 and 8.38E-11 respectively which is greater than the target reliability index value of 3.8 for a 50 years reference period with a probability of failure of approximately 7.2×10^{-5}. Since the Eurocode (EN, 1990) specifies that if the calculated failure probability for a given limit state function is higher or lower than a pre-set target value, then it should be calibrated. Hence, the design values have to be calibrated to meet the target reliability index value and obtain an appropriate safety factor. However, in order to adjust the design variables to satisfactorily meet a pre-set target value, there is need to perform sensitivity analysis in order to determine how the increase/decrease of parameters used in the limit state equation affects the reliability index value.

4.6 Sensitivity Analysis

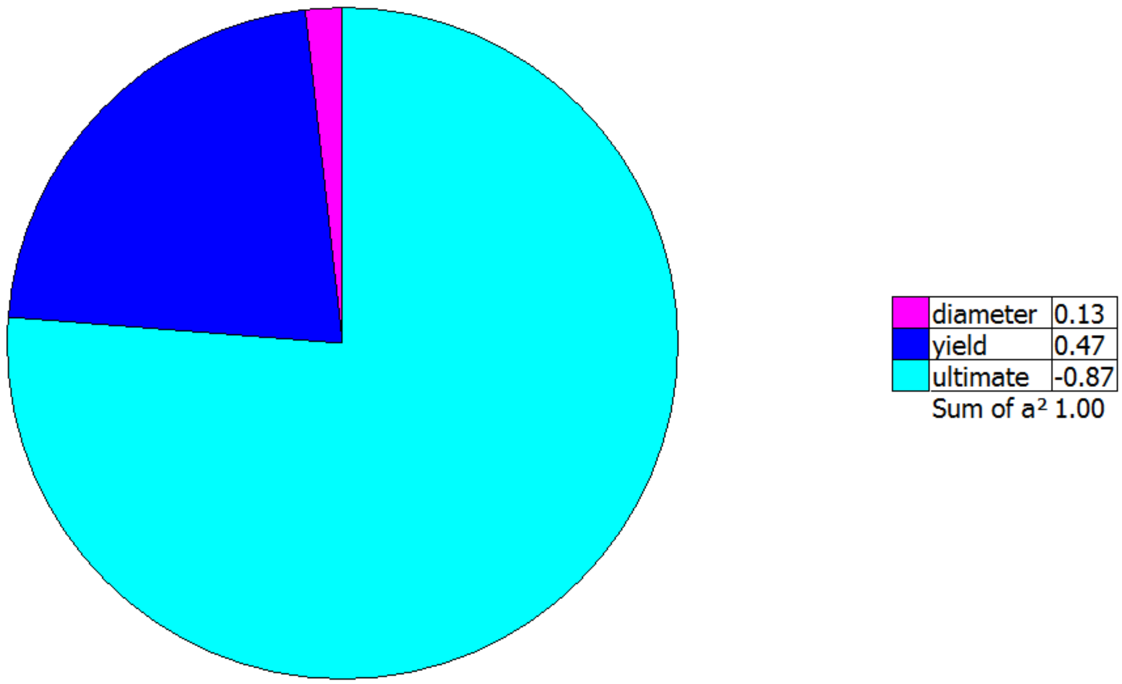


Figure 4.6: Steel Reinforcement Sensitivity Analysis

The result of the sensitivity analysis shows that the steel diameter have a sensitivity factor of +0.13, the ultimate yield strength have a sensitivity factor of +047 and the steel ultimate strength have a sensitivity factor of -0.87. However from the result, it is observed that the resistance parameters are positive while the load parameter is negative, which indicates that if the values of the resistance parameters increases (diameter and yield strength), the reliability index increases and if the values of the load parameter reduces (ultimate strength), the value of the reliability index increases and vice-versa (Appendix C). The outcome of the sensitivity analysis shows that the steel reinforcement barsultimate and yield strengthvalues are the most sensitive parameter that can significantly influence the increase and decrease of the reliability index value (target). However, these values cannot be reduced by judgment alone since it will be a deterministic approach. Hence, the design values are going to be calibrated based on the procedures given in the (EN, 1990).

4.7 Calibration of Partial Safety Factor

According to the Eurocode (EN 1990:2002), the target reliability index value for ultimate limit state of buildings over a 50 years design working life should be taken as $\beta_T = 3.8$. Hence, it is expected that calibration of safety factor for various design situations should have a target reliability index value approximately ≥ 3.8. Since the reliability index value obtained in this study is 6.4 which is greater than 3.8, the design values for the load and resistance parameters will be calibrated according to procedures described in the EuroCode (EN, 1990).

4.7.1 Calibration of Design Values of Steel reinforcement bars

The design values are calibrated such that the probability of having a more unfavorable value is as follows;

$$P(E > E_d) = \Phi(+\alpha_E \beta) \qquad (4.2)$$

$$P(R \leq R_d) = \Phi(+\alpha_R \beta) \qquad (4.3)$$

Where β is the target reliability index.

α_E and α_R, with $|\alpha| \leq 1$, are the values of the sensitivity factors.

The design value for a material property having various distributions according to Eurocode (EN 1990:2002) is given in Table 4.5; hence, the design values will be calibrated using these equations

Table 4.5: Design Values Equation for Various Parameters(EN 1990:2002)

Distribution	Design Values
Normal	$\mu - \alpha\beta\sigma$
Log-normal	$\mu \exp(-\alpha\beta V)$ for $V = \sigma/\mu < 0.2$
Gumbel	$u - \frac{1}{a}\ln\{-\ln\Phi(-\alpha\beta)\}$ where $u = \mu - \frac{0.577}{a}$; $a = \frac{\pi}{\sigma\sqrt{6}}$

Note: μ, σ, and V are the mean, standard deviation, and coefficient of variation

4.7.1.1 Calibration of Diameter Design Value (D_d)

The design value for steel reinforcement bars diameter with a mean value of 2.214mm as shown in Figure 4.3 having a log normal distribution is calculated with the equation given in Table 4.5 since the coefficient of variation is less than 0.2 i.e. 0.014 (Figure 4.3).

$$\text{Hence, } D_d = \mu \exp(-\alpha\beta V) \qquad (4.4)$$

Where, $\beta = 3.8$ for a 50 years reference period

α_R = sensitivity factor for resistance = 0.8 (EN 1990:2002)

However, the value of $\alpha_R = 0.8$ holds only if $0.16 < \sigma_E/\sigma_R < 7.6$; Therefore, $\frac{\sigma_E}{\sigma_R} = \frac{49.68}{0.03} = 1656$, since 1656> 7.6, it follows that α = 0.8 is not justified. Hence, α = ± 0.4 since the steel diameter has the smallest standard deviation compared to the yield strength (EN 1990:2002).

Therefore, $D_d = 2.214 \exp(-0.4 \times 3.8 \times 0.014)$

$= 2.214 \times 0.9789 = 2.167\text{mm}$

4.7.1.2 Yield Strength Design Value (F_y)

The design value for reinforcement yield strength with a mean value of 522.68N/mm^2 as shown in Figure 4.4 having a Gumbel distribution is calculated as expressed in equation (4.5).

$$F_d = u - \frac{1}{a}\ln\{-\ln\Phi(-\alpha\beta)\} \qquad (4.5)$$

Where $a = \frac{\pi}{\sigma\sqrt{6}}$; and $u = \mu - \frac{0.577}{a}$

Also, $\beta = 3.8$ for a 50 years reference period

α_R = sensitivity factor for resistance = 0.8 (EN 1990:2002)

However, the value of $\alpha_R = 0.8$ holds only if $0.16 < \sigma_E/\sigma_R < 7.6$; Therefore, $\frac{\sigma_E}{\sigma_R} = \frac{49.68}{62.89} = 0.79$, since $0.16 < 0.79 < 7.6$, it follows that $\alpha = 0.8$ is justified.

Also, $a = \frac{\pi}{62.89\sqrt{6}} = 0.02$ and $u = 522.68 - \frac{0.577}{0.02} = 493.83$

Putting the value of (a) and (u) in equation 4.5, we have;

$$F_y = 493.83 - \frac{1}{0.02}\ln\{-\ln\Phi(-0.8 * 3.8)\}$$

$$493.83 - 50\ \ln\{-\ln\Phi(-3.04)\}$$

$$493.83 - 50\ \ln\{-\ln(0.00118)\}$$

$$493.83 - 50\ \ln\{6.74\}$$

$$493.83 - 50 * (1.91)$$

$$493.83 - 95.5 = 398.33$$

F_y = 398.33N/mm^2

4.7.1.3 Ultimate Strength Design Value (F_u)

The design value for reinforcement ultimate strength with a mean value of 650.24N/mm^2 as shown in Figure 4.5 having a Gumbel distribution is calculated with the equation given in Table 4.5. The calculation is expressed as shown in equation (4.6).

$$F_d = u - \frac{1}{a}\ln\{-\ln \Phi\,(-\alpha\beta)\} \qquad (4.6)$$

Where $a = \frac{\pi}{\sigma\sqrt{6}}$; and $u = \mu - \frac{0.577}{a}$

Also, β = 3.8 for a 50 years reference period

α_R = sensitivity factor for Load = -0.7 (EN 1990:2002)

However, the value of α_R = -0.7 holds only if $0.16 < \sigma_E/\sigma_R < 7.6$; Therefore, $\frac{\sigma_E}{\sigma_R} = \frac{49.68}{62.89} = 0.79$, since $0.16 < 0.79 < 7.6$, it follows that α = -0.7 is justified.

Also, $a = \frac{\pi}{49.68\sqrt{6}} = 0.0258$ and $u = 650.24 - \frac{0.577}{0.0258} = 627.88$

Putting the value of (a) and (u) in equation 4.6, we have;

$$F_u = 627.88 - \frac{1}{0.0258}\ln\{-\ln \Phi\,(-(-0.7) * 3.8)\}$$

$$627.88 - 38.76\ \ln\{-\ln \Phi(2.66)\}$$

$$627.88 - 38.76\ \ln\{-\ln(0.99609)\}$$

$$627.88 - 38.76\ \ln\{0.0039\}$$

$$627.88 - 38.76 * (-5.543)$$

$$627.88 + 214.84 = 842.7$$

F_u = 842.7 N/mm^2

Table 4.6: Calibrated Design Values for Reliability Analysis

Parameters	Design Values	COV	Std. Deviation	PDF
Diameter	2.167	0.01	0.03	Log-Normal
Yield Strength	398.33	0.12	62.89	Gumbel (Min)
Ultimate Strength	842.7	0.08	49.68	Gumbel (Max)

FromTable 4.6, the calibrated design values for the resistance and load parameters are 2.167 (8.73mm) for the steel diameter, 398.33N/mm^2 for the steel yield strength, and 842.7N/mm^2 for the steel ultimate strength. However, the calibrated design values were put into the limit state equation of (equation 4.1) to obtain the target reliability index value, probability of failure and partial safety factor. The reliability analysis for the calibrated values was also done for the First Order Reliability Method (FORM), and Second Order Reliability Method (SORM) as tabulated in Table 4.7.

Table 4.7: Reliability Analysis for Calibrated Design Values

Method	Reliability Index	Probability of Failure
FORM	3.805	7.08E-5
SORM	3.804	7.12E-5

The result from Table 4.7 shows that the reliability index for the FORM and SORM reliability analysis are 3.805 and 3.804 with a probability of failure of 7.08E-5 and 7.12E-5 respectively which is approximatelyequal to the target reliability index value of 3.8 for a 50 years reference period with a probability of failure of approximately $7.2x10^{-5}$. Since the calibrated values yield a reliability index value close to the target reliability index value of 3.8, the calibrated design values is adequate to yield an appropriate partial safety factor for the steel reinforcement bars.However, the partial safety factor for a probabilistic reliability based approach considers all uncertainties associated with the limit state equation as shown in Figure 4.7. Hence, the global safety factor is as a result of the combination of individual safety factors from the steel diameter, yield strength and ultimate strength.

4.8 Partial Safety Factor

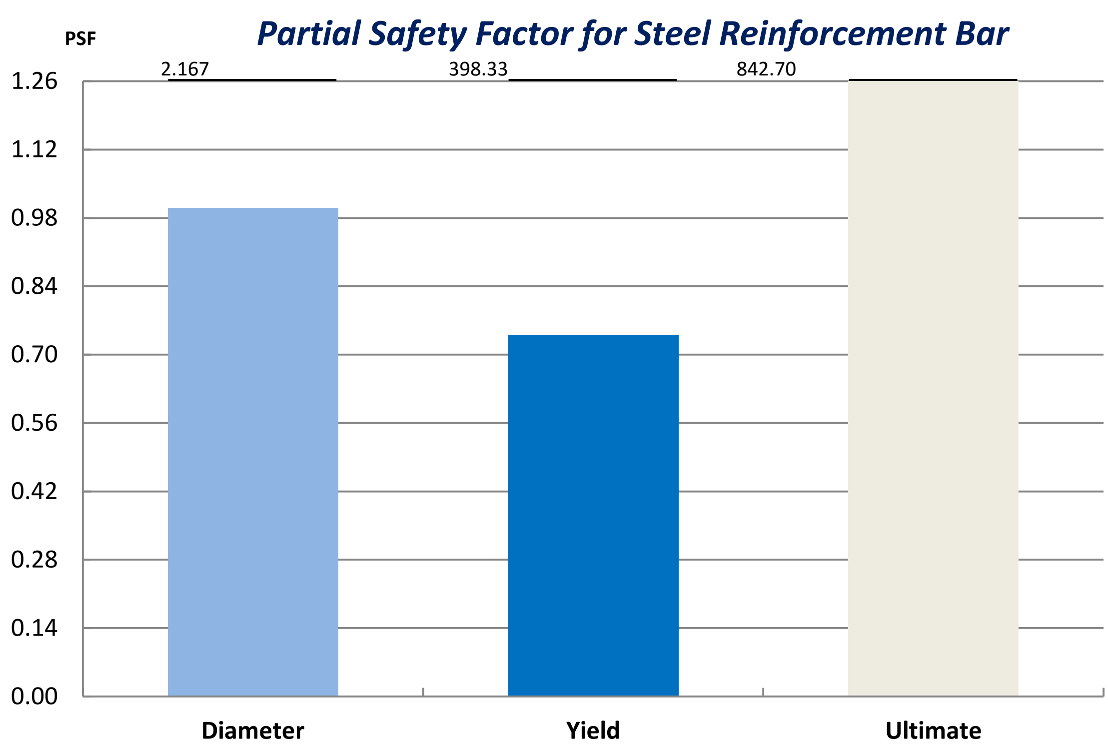


Figure 4.7: Steel reinforcement bars Partial Safety Factor

The partial safety factor is computed by multiplying the individual safety factors of the variables considered in the reliability analysis (i.e. steel diameter, steel yield strength and steel ultimate strength) as shown if Figure 4.7.

Therefore, Steel diameter safety factor = 0.74

Yield strength safety factor = 1.00

Ultimate strength safety factor = 1.26

Hence, PSF = 0.74 x 1.00 x 1.26 = 0.9324

Hence, PSF = 0.9324 or (1/0.93) = 1.07

Therefore, partial safety factor for the steel reinforcement barsshould be taken as 1.07 for load parameters and 0.93 (i.e. 1/1.07) for resistance parameters.

Since the design value for the steel reinforcement bars yield strength is approximately 398N/mm^2 and the partial safety factor is 1.07(0.93), the characteristic strength of the steel reinforcement bars is obtained by multiplying the design yield strength and the safety factor;

$f_d = \frac{f_c}{PSF} = f_c = 398 \text{ x } 1.07 = 425.86\text{N/mm}^2$.

Hence, the characteristic strength of the steel reinforcement bars should be taken as 426N/mm^2 for increased safety.

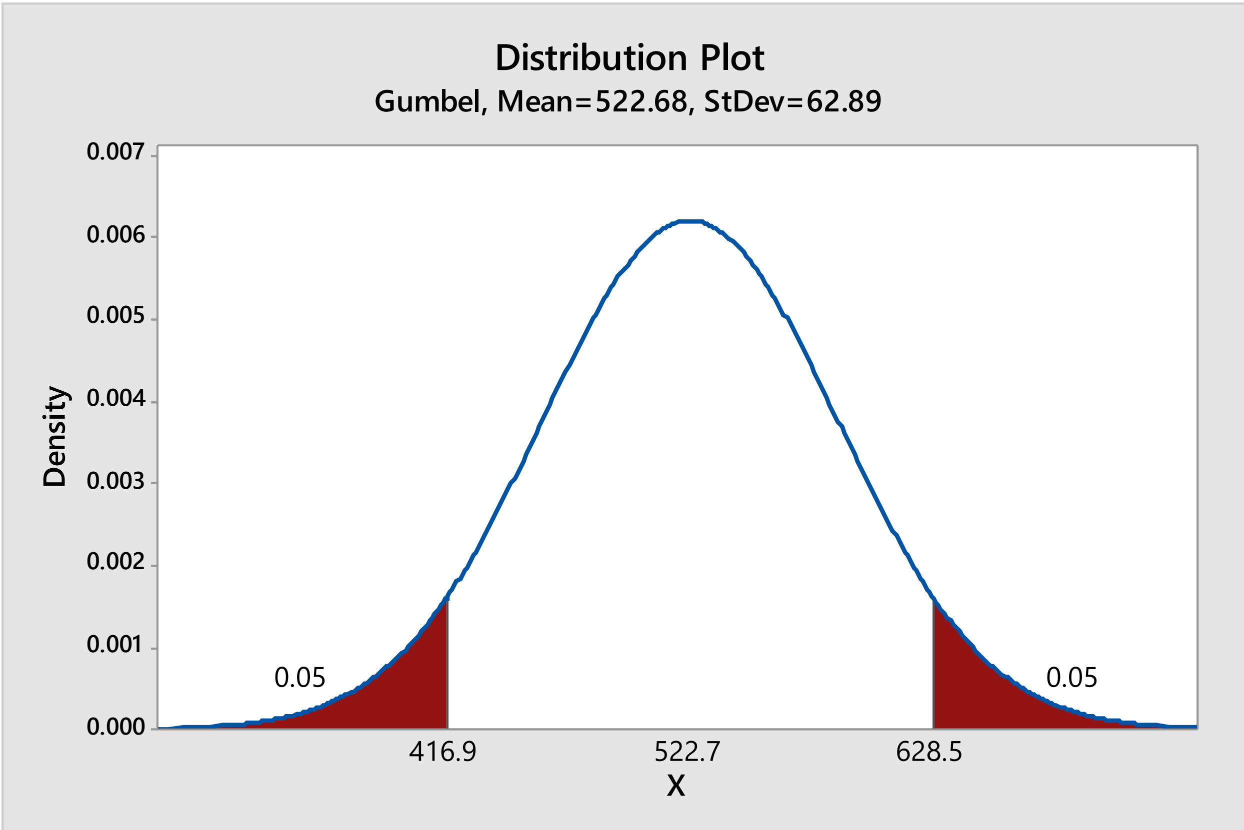


Figure 4.8: Characteristic Strength Distribution of Steel reinforcement bars

From Figure 4.8, it can be seen that 5% of the steel reinforcement bars yield strength below the mean value is less than approximately 417N/mm^2 and from the definition of characteristics yield strength which states that it must be a value below which not more than 5% of the test result is expected to fall, it shows that a characteristics value of 425N/mm^2 is justified since its greater than 417N/mm^2.

4.9 Discussion of Findings

The result of the analysis from Table 4.1 with T-test analysis in Appendix A1 shows that the mean diameter of the steel reinforcement barsare significantly lower than specified standard sizesrecommended by ISO 6935-2:2007 and BS4449:1997-2005. (i.e. 9.68<10mm, 11.40<12mm, 15.35<16mm, 19.18<20mm, 24.28<25mm). Also from the Table, the mean yield strength of different bar sizes are all tabulated with a minimum value of 326N/mm^2 and a maximum value of 697.9N/mm^2. However, the high values of the reinforcement yield strength can be attributed to influx of imported steel reinforcement bars from various parts of the world into the country while the low characteristics strength value of steel reinforcement bars can be attributed to locally produced steel from the country. The result shows that the mean elongation of the steel reinforcement bars ranges from 11.89-14.48% with a minimum and maximum value of 6.07% and 21.70% respectively. However, the implication of the outcome of the steel reinforcement bars elongation result shows that only diameter 20mm and diameter 25mm bars meets the minimum of 14% as specified by BS4449:1997-2005 and ISO 6935-2:2007. Hence, diameter 10, 12, and 16mm bars are less ductile and are significantly lower than the specified standard minimum of 14% (see Appendix A2).

The check for outliers result from Figure 4.1 and 4.2 was aimed at eliminating extreme lower and higher values which can ultimately affect the distribution of the data as well as the statistical parameters of the steel reinforcement bars. The outcome of the outlier analysis reduces the data from 1150 samples to 1134 samples, thereby eliminating 16 extreme lower and higher values that would affect the analysis of the data set.

With regard to the probability density function of the steel reinforcement bars, the result from Figure 4.3 to Figure 4.5 shows that the diameter of steel reinforcement bars significantly fits a Log-Normal distribution, the yield strength of the steel reinforcement bars significantly fits with a Gumbel (Min) distribution and the ultimate strength of the steel reinforcement bars fits significantly with a Gumbel (Max) distribution.

The result of the analysis from Table 4.2 is based on the recommendations from the Eurocode (EN1990:2002) where is states that material properties for resistance should be reduced to the 5^{th} percentile while the load action should be increased to the 95^{th} percentile. The interpretation of this is that if the material resistance property fails to meet a required value/standard due to various circumstances, such error/deficit should not be less that the 5^{th} percentile of the expected mean/standard value, and the load such a material must be able to withstand during its lifetime should be greater than the 95^{th} percentile of its intended load carrying capacity to ensure adequate safety and reliability of the material. Hence, the 5^{th} percentile of the steel reinforcement bars resistance parameters of diameter and yield strength were 2.18mm and 411.42N/mm^2 respectively, while the 95^{th} percentile value for the load parameter is 734.79N/mm^2.

The result of the analysis from Table 4.4 shows that the reliability index for the FORM and SORM reliability analysis are 6.370, and 6.389 with a probability of failure of 9.53E-11 and 8.38E-11 respectively which is greater than the preset reliability target value of 3.8. Hence, the need for calibrating the design value in a probabilistic manner to be close to the target value in order to increase and ensure adequate safety. The implication of this finding is that if a partial safety factor associated to this reliability index is used for structural design, the structure will stand after design but will not reach its intended design life i.e. 50 years.

The result of the sensitivity analysis from Figure 4.6 shows that reinforcement ultimate strength is the most sensitive with a value of +0.87 followed by the reinforcement yield strength with a value of -0.47. The values of the sensitivity analysis have direct impact on the reliability index value and the probability of failure i.e. as the sensitivity values increases, the reliability index value increases while the probability of failure reduces and vice-versa. Hence, increased reliability index increases the safety of a structure or material property. However, with regards to steel reinforcement bars and from Figure 4.6, as the resistance parameters increases (i.e. diameter and yield strength), the reliability index increases and as the load action parameters increases (i.e. ultimate yield strength), the reliability index reduces (See Appendix C).

The calibration of partial safety factor for steel reinforcement bars was necessary since the reliability index value calculated in this research is greater than the target reliability index value recommended in the Eurocode i.e. 6.37 > 3.80. Therefore, in the calibration of the various design variables, the equations 4.2 and 4.3 ensures the probability of the design load (E_d) exceeding the normal load (i.e. ultimate strength) and the design resistance (R_d) not exceeding the normal resistance parameters in practice (i.e. diameter, and yield strength). However, the calibrated design values for the resistance parameters are 2.167 (8.7mm) for the steel diameter and 398.33N/mm^2 for the yield strength while the design value for the ultimate strength is 842.7N/mm^2 which yielded a target reliability index value of approximately 3.8 with probability of failure of 7.08E-5. The outcome of this finding shows that unlike the first reliability analysis, the design parameters after calibration reduced the resistance parameters (i.e. 398.33<411.2N/mm^2) and increased the load parameter (842.7>735.0N/mm^2) which is a clear indication of increased safety and the possibility of this design parameters satisfying the conditions of equation 4.2 and 4.3.

The partial safety factor obtained for the individual parameters (Figure 4.7) used in the limit state equation after calibrating the design values is 1.00 for the steel diameter, 0.74 for the yield strength, and 1.26 for the ultimate yield strength with a combined safety factor value of 1.07. Since the design value for the steel reinforcement bars yield strength is approximately 398N/mm^2 and the partial safety factor is 1.07(0.93), the characteristic strength of the steel reinforcement bars is obtained by multiplying the design yield strength and the safety factor for load action and dividing the design yield strength and the safety factor for resistance action. Hence, the characteristics value for the steel reinforcement bars is approximately 425N/mm^2 which is confirmed from the analysis in Figure 4.8 since 5% of the steel reinforcement bars yield strength below the mean value is less than approximately 417N/mm^2.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

i. The statistical parameters of the steel reinforcement bars tested in the laboratorywere significantly lower than standards recommendedand needs to be improved upon.

ii. The probability density function of the steel reinforcement bars follows a Log-Normal and Gumbel distribution, where the diameter follows a Log-Normal distribution, the yield strength and ultimate yield strength fits a Gumbel(Min) and Gumbel(Max) distribution respectively.

iii. The most sensitive factor ofthe steel reinforcement bars is the steel reinforcement ultimate strength followed by the steel reinforcement yield strength.

iv. The calibrated design values of the steel reinforcement bars for diameter, yield and ultimate strength are 2.167 (8.7mm), 398.33N/mm^2, 842.7N/mm^2 respectively with a reliability index value of 3.805 and probability of failure of 7.08E-5

v. The partial safety factor of the steel reinforcement barsobtained was1.07(0.93)with a characteristics value of approximately 426N/mm^2.

5.2 Recommendation

- The characteristic strength of the steel reinforcement bars should be set to 426N/mm^{2}for reinforcement bar sizes of 10mm to 25mm.
- The partial safety factor of the steel reinforcement bars should be set at 0.93 for resistance and (1.07) for loading.
- The quality of steel reinforcement bars should be improved with less variation in characteristics strength at the manufacturing stage.

REFERENCES

Akinsola, S. I., Issa, I. O., Adegoke, M. A., Ojo, V. O., and Alabi, A. B. (2016). Characterization of Low Carbon Steel Bar Used For ReinforcingAnimal House Building. *FUTY Journal of the Environment*, Vol. 10: No. 1.

Allen D. E. (1972). Statistical study of the mechanical properties of reinforcing bars tech. rep., Division of Building Research, National Research Council of Canada.

ASTM A706/A706M-16. (206). Standard Specification for Deformed and Plain Low-Alloy Steel Bars for Concrete Reinforcement. *ASTM Intern*ational, West Conshohocken.

Baji, H. (2014). The effect of uncertainty in material properties and model error on the reliability of strength and ductility of reinforced concrete members.

Balogun, S., Esezobor, D., Adeosun, S., & Sekunowo, O. (2009). Challenges of producing quality construction steel bars in West Africa: Case study of Nigeria steel industry. *Journal of Minerals & Materials Characterization & Engineering*, *8*(4), 283-292.

Beeby A W. (1999). Safety of structures, and a new approach to robustness. *The Structural Engineer*, 77, No. 4, 1999.

Beeby, A., & Jackson, P. (2016). Partial safety factor for reinforcement. In *Structures* (Vol. 5, pp. 101-111). Elsevier.

BOMEL. (2001). Probabilistic methods: Uses and abuses in structural integrity. Printed and published by the Health and Safety Executive.

BS 8110. (1997). *The Structural Use of Concrete*, Her Majesty Stationary Office, London.

BS 8110: Part 2 (1985). Structural Use of Concrete: Code of Practice for Design and Construction, British Standards Institution. London.

BS 4449:1997: Specification for Carbon Steel Bars for Reinforcement of Concrete: *British Standard Institution*.

BS 4449:2005: Steel for the reinforcement of concrete. Weldable reinforcing steel. Bar, coil and decoiled product. Specification. BSI, London, UK.

Bulleit, W. M. (2008). Uncertainty in structural engineering. *Practice Periodical on Structural Design and Construction*, *13*(1), 24-30.

Buro, B., Taszner, Z., and Torok, T. (2006). Experimental results tackling the tramp element problem in steelmaking. *Acta Metallurgica Slovaca*, vol. 12, pp. 51–1515.

Chatterjee, A. (1995). Recent Developments in iron making and steel making. *Iron and Steel making,* pp. 100-104.

Clifton, F. (1969). Strength variability in structural material. Tech. rep., Aeronautical Research Council, Structures Dept., Farnborough, London.

Ede, A. N., Egunjobi, E. O., Bamigboye, G., & Ogundeji, J. (2015). Assessment of quality of steel reinforcing bars used in Lagos, Nigeria. *International Research Journal of Innovative Engineering*, *1*(3).

Efron, B. (1979). Bootstrap methods: another look at the jackknife. Annals of Statistics, Vol 7, No. 1, 1979.

EN 1990: (2002). Basis of Structural Design. , (Transformation ENV 1991-1, 1994) Brussels.

EN 1992-1-2. (2004). Eurocode 2: Design of concrete structures-Part 1-2: General rules-structural fire design. *European Committee for Standardization*.

Faber, M. H., & Sørensen, J. D. (2002). Reliability based code calibration. *MH Faber, JD Serensen//Joint Committee on Structural Safety.-2002.-C*, 1-17.

Farrell-Singleton, P. A. (2010). Critical Values For The Two Independent Samples Winsorized T Test.

Friis-Hansen, P., & Sørensen, J. D. (2002). Reliability-based code calibration of partial safety factors. In *Joint Committee of Structural Safety, JCCS-Workshop on Code calibration*. University of Zurich.

Gu, X., Jin, X., & Zhou, Y. (2016). *Basic principles of concrete structures*. Springer Berlin Heidelberg.

ISO 6935-2:2007. (2007). Steel for the reinforcement of concrete.

ISO 2394: (1997). General Principles on Reliability for Structures. Zurich.

JCSS. (2001). JCSS: Probabilistic model code. The Joint Committee on Structural Safety.

Kankam, C. K., & Adom-Asamoah, M. (2002). Strength and ductility characteristics of reinforcing steel bars milled from scrap metals. *Materials & design*, *23*(6), 537-545.

Matousek, M. (1977). Outcome of a survey of 800 construction failures. Proc. IABSE Colloq. On Inspection and Quality Control. Swiss Federal Inst. of Technology, Zürich.

Melchers, R.E. (1999). Structural Reliability Analysis and Prediction. John Wiley & Sons, Hoboken, New Jersey.

Mirza, S.A. (1979). Variability of mechanical properties of reinforcing bars. *Journal of the Structural Division*, vol. 105, pp. 921 – 937.

Munyazikwiye, B. B. (2013). *Investigation on characterization and variability of mechanical properties of reinforcing steel bars made from scrap* (Doctoral dissertation).

Ndaliman, M. B. (2006). An assessment of mechanical properties of medium carbon steel under different quenching media. *AU JT*, *10*(2), 100-104.

Novák, D., Vořechovský, M., & Teplý, B. (2014). FReET: Software for the statistical and reliability analysis of engineering problems and FReET-D: Degradation module. *Advances in Engineering Software*, *72*, 179-192.

Osborne, J. W., & Overbay, A. (2004). The power of outliers (and why researchers should always check for them). *Practical Assessment, Research, and Evaluation*, *9*(1), 6.

Roy, R. (1996). Benefit of steel reinforcement in concrete slabs. Tech. rep., Wire Reinforcement Institute, Findlay, Ohio.

Shadab,F. M., & Huang, H. (2019). Simplified algorithm for reliability sensitivity analysis of structures: A spreadsheet implementation. *Plos one*, *14*(3), e0213199.

Shunichi, H., and Morifumi, N. (2006). Effect of Microalloying Elements on mechanical Properties of Reinforcing Bars. *ISIJ International*, *46*(10), 1510-1515.

Singh, B., & Kaushik, S. K. (2002). Influence of steel-making processes on the quality of reinforcement. *Indian Concrete Journal*, *76*(7), 407-412.

Sudret, B. (2008). Global sensitivity analysis using polynomial chaos expansions. *Reliability engineering & system safety*, *93*(7), 964-979.

Thoft-Christensen, P., & Baker, M. J. (1982). Reliability of structural systems. In *Structural Reliability Theory and Its Applications* (pp. 113-127). Springer, Berlin, Heidelberg.

Ullah, A., Ikram, N., & Salam, A. (2008). Improvement in tensile properties of a structural steel by heat treatment. *International Journal of Material Science, ISSN0973-4589, 3*(1), 8994-9004.

World Steel Association. (2012). Sustainable steel: At the core of a green economy. *Rue Colonel Bourg, 120*.

Zhang, X. (2013). Efficient Computational Methods for Structural Reliability and Global Sensitivity Analyses. Retrieved January 5, 2019, from https://uwspace.uwaterloo.ca/bitstream/handle/10012/7509/Zhang_Xufang.pdf?sequence=1.

LIST OF APPENDICES

Appendix A1: One Sample T-test for Steel reinforcement bars Diameter

Diameter (mm)	N	Mean Diameter	Std. Deviation	Mean difference	df	t	Sig
10mm	240	9.676	0.204	0.013	239	24.562	0.000
12mm	249	11.398	0.525	0.602	248	18.084	0.000
16mm	275	15.349	0.625	0.652	274	17.279	0.000
20mm	194	19.180	0.790	0.820	193	14.447	0.000
25mm	110	24.283	0.802	0.717	109	9.375	0.000

Appendix A2: One Sample T-test for Steel reinforcement bars Elongation

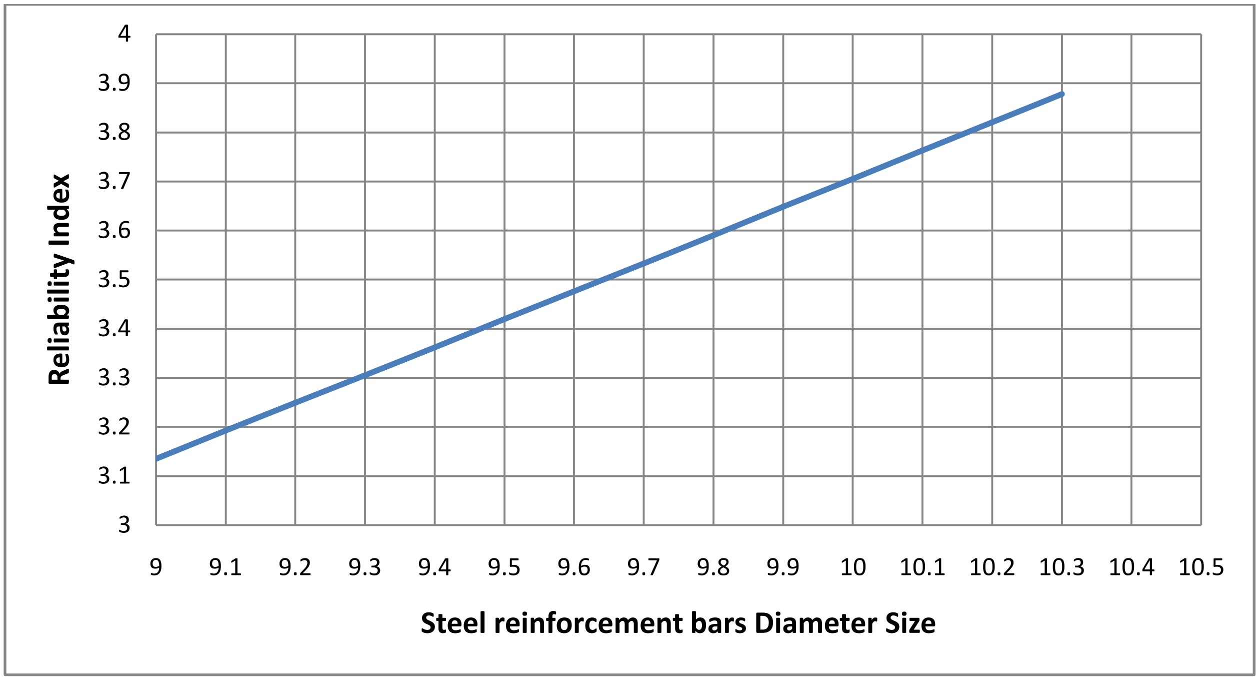
Diameter (mm)	N	Mean Elongation	Std. Deviation	Mean Difference	df	t	Sig.
10	240	11.891	3.342	2.109	239	9.777	0.000
12	249	12.925	3.314	1.075	248	5.120	0.000
16	275	12.572	2.578	1.428	274	9.185	0.000
20	194	14.602	2.879	0.603	193	2.915	0.004
25	110	14.484	2.683	0.484	109	1.890	0.061

Appendix B: Extreme Lower and Higher Outlier Values

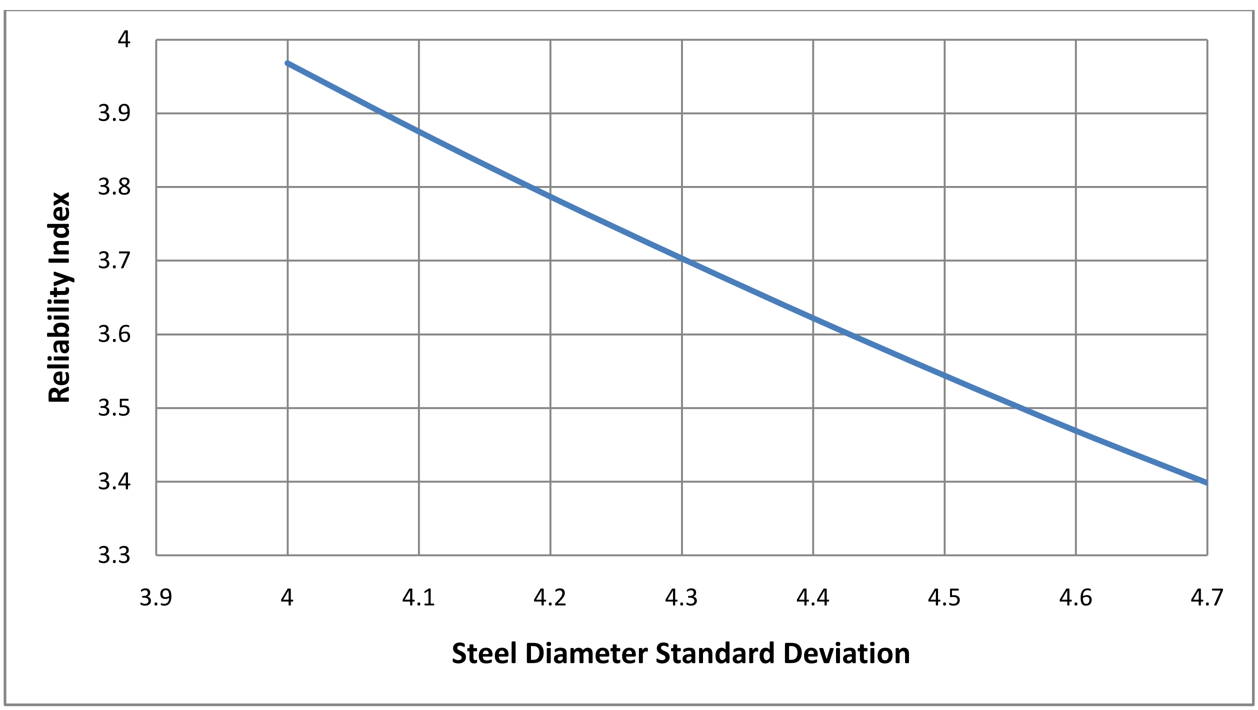
Extreme Outlier Values				
Test Variable	**Region**	**S/N**	**Case Number**	**Value**
Yield Strength	Highest	1	154	783.90
		2	153	727.90
Ultimate Strength	Highest	1	154	1304.60
		2	153	1215.00
		3	152	1209.40
		4	717	890.90
		5	676	835.50
		6	680	829.60
		7	685	815.30
		8	687	815.30
		9	1029	803.00
	Lowest	1	995	436.00
		2	994	436.00
		3	993	436.00
		4	991	436.00
		5	86	449.10

Total =	Yield strength	= 2
Total =	Ultimate strength	= 14
Grand Total =	**Total Outliers**	**= 16**

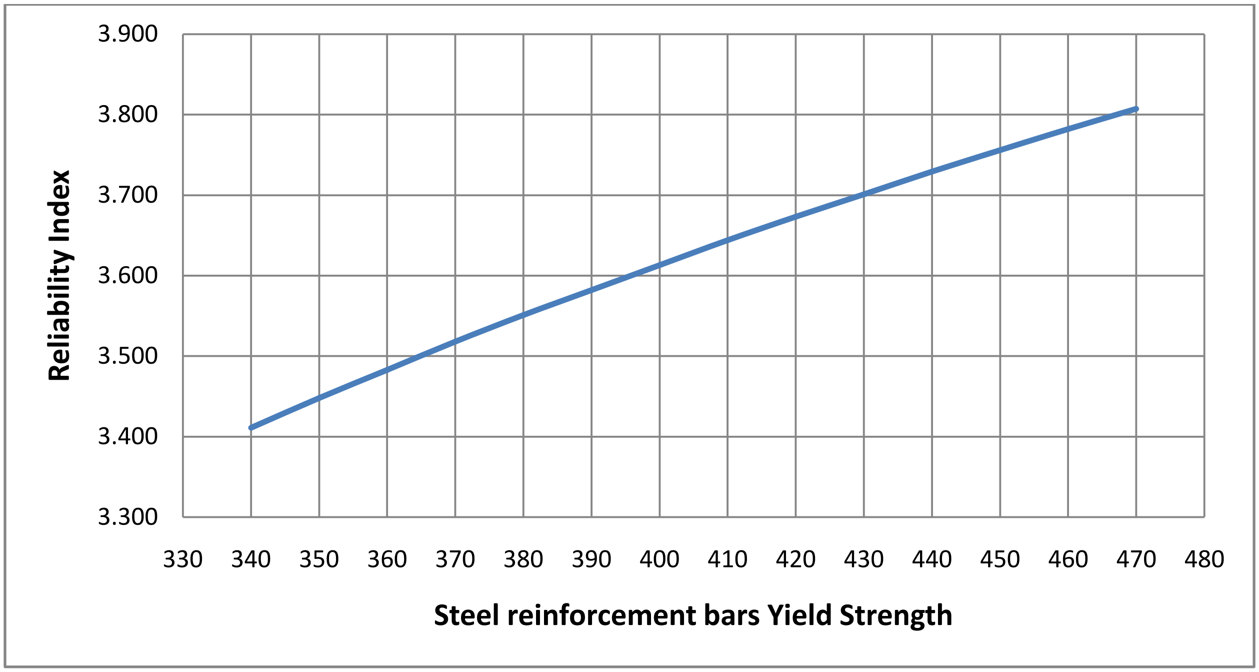
Appendix C1: Trend in Steel Diameter Size and Reliability Index



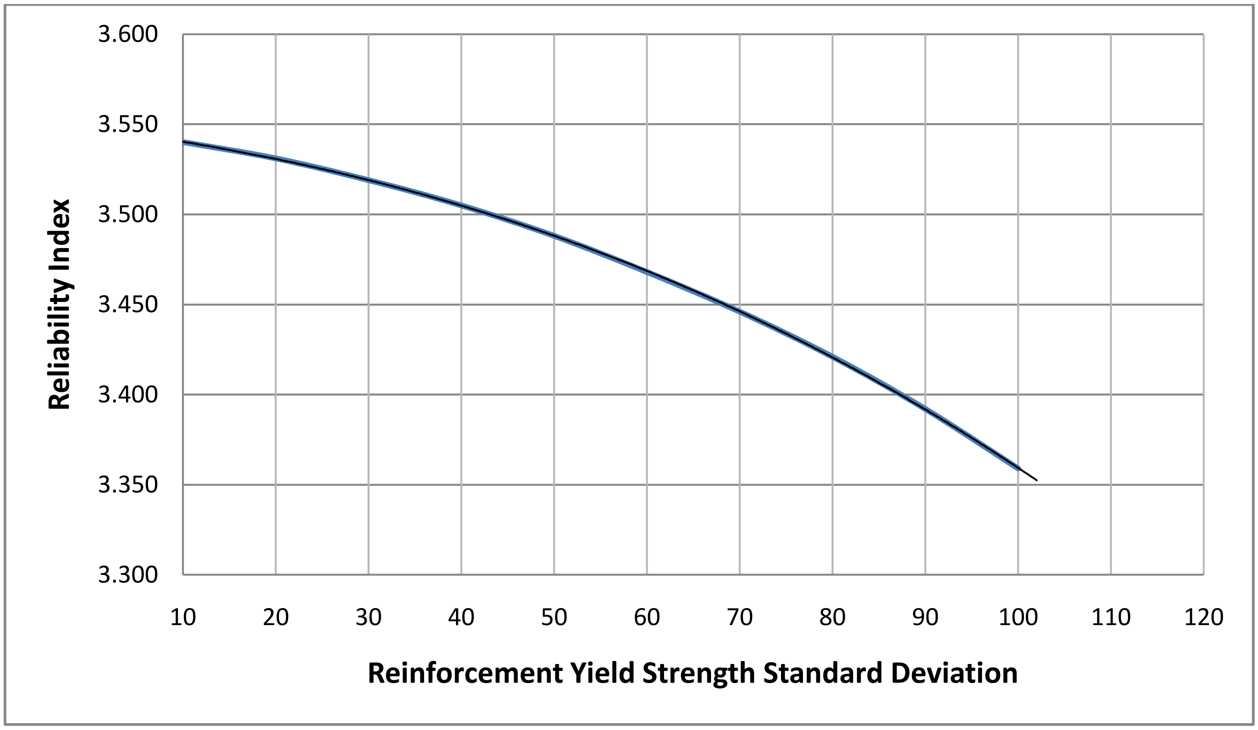
Appendix C2: Trend in Steel Diameter Standard Deviation and Reliability Index



Appendix C3: Trend in Steel Yield Strength and Reliability Index



Appendix C4: Trend in Steel Yield Strength Standard Deviation and Reliability Index



Appendix D: Kolmogorov Smirnov Goodness of Fit Test Summary

Goodness of Fit Summary for Steel Diameter

			Kolmogorov Smirnov	
Distribution	**Parameters**	**P-value**	**Statistic**	**Rank**
Lognormal	σ=0.030555 μ=2.214	0.790	0.01977	1
Weibull	α=3.7339 β=16.417	0.070	0.06965	2
Gumbel Min	σ=3.6399 μ=16.87	4.57E-16	0.12948	3

Goodness of Fit Summary for Steel Yield Strength

			Kolmogorov Smirnov	
Distribution	**Parameters**	**P-value**	**Statistic**	**Rank**
Gumbel Min	σ=62.886 μ=522.68	0.918	0.017	1
Lognormal	σ=0.17342 μ=6.1724	3.49E-17	0.134	2
Weibull	α=7.124 β=519.44	2.87E-18	0.138	3

Goodness of Fit Summary for Steel Ultimate Strength

			Kolmogorov Smirnov	
Distribution	**Parameters**	**P-value**	**Statistic**	**Rank**
Gumbel Min	σ=49.682 μ=650.24	0.709	0.02	1
Normal	σ=63.72 μ=621.56	1.00E-6	0.082	2
Lognormal	σ=0.10449 μ=6.4268	4.50E-10	0.102	3